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# Degree Sequence of Isomorphic Fuzzy Graphs

Research Article

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**Abstract:** In this paper degree sequence of isomorphic fuzzy graphs are considered and some of its properties are studied and also gave a sufficient condition for a fuzzy graph and its $\mu-$complement have an identical degree sequence.

**MSC:** 05C07, 05C38.

**Keywords:** Degree of a vertex, degree sequence of a fuzzy graph, degree sequence of isomorphic fuzzy graphs.



## 1. Introduction

The phenomena of uncertainty in real life situation was described in a mathematical framework by Zadeh in 1965. He also introduced the concept of fuzzy relations which has a widespread application in pattern recognition.K.R. Bhutani also introduced the concepts of weak, co-weak isomorphism and isomorphism between fuzzy graphs in [12]. M.S. Sunitha and A.Vijayakumar discussed the complement of a fuzzy graph in [13] and in [10] the $\mu-$complement was discussed by A.Nagoorgani and J. Malarvizhi. In [9] K.Radha and A.Rosemine introduced degree sequence of fuzzy graph. In this paper, we discussed about the degree sequence of isomorphic, co-weak and weak isomorphic fuzzy graphs.

## 2. Preliminaries

A summary of basic definitions is given , which are represented in [1–13].

A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where $\sigma$ is a fuzzy subset of a non empty set V and $\mu$ is a symmetric fuzzy relation on $\sigma$ (ie) $\mu(xy)\leq\sigma(x)\wedge\sigma(y) \quad \forall\, x,y\in V$. The underlying crisp graph of $G:(\sigma,\mu)$ is denote by $G^*:(V,E)$ where $E\subseteq V\times V$. In a fuzzy graph $G:(\sigma,\mu)$ degree of vertex $u\in V$ is $d(u)=\sum_{u\neq v}\mu(uv)$, the minimum degree of G is $\delta(G)=\wedge\{d_G(u)/u\in V\}$, the maximum degree of G is $\Delta(G)=\vee\{d_G(u)/u\in V\}$.

The degree of a vertex u of a graph G is the number of edges of G which are incident with v (ie) $d(v) = |\{e\in E;\ \ e=uv \text{ for some } u\in V\}|$. A sequence of real numbers $(d_1,d_2,d_3,\ldots,d_n)$ with $d_1\geq d_2\geq\ldots d_n$, where $d_i$ is equal to $d(v_i)$, is the degree sequence of a fuzzy graph G.

A sequence $\xi=(d_1,d_2,d_3,\ldots,d_n)$ of real numbers is said to be fuzzy graphic sequence if there exists a graph G whose vertices have degree $d_i$ and G is called realization of $\xi$. A degree sequence of real numbers in which no two of its elements are equal is called perfect degree sequence.

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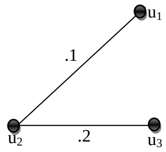


**Figure 1.** $G:(\sigma,\mu)$, **Degree sequence of G is (.3, .2, .1)**

In crisp graph theory there is no perfect degree sequence. But fuzzy graphs may have perfect degree sequence. A degree sequence of real numbers in which exactly two of its elements are same is called quasi- perfect. A homomorphism of fuzzy graphs $h : G \to G'$ is a map $h : V \to V'$ such that $\sigma(x) \leq \sigma'(h(x))\ \ \forall\ \ x \in V, \mu(xy) \leq \mu'(h(x)h(y))\ \ \forall\, ; x, y \in V$.
A weak isomorphism of fuzzy graphs $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies $\sigma(x) = \sigma'(h(x)\ \ \forall x \in V, \mu(xy) \leq \mu'(h(x)h(y))\ \ \ \forall x, y \in V$. A co-weak isomorphism of fuzzy graphs $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies $\sigma(x) \leq \sigma'(h(x))\ \ \forall\, x \in V, \mu(xy) = \mu'(h(x)h(y))\ \ \forall\, x, y \in V$.
An isomorphism $h : G \to G'$ is a map $h : V \to V'$ which is a bijective that satisfies $\sigma(x) = \sigma'(h(x))\ \ \forall\ x \in V, \mu(xy) = \mu'(h(x)h(y))\ \ \forall x, y \in V$.

# 3. Degree Sequence of Isomorphic Fuzzy Graphs

**Theorem 3.1.** *If G and $G'$ are isomorphic fuzzy graphs then the degree sequence of G and $G'$ are same.*

*Proof.* Since $G : (\sigma, \mu)$ and $G' : (\sigma', \mu')$ are two isomorphic fuzzy graphs there exists a bijective map $h : V \to V'$ such that $\sigma(x) = \sigma'(h(x))\ \ \forall x \in V, \mu(xy) = \mu'(h(x)h(y))\ \ \forall x, y \in V$. Let u be a vertex of G such that $h(u) = v$. We have to prove that $d_G(u) = d_{G'}(v)$.
**Case i :** $d_G(u) = 0$. Then no vertex of G is adjacent to u. Let y be a vertex of $G'$ different from v such that $h(w) = y$ for some $w \in V$. Since u and w are not adjacent in G, y and v are also not adjacent in $G'$. Since $y \in V'$ arbitrarily chosen, v is not adjacent with any of the vertex of $G'$. Therefore $d_{G'}(v) = 0$. Hence $d_G(u) = 0 = d_{G'}(v)$.
**Case ii :** $d_G(u) > 0$. Let $\{w_1, w_2, w_3, \ldots, w_n\}$ be the set of vertices which are adjacent to u and let $h(w_i) = z_i;\ 1 \leq i \leq n$ where $z_i \in V'$. Since G and $G'$ are isomorphic to each other, the vertices v and $z_i$ are adjacent in $G'$ such that $\mu'(vz_i) = \mu(uw_i)$ and if z is a vertex of $G'$ such that $\mu'(vz) = 0$ then there is a vertex w in G such that $f(w) = z$ and $\mu(uw) = \mu'(vz) = 0$. Hence

$$\begin{aligned} d_{G'}(v) &= \sum_{i=1}^{n} \mu'(vz_i) \\ &= \sum_{i=1}^{n} \mu(uw_i) \\ &= d_G(u). \end{aligned}$$

Since $u \in V$ is arbitrarily chosen, $d_G(u) = d_{G'}(h(u))\ \ \forall\, u \in V$. Thus the degree sequences of G and $G'$ are same. □

**Remark 3.2.** *Two fuzzy graphs with same degree sequence need not be isomorphic. In the following figure 2 the degree sequence of both G and $G'$ is (0.6, 0.5, 0.4, 0.4, 0.4, 0.3, 0.2, 0.2). Let all the vertices be of membership value 1. Suppose $G \cong G'$. Then by the Theorem 3.1, under any isomorphism $v_2$ must correspond to $w_1$ and $v_4$ must correspond to $w_4$. Since $v_1$ must adjacent to $v_2$ and $v_4$, it must be mapped to a vertex which is adjacent to both $w_1$ and $w_4$ in $G'$. But there is no such vertex in $G'$. Hence G is not isomorphic to $G'$.*

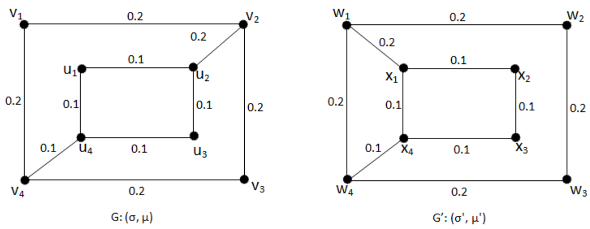


**Figure 2.**

**Theorem 3.3.** *Co-weak isomorphic fuzzy graphs preserve degree sequence.*

*Proof.* Since G is co-weak isomorphic to $G'$, there is a bijective map $h : V \to V'$ such that $\sigma(x) \leq \sigma'(h(x) \;\forall\; x \in V, \mu(xy) = \mu'(h(x)h(y)) \;\; \forall\; x, y \in V$. Then as in the proof of Theorem 3.1, $d_G(u) = d_{G'}(h(u)) \;\; \forall\; u \in V$. Thus G and $G'$ have identical degree sequence. □

**Remark 3.4.** *Two fuzzy graphs with same degree sequence need not be co-weak isomorphic. In figure 3 the degree sequence of both G and $G'$ is (1.1, 0.9, 0.8, 0.1). Suppose G is co-weak isomorphic to $G'$. Then by Theorem 3.3 under any coweak isomorphism $v_4$ must correspond to $u_4$; $v_3$ must correspond to $u_3$. But $\sigma(v_3) = 0.7 > 0.6 = \sigma'(h(u_3)) = \sigma'(v_3)$ and $\sigma(v_4) = 0.1 < 0.5 = \sigma'(h(u_4)) = \sigma'(v_4)$. Hence G is not co-weak isomorphic with $G'$.*

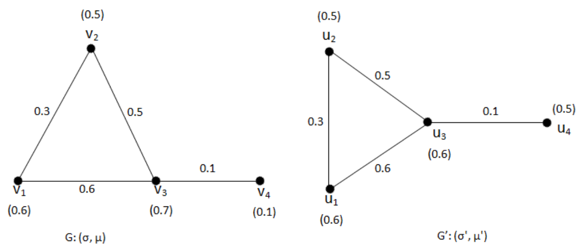


**Figure 3.**

**Remark 3.5.** *Weak isomorphic fuzzy graphs need not preserve the degree sequence. For example consider the following figure 4 The bijective map $h : V_1 \to V_2$ defined by $h(v_1) = u_1$, $h(v_2) = u_5$, $h(v_3) = u_4$, $h(v_4) = u_3$, $h(v_5) = u_2$ satisfies $\sigma_1(v_i) = \sigma_2(h(v_i)) \;\; \forall\; v_i \in V$, $\mu_1(uv) \leq \mu_2(h(u), h(v)) \;\; \forall\; u, v \in V$. Hence $G_1$ is weak isomorphic with $G_2$. But the degree sequence of $G_1$ is (0.8, 0.7, 0.7, 0.5, 0.5) and the degree sequence of $G_2$ is (1.1, 0.9, 0.8, 0.7, 0.5) which are not identical.*

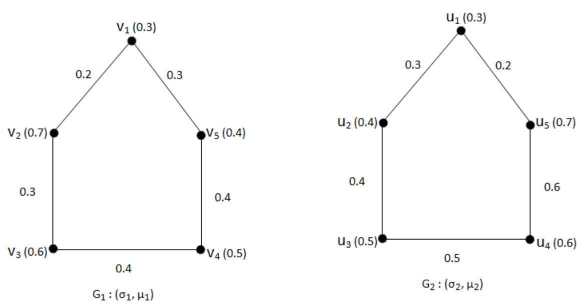


**Figure 4.**

**Theorem 3.6.** *If G is a self complementary fuzzy graph, then the degree sequences of G and $\bar{G}$ are identical.*

*Proof.* Since G is a self complementary fuzzy graph, $G \cong \bar{G}$. Hence the theorem follows by Theorem 3.1. □

**Remark 3.7.** *The converse of the above theorem need not be true. In figure 5 the degree sequence of both G and $\bar{G}$ is (1, 1, 1, 1). But $\mu(v_1v_2) = 0.5 \neq 0 = \bar{\mu}(v_1v_2)$. Also $\mu(v_3v_4) \neq \bar{\mu}(v_3v_4)$. Hence G is not self complementary fuzzy graph.*

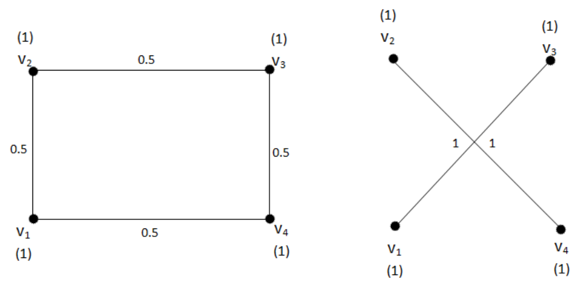


**Figure 5.** $G : (\sigma, \mu)$, $\bar{G} : (\bar{\sigma}, \bar{\mu})$

**Theorem 3.8.** *Let $G : (\sigma, \mu)$ be a fuzzy graph such that $\mu(xy) = \frac{1}{2}(\sigma(x) \wedge \sigma(y))$ $\forall$ $xy \in E$. Then G and $G^\mu$ have same degree sequence.*

*Proof.* Here $\mu(xy) = \frac{1}{2}(\sigma(x) \wedge \sigma(y))$ $\forall$ $(xy) \in E$. By the definition of $\mu-$complement of G, we have

$$\begin{aligned} \mu^\mu(xy) &= (\sigma(x) \wedge \sigma(y)) - \frac{1}{2}(\sigma(x) \wedge \sigma(y)) && \forall\ xy \in E \\ &= \frac{1}{2}((\sigma(x) \wedge \sigma(y)) && \forall\ xy \in E \\ \mu^\mu(xy) &= \mu(xy) && \forall\ (xy) \in E \end{aligned}$$

$\sum_{y \neq x} \mu^\mu(xy) = \sum_{y \neq x} \mu(xy)$. Hence $d_{G^\mu}(x) = d_G(x)$ $\forall$ $x \in V$. □

**Remark 3.9.** *The converse of the Theorem 3.8 need not be true. For example in the following figure 6 the degree sequence of both G and $G^\mu$ is (0.85, 0.65, 0.5, 0.5). But $\mu(uv) \neq \frac{1}{2}(\sigma(u) \wedge \sigma(v))$ $\forall$ $uv \in E$.*

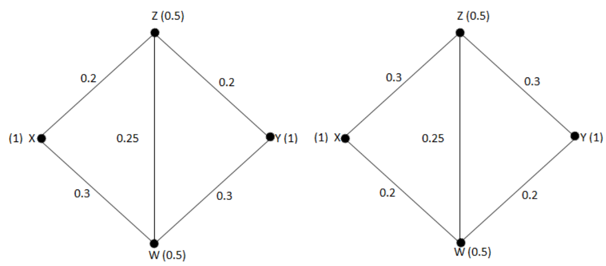


**Figure 6.** $G : (\sigma, \mu)$, $G^\mu : (\sigma, \mu^\mu)$

**Corollary 3.10.** *If $G : (\sigma, \mu)$ is a fuzzy graph such that $\mu(xy) = \frac{1}{2}(\sigma(x) \wedge \sigma(y))$ $\forall$ $x, y \in S$ then G and $G^\mu$ have same degree sequence.*

**Theorem 3.11.** *Let $G : (\sigma, \mu)$ be a fuzzy graph such that such that $\sigma(v) = c$ $\forall$ $v \in V$. Then $\sum_{i=1}^{n} d_i = 2mc$ if and only if G is an effective fuzzy graph, where m is the number of edges in G.*

*Proof.* Let $G : (\sigma, \mu)$ be a fuzzy graph such that such that $\sigma(v) = c$ $\forall$ $v \in V$. Assume that $\sum_{i=1}^{n} d_i = 2mc$. Suppose that G is not an effective fuzzy graph. Then there is an edge uv such that $\mu(uv) < \sigma(u) \wedge \sigma(v) = c$.

Therefore $d_G(v) = \sum_{uv \in E} \mu(uv) < \sum_{uv \in E} c = c.d_{G^*}(v)$. $\sum_{v \in V} d_G(v) < \sum_{v \in V} c.d_{G^*}(v) = 2mc$, which is a contradiction. Hence G is effective.

Conversely assume that $G : (\sigma, \mu)$ is an effective fuzzy graph. Then $\mu(uv) = \sigma(u) \wedge \sigma(v) = c, \ \forall \ uv \in E$. Therefore $d_G(v) = c.d_{G^*}(v), \forall \ v \in V$. $\sum_{v \in V} d_G(v) = c. \sum_{u \in V} d_{G^*}(v) = 2mc$. □

**Theorem 3.12.** *If $G : (\sigma, \mu)$ be a fuzzy graph such that $\mu$ is constant function with constant value c, then $\sum d_i = 2mc$.*

*Proof.* Since $\mu(uv) = c, \ \ uv \in E$,

$$\begin{aligned} d_G(v) &= \sum_{uv \in E} \mu(uv) \\ &= \sum_{uv \in E} c \\ &= c.d_{G^*}(v) \\ \sum_{u \in V} d_G(v) &= \sum_{u \in V} c.d_{G^*}(v) \\ &= 2mc \end{aligned}$$

□

**Remark 3.13.** *Converse of the above theorem need not be true. For example consider the figure 7. The degree sequence is (1.1, 1, 0.9). Then $\sum_{i=1}^{n} d_i = 1.1 + 1 + 0.9 = 3 = 2 \times 3 \times (1/2) = 2mc$, where $c = 1/2 = 0.5$. But $\mu$ is not a constant function.*

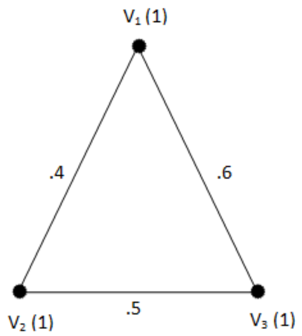


**Figure 7.** $G : (\sigma, \mu)$

**Theorem 3.14.** *If $G : (\sigma, \mu)$ be a fuzzy graph such that $r = \wedge\{\mu(e); e \in E\}$, $s = \vee\{\mu(e); e \in E\}$, then $2mr \leq \sum d_i \leq 2ms$.*

*Proof.* Here we have $r \leq \mu(uv) \leq s \ \ \forall \, uv \in E$. Therefore

$$\begin{aligned} \sum_{uv \in E} r \leq \sum_{uv \in E} \mu(uv) &\leq \sum_{uv \in E} s \\ d_{G^*}(u).r \leq d_G(u) &\leq d_{G^*}(u).s \\ \Rightarrow \sum_{u \in V} d_{G^*}(u).r \leq \sum_{u \in V} d_G(u) &\leq \sum_{u \in V} d_{G^*}(u).s \end{aligned}$$

Hence $2mr \leq \sum_{u \in V} d_G(u) \leq 2ms$. □

**Corollary 3.15.** *If G is a complete fuzzy graph on n vertices, then $n(n-1)r \leq \sum_{i=1}^{n} d_i \leq n(n-1)s$ where $r = \wedge\{\mu(e); e \in E\}$, $s = \vee\{\mu(e); e \in E\}$.*

*Proof.* By Theorem 3.14, we have $2mr \leq \sum d_i \leq 2ms$. Since G is complete, $m = n(n-1)/2$. Hence

$$n(n-1)r \leq \sum_{i=1}^{n} d_i \leq n(n-1)s$$

. □

# 4. Conclusion

In fuzzy graph theory degree of a vertex is a parameter. In this paper we made a study about that parameter in isomorphic fuzzy graphs and in the $\mu-$complement of a fuzzy graph.

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