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Generalization of Pairwise Weakly Continuous Functions

Research Article

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Abstract: As a generalization of δ-b-continuous functions, we introduce the notion of weakly δ-b-continuous functions in bitopological spaces and obtain several characterizations and some properties of weakly δ-b-continuous functions.

MSC: 54A40.

Keywords: Bitopological spaces, (i, j)-δ-b-open sets, weakly (i, j)-δ-b-continuous function.

1. Introduction

The concept of bitopological spaces was first introduced by Kelly [6]. After the introduction of the definition of a bitopological space by Kelly, a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. In this paper, we introduce and study the concept of weakly δ-b-continuous functions in bitopological spaces. Throughout this paper, the triple (X, τ_1, τ_2) where X is a set and τ_1 and τ_2 are topologies on X, will always denote a bitopological space. For a subset A of a bitopological space (X, τ_1, τ_2), the closure of A and the interior of A with respect to τ_i are denoted by $i\,\mathrm{Cl}(A)$ and $i\,\mathrm{Int}(A)$, respectively, for $i = 1, 2$.

2. Preliminaries

Definition 2.1 ([9]). *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be pairwise continuous (resp. pairwise open) if the induced functions $f : (X, \tau_i) \to (Y, \sigma_i)$ are continuous (resp. open) for $i = 1, 2$.*

Definition 2.2. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be*

(1) (i, j)-regular open [3] if $A = i\,\mathrm{Int}(j\,\mathrm{Cl}(A))$, where $i \neq j$, $i, j = 1, 2$,

(2) (i, j)-δ-b-open [1] if $A \subset j\,\mathrm{Cl}(i\,\mathrm{Int}_\delta(A)) \cup i\,\mathrm{Int}(j\,\mathrm{Cl}_\delta(A))$, where $i \neq j$, $i, j = 1, 2$.

The complement of an (i, j)-regular open (resp. (i, j)-δ-b-open) set is called an (i, j)-regular closed (resp. (i, j)-δ-b-closed).

Definition 2.3 ([1]). *The intersection (resp. union) of all (i, j)-δ-b-closed (resp. (i, j)-δ-b-open) sets of X containing (resp. contained in) $A \subset X$ is called the (i, j)-δ-b-closure (resp. (i, j)-δ-b-interior) of A and is denoted by (i, j)-$b\,\mathrm{Cl}_\delta(A)$ (resp. (i, j)-$b\,\mathrm{Int}_\delta(A)$).*

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Lemma 2.4 ([1]). *Let (X, τ_1, τ_2) be a bitopological space and A a subset of X. Then*

(1) (i,j)-$b\operatorname{Int}_\delta(A)$ *is* (i,j)-δ-b-*open;*

(2) (i,j)-$b\operatorname{Cl}_\delta(A)$ *is* (i,j)-δ-b-*closed;*

(3) A *is* (i,j)-δ-b-*open if and only if* $A = (i,j)$-$b\operatorname{Int}_\delta(A)$;

(4) A *is* (i,j)-δ-b-*closed if and only if* $A = (i,j)$-$b\operatorname{Cl}_\delta(A)$;

(5) (i,j)-$b\operatorname{Int}_\delta(X\backslash A) = X\backslash(i,j)$-$b\operatorname{Cl}_\delta(A)$;

(6) (i,j)-$b\operatorname{Cl}_\delta(X\backslash A) = X\backslash(i,j)$-$b\operatorname{Int}_\delta(A)$.

Lemma 2.5 ([1]). *Let (X, τ_1, τ_2) be a bitopological space and $A \subset X$. A point $x \in (i,j)$-$b\operatorname{Cl}_\delta(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$B\delta O(X, x)$.*

Definition 2.6 ([5]). *A subset A of X is said to be (i,j)-θ-closed if $A = (i,j)$-$\operatorname{Cl}_\theta(A)$. A subset A of X is said to be (i,j)-θ-open if $X\backslash A$ is (i,j)-θ-closed. The (i,j)-θ-interior of A, denoted by (i,j)-$\operatorname{Int}_\theta(A)$, is defined as the union of all (i,j)-θ-open sets contained in A. Hence $x \in (i,j)$-$\operatorname{Int}_\theta(A)$ if and only if there exists a τ_i-open set U containing x such that $x \in U \subset j\operatorname{Cl}(U) \subset A$.*

Lemma 2.7 ([5]). *Let (X, τ_1, τ_2) be a bitopological space and A a subset of X. Then*

(1) (i,j)-$\operatorname{Int}_\theta(X\backslash A) = X\backslash(i,j)$-$\operatorname{Cl}_\theta(A)$;

(2) (i,j)-$\operatorname{Cl}_\theta(X\backslash A) = X\backslash(i,j)$-$\operatorname{Int}_\theta(A)$.

Lemma 2.8 ([5]). *Let (X, τ_1, τ_2) be a bitopological space. If U is a τ_j-open set of X, then (i,j)-$\operatorname{Cl}_\theta(U) = i\operatorname{Cl}(U)$.*

Definition 2.9 ([1]). *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-δ-b-continuous if for each $x \in X$ and each σ_i-open set V of Y containing $f(x)$, there exists an (i,j)-δ-b-open set U containing x such that $f(U) \subset V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be pairwise b-continuous if f is $(1,2)$-b-continuous and $(2,1)$-b-continuous.*

3. Weakly (i,j)-δ-b-contiuous Functions

In this section, we define weakly (i,j)-b-continuous function in bitopological space and study some of their properties on them.

Definition 3.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be weakly (i,j)-δ-b-continuous if for each $x \in X$ and each σ_i-open set V of Y containing $f(x)$, there exists an (i,j)-δ-b-open set U containing x such that $f(U) \subset j\operatorname{Cl}(V)$.*
A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be pairwise weakly δ-b-continuous if f is weakly $(1,2)$-δ-b-continuous and weakly $(2,1)$-δ-b-continuous.

Proposition 3.2. *Every (i,j)-δ-b-continuous function is weakly (i,j)-δ-b-continuous.*

Proof. Straightforward. □

Theorem 3.3. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent;*

(1) f *is weakly* (i,j)-δ-b-*continuous;*

(2) $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(i\,\mathrm{Cl}(B)))) \subset f^{-1}(i\,\mathrm{Cl}(B))$ *for every subset* B *of* Y*;*

(3) $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(F))) \subset f^{-1}(F)$ *for every* (i,j)*-regular closed set* F *of* Y*;*

(4) $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(V)) \subset f^{-1}(i\,\mathrm{Cl}(V))$ *for every* σ_j*-open set* V *of* Y*;*

(5) $f^{-1}(V) \subset (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$ *for every* σ_i*-open set* V *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Suppose that $x \in X \backslash f^{-1}(i\,\mathrm{Cl}(B))$. Then $f(x) \in Y \backslash i\,\mathrm{Cl}(B)$ and ther exists a σ_i-open set V of Y containing $f(x)$ such that $V \cap B = \emptyset$. Therefore, $V \cap j\,\mathrm{Int}(i\,\mathrm{Cl}(B)) = \emptyset$ and hence $j\,\mathrm{Cl}(V) \cap j\,\mathrm{Int}(i\,\mathrm{Cl}(B)) = \emptyset$. Therefore, there exists an (i,j)-δ-b-open set U containing x such that $f(U) \subset j\,\mathrm{Cl}(V)$. Hence, we have $U \cap f^{-1}(j\,\mathrm{Int}(i\,\mathrm{Cl}(B))) = \emptyset$ and $x \in X \backslash (i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(i\,\mathrm{Cl}(B))))$. Thus, we obtain (i,j)-$b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(i\,\mathrm{Cl}(B)))) \subset f^{-1}(i\,\mathrm{Cl}(B))$.

(2) $\Rightarrow$ (3): Let F be an (i,j)-regular closed set of Y. Then $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(F))) = (i,j)$-$b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(i\,\mathrm{Cl}(j\,\mathrm{Int}(F))))) \subset f^{-1}(i\,\mathrm{Cl}(j\,\mathrm{Int}(F))) = f^{-1}(F)$.

(3) $\Rightarrow$ (4): Let V be a σ_j-open set of Y. Then $i\,\mathrm{Cl}(V)$ is (i,j)-regular closed. Then we obtain $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(V)) \subset (i,j)$-$b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(i\,\mathrm{Cl}(V)))) \subset f^{-1}(i\,\mathrm{Cl}(V))$.

(4) $\Rightarrow$ (5): Let V be a σ_i-open set of Y. Then $Y \backslash i\,\mathrm{Cl}(V)$ is σ_j-open and we have $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(Y \backslash j\,\mathrm{Cl}(V))) \subset f^{-1}(i\,\mathrm{Cl}(Y \backslash j\,\mathrm{Cl}(V)))$ and hence $X \backslash (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V))) \subset X \backslash f^{-1}(i\,\mathrm{Int}(j\,\mathrm{Cl}(V))) \subset X \backslash f^{-1}(V)$. Therefore, we obtain $f^{-1}(V) \subset (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$.

(5) $\Rightarrow$ (1): Let $x \in X$ and V be a σ_i-open set containing $f(x)$. We have $x \in f^{-1}(V) \subset (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$. Put $U = (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$. Then U is an (i,j)-δ-b-open set containing X and $f(U) \subset j\,\mathrm{Cl}(V)$. This shows that f is weakly (i,j)-δ-b-continuous. □

Theorem 3.4. *For a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) f *is weakly* (i,j)*-*δ*-*b*-continuous;*

(2) $f((i,j)\text{-}b\,\mathrm{Cl}_\delta(A)) \subset (i,j)\text{-}\mathrm{Cl}_\theta(f(A))$ *for every subset* A *of* X*;*

(3) $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(B)) \subset (f^{-1}(i,j)\text{-}\mathrm{Cl}_\theta(B))$ *for every subset* B *of* Y*;*

(4) $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}(i,j)\text{-}\mathrm{Cl}_\theta(B)))) \subset f^{-1}((i,j)\text{-}\mathrm{Cl}_\theta(B))$ *for every subset* B *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Assume that f is weakly (i,j)-δ-b-continuous. Let A Be any slubset of X, $x \in (i,j)\text{-}b\,\mathrm{Cl}_\delta(A)$ and V be a σ_i-open set of Y containing $f(x)$. Then, ther exists an (i,j)-δ-b-open set U containing x such that $f(U) \subset j\,\mathrm{Cl}(V)$. Since $x \in (i,j)\text{-}b\,\mathrm{Cl}_\delta(A)$, we obtain $U \cap A \neq \emptyset$ and hence $\emptyset \neq f(U) \cap f(A) \subset j\,\mathrm{Cl}(V) \cap f(A)$. Therefore, we obtain $f(x) \in (i,j)$-$\mathrm{Cl}_\theta(f(A))$.

(2) $\Rightarrow$ (3): Let B be any subset of Y.Then we have $f((i,j)\text{-}b\,\mathrm{Cl}_\delta\, f^{-1}(B))) \subset (i,j)\text{-}\mathrm{Cl}_\theta(f(f^{-1}(B))) \subset (i,j)\text{-}\mathrm{Cl}_\theta(B)$ and hence $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(B)) \subset f^{-1}((i,j)\text{-}\mathrm{Cl}_\theta(B))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. Since $(i,j)\text{-}\mathrm{Cl}_\theta(B)$ is σ_i-closed in Y, by Lemma 2.8 $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}((i,j)$-$\mathrm{Cl}_\theta(B)))) \subset f^{-1}((i,j)\text{-}\mathrm{Cl}_\theta(j\,\mathrm{Int}((i,j)\text{-}\mathrm{Cl}_\theta(B)))) = f^{-1}(i\,\mathrm{Cl}(j\,\mathrm{Int}((i,j)\text{-}\mathrm{Cl}_\theta(B)))) \subset f^{-1}(i\,\mathrm{Cl}(i,j)\text{-}\mathrm{Cl}_\theta(B))) = f^{-1}((i,j)$-$\mathrm{Cl}_\theta(B))$.

(4) $\Rightarrow$ (1): Let V be any σ_j-open set of Y. Then by Lemma 2.8 $v \subset j\,\mathrm{Int}(i\,\mathrm{Cl}(V)) = j\,\mathrm{Int}((i,j)\text{-}\mathrm{Cl}_\theta(V))$ and we have $(i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(V)) \subset (i,j)\text{-}b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Int}((i,j)\text{-}\mathrm{Cl}_\theta(V)))) \subset f^{-1}((i,j)\text{-}\mathrm{Cl}_\theta(V)) = f^{-1}(i\,\mathrm{Cl}(V))$. Thus we obtain (i,j)-$b\,\mathrm{Cl}_\delta(f^{-1}(V)) \subset f^{-1}(i\,\mathrm{Cl}(V))$. It follows from Theorem 3.3 that f is weakly (i,j)-δ-b-ocntinuous. □

Theorem 3.5. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is weakly (i, j)-δ-b-continuous;

(2) (i, j)-$b\,\mathrm{Cl}_\delta(f^{-1}(V)) \subset f^{-1}(i\,\mathrm{Cl}(V))$ for every (j, i)-preopen set V of Y;

(3) $f^{-1}(V) \subset (i, j)$-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$ for every (i, j)-preopen set V of Y.

Proof. (1) $\Rightarrow$ (2): Let V be any (j, i)-preopen set of Y. Suppose that $x \notin f^{-1}(i\,\mathrm{Cl}(V))$. Then there exists a σ_i-open set W containing $f(x)$ such that $W \cap V = \emptyset$. Hence we have $i\,\mathrm{Cl}(W \cap V) = \emptyset$. Since V is (j, i)-preopen, we have $V \cap j\,\mathrm{Cl}(W) \subset j\,\mathrm{Int}(i\,\mathrm{Cl}(V)) \cap j\,\mathrm{Cl}(W) \subset j\,\mathrm{Cl}(j \int(i\,\mathrm{Cl}(V)) \cap W) \subset j\,\mathrm{Cl}(i\,\mathrm{Cl}(V)) \cap W) \subset j\,\mathrm{Cl}(i\,\mathrm{Cl}(V \cap W)) = \emptyset$. Since f is weakly (i, j)-δ-b-continuous and W is a σ_i-open set containing $f(x)$, there exists $U \in (i, j)$-$B\delta O(X, x)$ such that $f(U) \subset j\,\mathrm{Cl}(W)$. Then $f(U) \cap V = \emptyset$ and hence $U \cap f^{-1}(V) = \emptyset$. This shows that $x \notin (i, j)$-$b\,\mathrm{Cl}_\delta(f^{-1}(V))$. Therefore, we obtain (i, j)-$b\,\mathrm{Cl}_\delta(f^{-1}(V)) \subset f^{-1}(i\,\mathrm{Cl}(V))$.
(2) $\Rightarrow$ (3): Let V be any (i, j)-preopen set of Y. By (2), we have $f^{-1}(V) \subset f^{-1}(i\,\mathrm{Int}(j\,\mathrm{Cl}(V))) = X \backslash f^{-1}(i\,\mathrm{Cl}(Y \backslash j\,\mathrm{Cl}(V))) \subset X \backslash (i, j)$-$b\,\mathrm{Cl}_\delta(f^{-1}(Y \backslash j\,\mathrm{Cl}(V))) = (i, j)$-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$.
(3) $\Rightarrow$ (1): Let V be any σ_i-open set of Y. Then V is (i, j)-preopen set in Y and $f^{-1}(V) \subset (i, j)$-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$. By Theorem 3.3, f is weakly (i, j)-δ-b-continuous. □

Lemma 3.6. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly (i, j)-δ-b-continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is pairwise continuous, then the composition $g \circ f : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is weakly (i, j)-δ-b-continuous.*

Proof. Let $x \in X$ and W be an η_i-open set of Z containing $g(f(x))$. Then $g^{-1}(W)$ is a σ_i-open set of Y containing $f(x)$ and there exists $U \in (i, j)$-$B\delta O(X, x)$ suhc that $f(U) \subset j\,\mathrm{Cl}(g^{-1}(W))$. Since g is pairwise continuous, we obtain $(g \circ f)(U) \subset g(j\,\mathrm{Cl}(g^{-1}(W))) \subset g(g^{-1}(j\,\mathrm{Cl}(W))) \subset j\,\mathrm{Cl}(W)$. □

Definition 3.7. *A bitopological space (X, τ_1, τ_2) is said to be (i, j)-regular [6] if for each $x \in X$ and each τ_i-open set U containing x, there eists a τ_i-open set V such that $x \in V \subset j\,\mathrm{Cl}(V) \subset U$.*

Lemma 3.8 ([8]). *If a bitopological space (X, τ_1, τ_2) is (i, j)-regular, then (i, j)-$\mathrm{Cl}_\theta(F) = F$ for every τ_i-closed set F.*

Theorem 3.9. *Let (Y, σ_1, σ_2) be an (i, j)-regular bitopological space. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) f is (i, j)-δ-b-continuous;

(2) $f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))$ is (i, j)-δ-b-closed in X for every lsubset B of Y;

(3) f is weakly (i, j)-δ-b-continuous;

(4) $f^{-1}(F)$ is (i, j)-δ-b-closed in X for every (i, j)-θ-closed set F of Y;

(5) $f^{-1}(V)$ is (i, j)-δ-b-open in X for every (i, j)-θ-closed set V of Y.

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Since (i, j)-$\mathrm{Cl}_\theta(B)$ is σ_i-closed in Y, $f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))$ is (i, j)-δ-b-closed in X.
(2) $\Rightarrow$ (3): Let B be any subset of Y. Then we have (i, j)-$b\,\mathrm{Cl}_\delta(f^{-1}(B)) \subset (i, j)$-$b\,\mathrm{Cl}_\delta(f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))) = f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))$. By Theorem 3.4, f is weakly (i, j)-δ-b-continuous.
(3) $\Rightarrow$ (4):Let F be any (i, j)-θ-closed set of Y. Then by Theorem 3.4, (i, j)-$b\,\mathrm{Cl}_\delta(f^{-1}(F)) \subset f^{-1}((i, j)$-$\mathrm{Cl}_\theta(F)) = f^{-1}(F)$.

Therefore, by Lemma 2.4, $f^{-1}(F)$ is (i,j)-δ-b-closed in X.

(4) $\Rightarrow$ (5): Let V be any (i,j)-θ-open set of Y. By (4) $f^{-1}(Y-V) = X\backslash f^{-1}(V)$ is (i,j)-δ-b-closed lin X and hence $f^{-1}(V)$ is (i,j)-δ-b-open in X.

(5) $\Rightarrow$ (1): Since Y is (i,j)-regular, by Lemma 3.8 (i,j)-$\mathrm{Cl}_\theta(B) = B$ for every σ_i-closed set B of Y and hence every σ_i-open set is (i,j)-θ-open. Therefore, $f^{-1}(V)$ is (i,j)-δ-b-open for every σ_i-open set V of Y. Hence f is (i,j)-δ-b-continuous. □

Definition 3.10. *A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be weakly (i,j)-* quasicontinuous [8] if for every σ_i-open set V of Y, $f^{-1}(j\,\mathrm{Cl}(V)\backslash V)$ is biclosed in X.*

Theorem 3.11. *If a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is weakly (i,j)-δ-b-continuous and weakly (i,j)-* quasicontinuous, then f is (i,j)-δ-b-continuous.*

Proof. Let $x \in X$ and V be any σ_i-open set of Y containing $f(x)$. Since f is weakly (i,j)-δ-b-continuous, there exists an (i,j)-δ-b-open set U of X containing x such that $f(U) \subset j\,\mathrm{Cl}(V)$. Hence $x \notin f^{-1}(j\,\mathrm{Cl}(V)\backslash V)$. Therefore, $x \in U\backslash f^{-1}(j\,\mathrm{Cl}(V)\backslash V) = U \cap (X\backslash f^{-1}(j\,\mathrm{Cl}(V)\backslash V)$. Since U is (i,j)-δ-b-open and $X\backslash f^{-1}(j\,\mathrm{Cl}(V)\backslash V)$ is biopen, $G = U \cap (X\backslash f^{-1}(j\,\mathrm{Cl}(V)\backslash V)$ is (i,j)-δ-b-open [1]. Then $x \in G$ and $f(G) \subset V$. For, if $y \in G$, then $f(y) \notin j\,\mathrm{Cl}(V)\backslash V$ and hence $f(y) \in V$. Therefore, f is (i,j)-δ-b-continuous. □

Definition 3.12. *A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to have (i,j)-b-Interiority condition if (i,j)-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V) \subset f^{-1}(V)$ for every σ_i-open set V of Y .*

Theorem 3.13. *If a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is weakly (i,j)-b-continuous and satisfies the (i,j)-δ-b-interiority condition, then f is (i,j)-δ-b-continuous.*

Proof. Let V be any σ_i-open set of Y. Since f is weakly (i,j)-δ-b-continuous, by Theorem 3.3, $f^{-1}(V) \subset (i,j)$-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$. By the (i,j)-δ-b-interiority condition of f, we have (i,j)-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V))) \subset f^{-1}(V)$ and hence $f^{-1}(V = (i,j)$-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$. By Lemma 2.4, $f^{-1}(V)$ is (i,j)-δ-b-open in X and thus f is (i,j)-δ-b-continuous. □

Definition 3.14. *Let (X,τ_1,τ_2) be a bitopological space and A be a subset of X. The (i,j)-δ-b-frontier of A is defined as follows: (i,j)-$bFr(A) = (i,j)$-$b\,\mathrm{Cl}(A) \cup (i,j)$-$b\,\mathrm{Cl}_\delta(X\backslash A) = (i,j)$-$b\,\mathrm{Cl}_\delta(A)\backslash(i,j)$-$b\,\mathrm{Int}_\delta(A)$.*

Theorem 3.15. *The set of all points x of X at which a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is not weakly (i,j)-δ-b-continuous is identical with the union of the (i,j)-δ-b-frontiers of the inverse images of the σ_i-closure of σ_i-open sets of Y contining $f(x)$.*

Proof. Let x be a point of X at which $f(x)$ is not weakly (i,j)-δ-b-continuous. Then, there exists a σ_i-open set V of Y containing $f(x)$ such that $U \cap (X\backslash f^{-1}(j\,\mathrm{Cl}(V))) \neq \emptyset$ for every (i,j)-δ-b-open set U of X containing x. By Lemma 2.5, $x \in (i,j)$-$b\,\mathrm{Cl}_\delta(X\backslash f^{-1}(j\,\mathrm{Cl}(V)))$. Since $x \in f^{-1}(j\,\mathrm{Cl}(V)))$, we have $x \in (i,j)$-$b\,\mathrm{Cl}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$ and hence $x \in (i,j)$-$bFr(f^{-1}(j\,\mathrm{Cl}(V)))$. Conversely, if f is weakly (i,j)-δ-b-continuous at x, then for each σ_i-open set V of Y containing $f(x)$, there exists an (i,j)-δ-b-open set U containing x suchthat $f(U) \subset j\,\mathrm{Cl}(V)$ and hence $x \in U \subset f^{-1}(j\,\mathrm{Cl}(V))$. Therefore, we obtain that $x \in (i,j)$-$b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V)))$. This contradicts that $x \in (i,j)$-$bFr(f^{-1}(j\,\mathrm{Cl}(V)))$. □

Definition 3.16. *A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be almost (i,j)-δ-b-continuous [2] if for each $x \in X$ and each σ_i-open set V containing $f(x)$, there exists an (i,j)-δ-b-open set U of X containing x such that $f(U) \subset i\,\mathrm{Int}(j\,\mathrm{Cl}(V))$.*

Lemma 3.17. *A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is almost (i,j)-δ-b-continuous if and only if $f^{-1}(V)$ is (i,j)-δ-b-open for each (i,j)-regular open set V of Y .*

Definition 3.18. *A bitopological space* (X, τ_1, τ_2) *is said to be* (i, j)*-almost regular [10] if for each* $x \in X$ *and each* (i, j)*-regular open set* U *containing* x*, there exists an* (i, j)*-regular open set* V *of* X *such that* $x \in V \subset j\,\mathrm{Cl}(V) \subset U$*.*

Theorem 3.19. *Let a bitopological space* $((Y, \sigma_1, \sigma_2)$ *be* (i, j)*-almost regular. Then a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is* (i, j)*-almost b-continuous if and only if it is weakly* (i, j)*-*δ*-b-continuous.*

Proof. Necessity. This is obvious. Sufficiency. Suppose that f is weakly (i, j)-δ-b-continuous. Let V be any (i, j)-regular open set of Y and $x \in f^{-1}(V)$. Then we have $f(x) \in V$. By the almost (i, j)-regularity of Y, there exists an (i, j)-regular open set V_0 of Y such that $f(x) \in V_0 \subset j\,\mathrm{Cl}(V_0) \subset V$. Since f is weakly (i, j)-δ-b-continuous, there exists an (i, j)-δ-b-open set U of X containing x such that $f(U) \subset j\,\mathrm{Cl}(V_0) \subset V$. This implies that $x \in U \subset f^{-1}(V)$. Therefore, we have $f^{-1}(V) \subset (i, j)\text{-}p\,\mathrm{Int}(f^{-1}(V))$ and hence $f^{-1}(V) = (i, j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(V))$. By Lemma 2.4, $f^{-1}(V)$ is (i, j)-δ-b-open and by Lemma 3.17 f is (i, j)-almost b-continuous. □

Definition 3.20. *A bitopological space* (X, τ_1, τ_2) *is said to be pairwise Hausdorff or pairwise* T_2 *[6] if for each pair of distinct points* x *and* y *of* X*, there exist a* τ_i*-open set* U *containing* x *and a* τ_j *-open set* V *containing* y *such that* $U \cap V = \emptyset$ *for* $i \neq j$*,* $i, j = 1, 2$*.*

Theorem 3.21. *Let* (X, τ_1, τ_2) *be a bitopological space. If for each pair of distinct points* x *and* y *in* X*, there exists a function* f *of* (X, τ_1, τ_2) *into a pairwise* T_2 *bitopological space* (Y, σ_1, σ_2) *such that*

(1) $f(x) \neq f(y)$*,*

(2) f *is weakly* (i, j)*-*δ*-b-continuous at* x*,*

(3) f *is almost* (j, i)*-b-continuous at* y

then for each pair of distinct points x *and* y *of* X*, there exist a* (i, j)*-*δ*-b-open set* U *containing* x *and a* (j, i)*-b-open set* V *containing* y *such that* $U \cap V = \emptyset$ *for* $i \neq j$*,* $i, j = 1, 2$*.*

Proof. Let x and y be a pair of distinct points of X. Since Y is pairwise T_2, there exists a σ_i-open set U containing $f(x)$ and a σ_j -open set V containing $f(y)$ such that $U \cap V = \emptyset$. Since U and V are disjoint, we have $j\,\mathrm{Cl}(U) \cap j\,\mathrm{Int}(i\,\mathrm{Cl}(V)) = \emptyset$. Since f is weakly (i, j)-δ-b-continuous at x, there exists an (i, j)-δ-b-open set U_x of X containing x such that $f(U_x) \subset j\,\mathrm{Cl}(U)$. Since f is (j, i)-almost b-continuous at y, there exists a (j, i)-b-open set U_y of X containing y such that $f(U_y) \subset j\,\mathrm{Int}(i\,\mathrm{Cl}(V))$. Hence we have $U_x \cap U_y = \emptyset$. □

Definition 3.22. *A bitopological space* (X, τ_1, τ_2) *is said to be pairwise Urysohn [4] if for each distinct points* x*,* y *of* X *there exist a* τ_i*-open set* U *and a* τ_j*-open set* V *such that* $x \in U$*,* $y \in V$ *and* $j\,\mathrm{Cl}(U) \cap i\,\mathrm{Cl}(V) = \emptyset$ $i \neq j$*,* $i, j = 1, 2$*.*

Theorem 3.23. *If* (Y, σ_1, σ_2) *is a pairwise Urysohn and* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a pairwise weakly b-continuous injection, then for each pair of distinct points* x *and* y *of* X*, there exist a* (i, j)*-*δ*-b-open set* U *containing* x *and a* (j, i)*-b-open set* V *containing* y *such that* $U \cap V = \emptyset$ *for* $i \neq j$*,* $i, j = 1, 2$*.*

Proof. Let x and y be any distinct points of X. Then $f(x) \neq f(y)$. Since Y is pairwise Urysohn, there exist a σ_i-open set U and a σ_j-open set V such that $f(x) \in U$, $f(y) \in V$ and $j\,\mathrm{Cl}(U) \cap i\,\mathrm{Cl}(V) = \emptyset$. Hence $f^{-1}(j\,\mathrm{Cl}(U)) \cap f^{-1}(i\,\mathrm{Cl}(V)) = \emptyset$. Therefore, $(i, j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(U))) \subset (j, i)\text{-}b\,\mathrm{Int}(f^{-1}(i\,\mathrm{Cl}(V))) = \emptyset$. Since f is pairwise weakly b-continuous, by Theorem 3.1 $x \in f^{-1}(U) \subset (i, j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(U)))$ and $y \in f^{-1}(V) \subset (j, i)\text{-}b\,\mathrm{Int}(f^{-1}(i\,\mathrm{Cl}(V)))$. □

Definition 3.24. *A bitopological space* (X, τ_1, τ_2) *is said to be pairwise connected [7] (resp. pairwise* δ*-*b*-connected) if it cannot be expressed as the union of two nonempty disjoint sets* U *and* V *such that* U *is* τ_i*-open and* V *is* τ_j*-open (resp.* U *is* (i,j)*-*δ*-*b*-open and* V *is* (j,i)*-*δ*-*b*-open).*

Theorem 3.25. *If a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is a pairwise weakly* b*-continuous surjection and* (X, τ_1, τ_2) *is pairwise* δ*-*b*-connected, then* (Y, σ_1, σ_2) *is pairwise connected.*

Proof. Suppose that (Y, σ_1, σ_2) is not pairwise connected. Then, there exists a σ_i-open set U and a σ_j-open set V such that $U \neq \emptyset$, $V \neq \emptyset$, $U \cap V = \emptyset$ and $U \cup V = Y$. Since f is surjective, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty. Moreover $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ and $f^{-1}(U) \cup f^{-1}(V) = X$. Since f is pairwise weakly δ-b-continuous, by Theorem 3.3 we have $f^{-1}(U) \subset (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(U)))$ and $f^{-1}(V) \subset (j,i)\text{-}b\,\mathrm{Int}(f^{-1}(i\,\mathrm{Cl}(V)))$. Since U and V are σ_j-closed and σ_i-clsoed, respectively, we have $f^{-1}(U) \subset (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(U))$ and $f^{-1}(V) \subset (j,i)\text{-}b\,\mathrm{Int}(f^{-1}(V))$. Hence $f^{-1}(U) = (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(U))$ and $f^{-1}(V) = (j,i)\text{-}b\,\mathrm{Int}(f^{-1}(V))$. By Lemma 2.4 $f^{-1}(U)$ is (i,j)-δ-b-open and $f^{-1}(V)$ is (j,i)-δ-b-open in (X, τ_1, τ_2). This shows that (X, τ_1, τ_2) is not pairwise δ-b-connected. □

Definition 3.26. *A subset* K *of a bitopological space* (X, τ_1, τ_2) *is said to be* (i,j)*-quasi* H*-closed relative to* X *[3] if for each cover* $\{U_\alpha : \alpha \in \Omega\}$ *of* K *by* τ_i*-open sets of* X*, there exists a finite subset* Ω_0 *of* Ω *such that* $K \subset \cup\{j\,\mathrm{Cl}(U_\alpha) : \alpha \in \Omega_0\}$*.*

Definition 3.27. *A subset* K *of a bitopological space* (X, τ_1, τ_2) *is said to be* (i,j)*-*δ*-*b*-compact relative to* X *if every cover of* K *by* (i,j)*-*δ*-*b*-open sets of* X *has a finite subcover.*

Theorem 3.28. *If* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is weakly* (i,j)*-*δ*-*b*-continuous and* K *is* (i,j)*-*δ*-*b*-compact relative to* X*, then* $f(K)$ *is* (i,j)*-quasi* H*-closed relative to* Y *.*

Proof. Let K be (i,j)-δ-b-compact relative to X and $\{V_\alpha : \alpha \in \Omega\}$ any cover of $f(K)$ by σ_i-open sets of (Y, σ_1, σ_2). Then $f(K) \subset \cup\{V_\alpha : \alpha \in \Omega\}$ and so $K \subset \cup\{f^{-1}(V_\alpha : \alpha \in \Omega\}$. Since f is weakly (i,j)-δ-b-continuous, by Theorem 3.3 we have $f^{-1}(V_\alpha) \subset (i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V_\alpha)))$ for each $\alpha \in \Omega$. Therefore, $K \subset \cup\{(i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V_\alpha))) : \alpha \in \Omega\}$. Since K is (i,j)-δ-b-compact relative to X and $(i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V_\alpha)))$ is (i,j)-δ-b-open for each $\alpha \in \Omega$, there exists a finite subset Ω_0 of Ω such that $K \subset \cup\{(i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V_\alpha))) : \alpha \in \Omega_0\}$. This implies that $f(K) \subset \cup\{f((i,j)\text{-}b\,\mathrm{Int}_\delta(f^{-1}(j\,\mathrm{Cl}(V_\alpha)))) : \alpha \in \Omega_0\} \subset \cup\{f(f^{-1}(j\,\mathrm{Cl}(V_\alpha))) : \alpha \in \Omega_0\} \subset \cup\{j\,\mathrm{Cl}(V_\alpha) : \alpha \in \Omega_0\}$. Hence $f(K)$ is (i,j)-quasi H-closed relative to Y. □

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