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A Deterministic Inventory Model for Perishable Items with Time Dependent Demand and Shortages

Research Article

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Abstract: In the present paper an attempt has been made to develop a deterministic inventory model for perishable items with time dependent demand. The problem of perishability plays an important role in the field of inventory control and managements. Shortages are allowed and partially backlogged. The backlogging rate of unsatisfied demand is assumed as a function of waiting time. The purpose of our study is to maximize the total profit during a given period of time. A numerical example is given to demonstrate the developed model and sensitivity analysis of the parameters is also given.

Keywords: Inventory, Perishability, Shortages and Partial Backlogging.

1. Introduction

Academicians as well as industrialists have great interest in the development of inventory control and their uses. There are many goods that are either deteriorates or become obsolete with the passage of time such perishable products have different modeling.The items that are stored for future use always loose part of their value with passage of time then this phenomenon is known as deterioration. Obsolescence of a product occurs due to the arrival of a new and better product in the market. In the existing literature several inventory models were developed by several researchers under the assumption that demand rate is either constant or an increasing or decreasing function of time and stock dependent. Extensive research work has been in this direction. But in the real life there are many situations in which these assumptions are not valid such as seasonal products, bakery products, electronic items and medicines. T.M. Whitin [15] introduced the concept of EOQ in inventory modeling. Dave and Patel [5] developed an inventory model for deteriorating items with time dependent linearly increasing demand. Hollier and Mac [10] proposed an inventory model for deteriorating items with exponentially decaying demand. Haringa and Benkherouf [9] generalized the Hollier and Mac [10] model by considering both exponentially growing and decaying markets. Haiping and Wang [8] proposed an economic ordering policy for an inventory model of deteriorating items with time dependent demand. H. Xu [17] considered an optimal ordering policy for an inventory model of perishable products with time dependent demand. Goswami and Chaudhuri [6] developed an EOQ model for deteriorating items with time dependent demand by allowing shortages. Benkherouf and Mahmoud [2] presented an inventory model for deteriorating items with time varying linearly increasing demand and shortages. Wee [16] developed an economic production lot size

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inventory model for deteriorating items with partial backlogging. Silver [13] proposed an inventory model for deteriorating items with linear trend in demand. Abad [1] proposed an inventory model for deteriorating items with shortages. Chang and Dye [3] presented an inventory model for decaying items with time varying demand and partial backlogging. Papachristos and Skouri [11] considered an optimal replenishment policy for decaying items with time dependent demand and exponential type backlogging rate. Tang et al. [14] determine an optimal replenishment policy for an inventory model of deteriorating items with time dependent linearly increasing demand rate and shortages. Dave and Patel [5] proposed a $[T_1, S_i]$ policy for an inventory model of deteriorating items with time proportional demand rate. Sachan [12] developed an inventory model for decaying items $[T_1, S_i]$ policy and time dependent demand. Yan et al. [18] presented an EOQ model for decaying items by considering the optimal production stopping and restarting time. Goyal and Giri [7] presented a survey on recent trends in inventory model of deteriorating items. Padmanabhan and Vrat [19] consider an inventory model for perishable items under stock dependent selling rate. Yan [20] proposed an EOQ model for perishable items with freshness dependent demand and partial backlogging. Bag and Chakraborty [21] developed an inventory model for deteriorating items with fuzzy random planning horizon. Nagare and Dutta [22] presented a continuous review inventory model for perishable products with inventory dependent demand. The figure 1 is correspond to developed model. The figures 2 show the variation of total profit with respect to the deterioration parameter θ.

In the present paper we developed a deterministic inventory model for perishable items with time dependent demand. Shortages are allowed and partially backlogged. The purpose of our study is to maximize the total profit during a given period of time.

1.1. Assumptions Notations

we consider the following assumptions and notations

(1). The demand rate is deterministic i.e $R(t) = \begin{cases} \alpha e^{-\beta t}, & 0 \le t \le T_1 \\ \alpha, & T_1 \le t \le T \end{cases}$, where α and β are constants.

(2). θ is the deterioration rate.

(3). δ is the backlogging parameter.

(4). A is the ordering cost per order.

(5). h_C is the holding cost per unit per unit time.

(6). S_C is the shortage cost per unit per unit time.

(7). p_C is the purchase cost per unit.

(8). Π_R is the selling price per unit.

(9). Π_L is the opportunity cost due to lost sales per unit.

(10). T is the replenishment cycle length.

(11). $I(t)$ is the inventory level at any time t in $[0, T]$.

(12). T_1 is the time at which shortage starts.

(13). $TP(T_1, T)$ is the total profit per unit time.

(14). The replenishment rate is infinite.

(15). The lead time is zero.

(16). There is no repair or replacement of the deteriorated items.

2. Mathematical Formulation

Suppose an inventory system consists the maximum inventory level Q in the beginning of each cycle. During the interval $[0, T_1]$ the inventory level decreases due to both demand and deterioration and it becomes zero at $t = T_1$. During the shortage interval $[T_1, T]$ the unsatisfied demand is backlogged at a rate of $\delta(T-t)$. The instantaneous inventory level at any time t in $[0, T]$ is given by the following differential equations

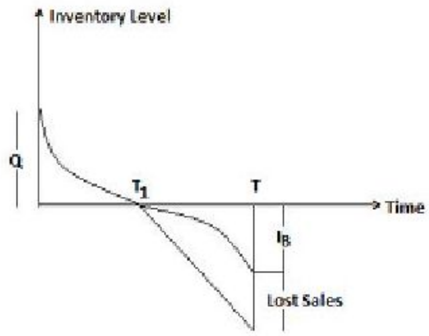

Figure 1. **corresponding to developed model**

$$\frac{dI}{dt} + \theta I = -\alpha e^{-\beta t},\ 0 \leq t \leq T_1 \tag{1}$$

With boundary condition $I(T_1) = 0$

$$\frac{dI}{dt} = -\alpha\delta(T-t),\ \ T_1 \leq t \leq T \tag{2}$$

With boundary condition $I(T_1) = 0$. For a first order approximation $e^{-\theta\, t}$. The solution of equation (1) is

$$I = \frac{\alpha}{(\theta-\beta)}[\theta T_1 + \beta(t-T_1)], \qquad 0 \leq t \leq T_1 \tag{3}$$

The solution of equation (2) is

$$I = 2a\delta[2TT_1 - 2Tt + t^2 - T_1^2], \qquad T_1 \leq t \leq T \tag{4}$$

The initial inventory level Q is obtained by putting $t = 0$ in the equation (3)

$$Q = \frac{\alpha(\theta-\beta)T_1}{(\theta-\beta)}$$

$$Q = \alpha T_1 \tag{5}$$

The ordering cost per cycle is

$$O_C = A \tag{6}$$

The holding cost per cycle is

$$H_C = h_C \int_0^{T_1} I(t)dt$$

$$H_C = \frac{\alpha h_C (2\theta - \beta)}{2(\theta - \beta)} T_1^2 \tag{7}$$

The shortage cost per cycle is

$$S_C = -s_C \int_{T_1}^{T} I(t)dt$$

$$S_C = -\frac{4a\delta s_C}{3}[3T^2 T_1 - 3TT_1^2 - T^3 + T_1^3] \tag{8}$$

The maximum backorder quantity is obtained by putting $t = T$ in equation (4)

$$I_B = 2a\delta[T^2 + T_1^2 - 2TT_1] \tag{9}$$

There the maximum order quantity is

$$Q^* = Q + I_B$$

$$Q^* = \alpha T_1 + 2a\delta[T^2 + T_1^2 - 2TT_1] \tag{10}$$

There the purchase cost per cycle

$$P_C = p_C[\alpha T_1 + 2a\delta(T^2 + T_1^2 - 2TT_1)] \tag{11}$$

Due to lost sales the opportunity cost in $[T_1, T]$ is

$$OP_C = \alpha\pi_L \int_{T_1}^{T} [1 - \delta(T - t)]dt$$

$$OP_C = 2\alpha\pi_L[2T - 2T_1 + 2\delta TT_1 - \delta T^2 - \delta T_1^2] \tag{12}$$

The sales revenue per cycle is

$$S_R = \pi_R[\int_0^{T_1} \alpha e^{-\beta t} dt + \int_{T_1}^{T} \alpha\delta(T - t)dt]$$

$$S_R = \frac{\alpha\pi_R}{2}[2T_1 + \delta(T^2 - 2TT_1 + T_1^2] \tag{13}$$

Therefore the total profit per unit time in $[0, T]$ is

$$\begin{aligned}
TP &= [\frac{1}{T}\{S_R - (O_C + H_C + S_C + P_C + OP_C)\}] \\
TP &= \frac{1}{T}[-A + \{\alpha(\pi_R + 4\pi_L - p_C)\} T_1 - 4\alpha\pi_L T + \{\frac{\alpha\delta}{2}(\pi_R + 4\pi_L) - 2a\delta p_C\} T^2 \\
&\quad + \{\frac{\alpha\delta}{2}(\pi_R + 4\pi_L) - \frac{\alpha(2\theta - \beta) h_C}{2(\theta - \beta)} - 2\alpha\delta p_C\} T_1^2 + \{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\} TT_1 \\
&\quad - \frac{4\alpha\delta s_C}{3} T^3 + \frac{4\alpha\delta s_C}{3} T_1^3 + 4\alpha\delta s_C T^2 T_1 - 4\alpha\delta s_C TT_1^2]
\end{aligned} \tag{14}$$

$$\begin{aligned}
\frac{\partial TP}{\partial T_1} &= \frac{1}{T}[\alpha(\pi_R + 4\pi_L - p_C) + \{\alpha\delta(\pi_R + 4\pi_L) - \frac{\alpha(2\theta - \beta)h_C}{(\theta - \beta)} - 4\alpha\delta p_C\} T_1 + \{4\alpha\delta p_C \\
&\quad - 4\alpha\delta\pi_L - \alpha\pi_R\}T + 4\alpha\delta s_C T_1^2 + 4\alpha\delta s_C T^2 - 8\alpha\delta s_C TT_1]
\end{aligned} \tag{15}$$

$$\frac{\partial^2 TP}{\partial T_1^2} = \frac{1}{T}[\{\alpha\delta(\pi_R + 4\pi_L) - \frac{\alpha(2\theta - \beta) h_C}{(\theta - \beta)} - 4\alpha\delta p_C\} + 8\alpha\delta s_C T_1 - 8\alpha\delta s_C T] \tag{16}$$

$$
\begin{aligned}
\frac{\partial TP}{\partial T} = & \frac{1}{T}[-4\alpha\pi_L + \{\alpha\delta(\pi_R+4\pi_L) - 4\alpha\delta p_C\}T + \{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\}T_1 \\
& - 4\alpha\delta s_C T^2 + 8\alpha\delta s_C TT_1 - 4\alpha\delta s_C T_1^2] - \frac{1}{T^2}[-A + \{\alpha(\pi_R+4\pi_L - p_C)\}T_1 \\
& - 4\alpha\pi_L T + \{\frac{\alpha\delta(\pi_R+4\pi_L)}{2} - 2\alpha\delta p_C\}T^2 + \{\frac{\alpha\delta(\pi_R+4\pi_L)}{2} - 2\alpha\delta p_C \\
& - \frac{\alpha(2\theta-\beta)h_C}{2(\theta-\beta)} - 2\alpha\delta p_C\}T_1^2 + \{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\}TT_1 - \frac{4\alpha\delta s_C}{3}T^3 \\
& + \frac{4\alpha\delta s_C}{3}T_1^2 + 4\alpha\delta s_C T^2 T_1 - 4\alpha\delta s_C TT_1^2]
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\frac{\partial^2 TP}{\partial T^2} = & \frac{1}{T}[\{\alpha\delta(\pi_R+4\pi_L) - 4\alpha\delta p_C\} - 8\alpha\delta s_C T + 8\alpha\delta s_C T_1] - \frac{1}{T^2}[-4\alpha\pi_L + \{\alpha\delta(\pi_R \\
& + 4\pi_L) - 4\alpha\delta p_C\}T + \{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\}T_1 - 4\alpha\delta s_C T^2 + 8\alpha\delta s_C TT_1 \\
& - 4\alpha\delta s_C T_1^2 - 4\alpha\pi_L + \{\alpha\delta(\pi_R+4\pi_L) - 4\alpha\delta s_C\}T + \{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\}T_1 \\
& - 4\alpha\delta s_C T^2 + 8\alpha\delta s_C TT_1 - 4\alpha\delta s_C T_1^2] + \frac{2}{T^3}[-A + \{\alpha(\pi_R+4\pi_L - p_C)\}T_1 \\
& - 4\alpha\pi_L T + \{\frac{\alpha\delta}{2}(\pi_R+4\pi_L) - 2\alpha\delta p_C\}T^2 + \{\frac{\alpha\delta}{2}(\pi_R+4\pi_L) - 2\alpha\delta p_C \\
& - \frac{\alpha(2\theta-\beta)h_C}{2(\theta-\beta)}\}T_1^2 + \{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\}TT_1 - \frac{4\alpha\delta s_C}{3}T^3 + \frac{4\alpha\delta s_C}{3}T_1^3 \\
& + 4\alpha\delta s_C T^2 T_1 - 4\alpha\delta s_C TT_1^2]
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\frac{\partial^2 TP}{\partial T_1\,\partial T} = & \frac{1}{T}[\{4\alpha\delta p_C - 4\alpha\delta\pi_L - \alpha\pi_R\} + 8\alpha\delta s_C T - 8\alpha\delta s_C T_1] - \frac{1}{T^2}[\{\alpha(\pi_R+4\pi_L \\
& - p_C)\} + \{\alpha\delta(\pi_R+4\pi_L) - \frac{\alpha(2\theta-\beta)h_C}{(\theta-\beta)} - 4\alpha\delta p_C\}T_1 + \{4\alpha\delta p_C - 4\alpha\delta\pi_L \\
& - \alpha\pi_R\}T + 4\alpha\delta s_C T_1^2 + 4\alpha\delta s_C T^2 - 8\alpha\delta s_C TT_1]
\end{aligned} \tag{19}
$$

3. Numerical Example

Let us consider the following parameters in the appropriate units

$$A = 100,\ \alpha = 50,\ \beta = 0.2,\ \delta = 0.3,\ \theta = 0.001,\ h_C = 0.6,\ s_C = 0.8,\ \pi_L = 10,\ \pi_R = 25,\ p_C = 5$$

When $\theta = 0.001$ then total profit is $TP = -4772.85902$. When $\theta = 0.005$ then total profit is $TP = -4770.91534$.

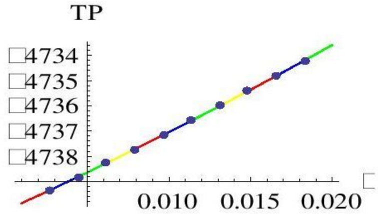

Figure 2. variation of total profit with respect to θ

4. Conclusion

In this paper we present an inventory model for perishable items with time dependent demand. Shortages are allowed and partially backlogged. The backlogging rate of unsatisfied demand is assumed as a function of waiting time. The results of the proposed model show that the total profit is deeply impacted by the deterioration parameter θ because the periodic products such as vegetables, milk, bakery products and news papers become necessarily to sold in the market as the cycle length decreases.

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