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MHD Boundary Layer Flow and Heat Transfer Over a Permeable Shrinking Sheet with Partial Slip with Connective Boundary Condition

Research Article

R.N.Jat[1], A.K.Jhankal[2] and Deepak Kumar[1*]

1 Department of Mathematics, University of Rajasthan, Jaipur, India.
2 Department of Mathematics, Army Cadet College, Indian Military Academy, Dehradun, India.

Abstract: An analysis is made for the two-dimensional laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid over a permeable shrinking sheet with partial slip. The governing boundary layer equations are transformed into ordinary differential equations using similarity transformation which are than solved numerically using shooting technique. The effects of various physical parameters, such as the magnetic parameter, slip parameter; Biot number and Prandtl number, on the flow and heat transfer characteristics are presented and discussed.

Keywords: MHD boundary layer flow, permeable shrinking sheet, partial slip, connective boundary condition.
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1. Introduction and Notations

Nomenclature

A	Biot number $\left(= -\frac{h_f}{k}\sqrt{\frac{\nu}{c}}\right)$, $[-]$
B_0	Constant applied magnetic field, $[Wbm^{-2}]$
c	Constants, $[-]$
C_p	Specific heat at constant pressure, $[JKg^{-1}K^{-1}]$
f	Dimensionless stream function, $[-]$
k	Constants, $[-]$
K	Slip parameter $(= k\sqrt{c\nu})$, $[-]$
M	Magnetic parameter $(= \frac{\sigma_e B_0^2}{\rho c})$, $[-]$
Pr	Prandtl number $(= \frac{\mu C_p}{)}$, $[-]$
S	Suction/injection parameter, $[-]$
T	Temperature of the fluid, $[K]$
u, v	Velocity component of the fluid along the x and y directions, respectively, $[ms^{-1}]$
x, y	Cartesian coordinates along the surface and normal to it, respectively, $[m]$

* E-mail: deepak120786@gmail.com

Greek symbols

ρ	Density of the fluid, $[Kgm^{-3}]$
μ	Viscosity of the fluid, $[Kgms^{-1}]$
σ_e	Electrical conductivity, $[m^2s^{-1}]$
η	Dimensionless similarity variable, $[=(\frac{c}{\nu})^{1/2}y]$
ν	Kinematic viscosity, $[m^2s^{-1}]$
κ	Thermal conductivity, $[Wm^{-2}K^{-4}]$
Ψ	Stream function, $[=(\nu c)^{1/2}xf(\eta)]$
θ	Dimensionless temperature, $[=\frac{(T-T_\infty)}{(T_w-T_\infty)}]$

Superscript

Derivative with respect to η

Subscripts

w	Properties at the plate
∞	Free stream condition

The fundamental governing equations for fluid mechanics are the Navier-Stokes equations. This inherently non-linear set of partial differential equations has no general solution, and only a small number of exact solutions have been found Wang [30]. Partial differential equations can describe many physical models in different fields of science. These linear and nonlinear models play important roles in applied science; therefore, finding their analytical solutions has fundamental significance in various fields of science and engineering Vleggaar [29]. These solutions may well describe various phenomena in nature, such as vibrations, solitons and propagation with a finite speed Nazar et al. [24] Exact solutions are important for the following reasons: (i) the solutions represent fundamental fluid-dynamic flows. Also, owing to the uniform validity of exact solutions, the basic phenomena described by the Navier-Stokes equations can be more closely studied. (ii) The exact solutions serve as standards for checking the accuracies of the many approximate methods, whether they are numerical, asymptotic, or empirical. Explicit solutions are used as models for physical or numerical experiments, and often reflect the asymptotic behaviour of more complicated solutions.

All explicit solutions for the boundary layer equations are seemingly similarity solutions in the sense that the longitudinal velocity component displays the same shape of profile across any transverse section of the layer, see Schlichting [27]. By an appropriate choice of the independent non-dimensional similarity variables, the boundary layer equations can therefore be reduced to ordinary differential equations. In the rare cases when these equations can be solved in closed form, the explicit solutions are obtained Nazar et al. [24]. The term “similarity solution” in fluid mechanics was first introduced for the solution of a problem of Prandtl’s boundary layer theory. The idea is to simplify the governing equations by reducing the number of independent variables, by a coordinate transformation. Analogous to dimensional analysis, instead of parameters, like the Reynolds number, the coordinates themselves are collapsed into dimensionless groups that scale the velocities Ishak et al. [16]. The terminology “similarity” is used because, despite the growth of the boundary layer with distance x from the leading edge, the velocity profile u/U remains geometrically similar. The same concept was then extended to the temperature profile. However, not all problems admit similarity solutions, since they depend on various factors, such as the surface geometries, boundary conditions, and the surface heating conditions.

The study of boundary layer flow over a shrinking sheet has generated much interest in recent years in view of its significant applications in industrial manufacturing such as glass-fibber and paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the Kinematics of shrinking and simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products Magyari and Keller [21]. The heat transfer in the flow due to a shrinking sheet is very important in practically.

This type of flow appears in many industrial and engineering processes and in those cases; the qualities at the final products depend to a great extent on the rate of cooling. In recent years, MHD flow problem have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is significant type of flow having considerable practical application in chemical engineering electrochemistry and polymer processing. In his pioneering work, Sakiadis [26] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [12] extended the work of Sakiadis [26] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being, the effects of heat and mass transfer and magnetic field under various physical conditions have been investigated by several authors such as Chen and Char [10], Chiam [11], Andersson [2], Ariel et al. [3], Jat and Chaudhary [18], Jhankal and Kumar [20], Hayat et al. [15], Fang et al. [13], Nadeem et al. [23], Bhattacharyya and Layek [8] etc.

The flow due to a shrinking boundary with partial slip has yet become relevance in many situations. For example, there is a slip regime where Navier-Stokes equation is valid but slip occurs in the rarefied gases as mentioned by Sharipov and Seleznev [28]. As the solid surface may be rough and porous, an equivalent slip exists. The no slip condition is replaced by Navier's partial slip condition, where the amount of relative slip is proportional to the local shear stress. Wang [31] has investigated the flow due to a stretching surface with partial slip. A few years later, Wang [32] continued the study on viscous flow due to a stretching sheet with suction and injection. Besides that, the magnetohydrodynamic (MHD) flow over a stretching sheet with partial slip was analyzed analytically by Fang et al. [14]. and recently by Aman and Ishak [15] studied the slip effects on permeable shrinking sheet, Bhattacharyya et al. [9] studied the slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet.

In recent years, investigations on the boundary layer flow problem with a convective surface boundary condition have gained much interest among researchers, since first introduced by Aziz [4], who considered the thermal boundary layer flow over a flat plate in a uniform free stream with a convective surface boundary condition. This problem was then extended by Bataller [7] by considering the Blasius and Sakiadis flows, both under a convective surface boundary condition and in the presence of thermal radiation. Ishak [17] obtained the similarity solutions for the steady laminar boundary layer flow over a permeable plate with a convective boundary condition. Very recently, Makinde and Aziz [22] investigated numerically the effect of a convective boundary condition on the two imensional boundary layer flows past a stretching sheet in a nano fluid. Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of two-dimensional laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid over a permeable shrinking sheet with partial slip with connective boundary condition.

2. Mathematical Formulation of the Problem

Let us consider laminar two-dimensional boundary layer flow over a shrinking boundary at a convective surface boundary condition where the lateral surface velocity is proportional to the distance x towards the origin i.e. $U = -cx$, where $c > 0$. The fluid is an electrically conducting incompressible viscous fluid. It is assumed that external fluid owing polarization of charges and Hall-effect are neglected. The stationary Cartesian coordinate system has its origin located at the leading edge of the sheet with the positive x-axis extending along the sheet, while y-axis is measured normal to the surface of the sheet. A transverse magnetic field of strength B_0 is assumed to be applied in the positive y-axis, normal to the sheet. The magnetic

Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy (Pai [25], Schlichting [27], Bansal [5]) under the influence of externally imposed transverse magnetic field (Jeffery [19], Bansal [6]) are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_e B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Accompanied by the boundary conditions:

$$y=0: u=U+k\nu\frac{\partial u}{\partial y}, v=V_w, T=T_w$$

$$y\to\infty: u\to 0, T\to T_\infty \tag{4}$$

Where T_w is constant surface temperature, V_w is the mass transfer velocity at the surface of the sheet with $V_w>0$ for injection (blowing), $V_w<0$ for suction and $V_w=0$ corresponds to an impermeable sheet. Further, k is a proportional constant and ν is the viscosity of the bulk fluid. The governing partial differential equations (1)-(3) can be reduced to ordinary differential equations by introducing the following transformation

$$\eta=(\frac{C}{\nu})^{1/2}y,\quad \Psi=(\nu c)^{\frac{1}{2}}xf(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

The continuity equation (1) is satisfied by introducing a stream function Ψ such that $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial x}$. The transformed nonlinear ordinary differential equations are:

$$f'''+ff''-f'^2-Mf'=0 \tag{6}$$

$$\theta''+Prf\theta'=0 \tag{7}$$

The transformed boundary conditions are:

$$f(0)=S,\quad f'(0)=-1+Kf''(0),\quad \theta'(0)=-A(1-\theta(0))\ \text{ and }\ f'(\infty)\to 0,\quad \theta(\infty)\to 0. \tag{8}$$

Where prime denotes differentiation with respect to η, $A=-\frac{h_f}{k}\sqrt{\frac{\nu}{C}}$ is the Biot number (the equivalent dimensionless convective heat transfer parameter), $K=k\sqrt{c\nu}$ is a non-dimensional parameter indicating the relative importance of partial slip. If $K=0$ there is no slip, and $K\to\infty$ the surface is stress-free, $M=\frac{\sigma_e B_0^2}{\rho c}$ is the magnetic parameter, and $Pr=\frac{\mu c_p}{\kappa}$ is the Prandtl number.

3. Numerical Solution and Discussion

The non-linear differential equations (6) and (7) subject to the boundary conditions (8) are solved by Runge-Kutta fourth order scheme with a systematic guessing of $f'(0)$ and $\theta'(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta=0.01$ is used while obtaining the numerical solution and accuracy upto the seventh decimal place i.e. 1×10^{-4}, which is very sufficient for convergence. The computations were done by a programme which uses a symbolic and computer language Matlab. It is observed from tables 1 and 2 that shear stress and Nusselt number respectively increase due to increase in slip parameter K, for the given values of Prandtl number Pr, Biot number A, magnetic parameter M and suction parameter S.

Figures 1 and 2 depict that the fluid velocity is negative and increases with the increase of magnetic parameter (M) and slip parameter (K). Figures 3 and 4 depict that the temperature of the fluid decreases with increases in slip parameter (K) and Biot number (A).

Figure 5 which illustrate the effect of Prandtl number (Pr) on the temperature profiles. We infer from this figure that the temperature decreases with an increase in Prandtl number, which implies viscous boundary layer thickness than the thermal boundary layer. From these plots it is evident that large values of Prandtl number result in thinning of the thermal boundary layer. In this case temperature asymptotically approaches to zero in free stream region.

4. Conclusion

The two-dimensional laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid over a permeable shrinking sheet with partial slip with connective boundary condition has been investigated. The governing partial differential equations are transformed into ordinary differential equations by means of similarity transformations.

The resulting non-linear ordinary differential equations are solved using Runge-Kutta fourth order method along with shooting technique. The velocity and temperature profiles are discussed numerically and presented through graphs. The numerical values of Skin-friction coefficient and Nusselt number are derived, for various values of slip parameter (K) and presented through tables. Some of the important finding are listed below:

- The effect of magnetic parameter (M) increases the fluid velocity.
- The effect of slip parameter (K) is to increase the fluid velocity and to decrease the temperature of the fluid.
- The effect of the Biot number (A) is to decreases the temperature of the fluid.
- The boundary layers are highly influenced by the Prandtl number (Pr). The effect of Pr is to decreases the thermal boundary layer thickness.

K	$\mathbf{f''}(0)$
0	2.2300
1	0.7198
2	0.4190

Table 1. **Numerical values of Skin friction coefficient, when M=1.0, A=0.5, Pr=1.0 and S=3.0**

K	$-\theta'(0)$
0	0.5680
1	0.4845
2	0.4195

Table 2. **Numerical values of Nusselt number, when M =1.0, A=0.5, Pr=1.0 and S=3.0**

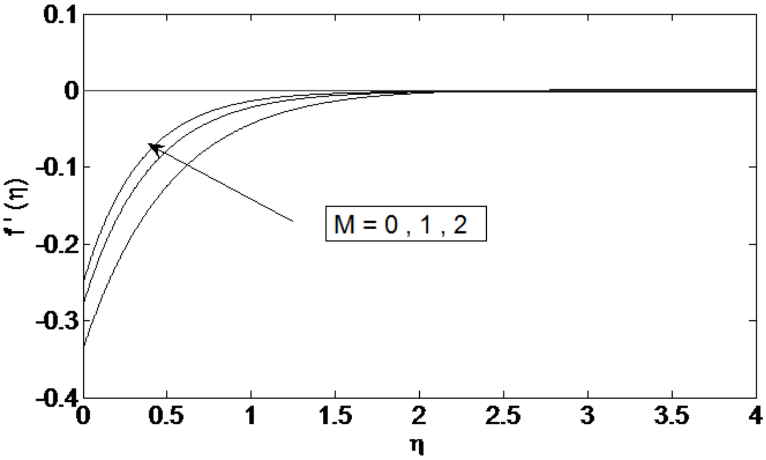

Figure 1. **Velocity profile for various values of M when K=1 and S=3.**

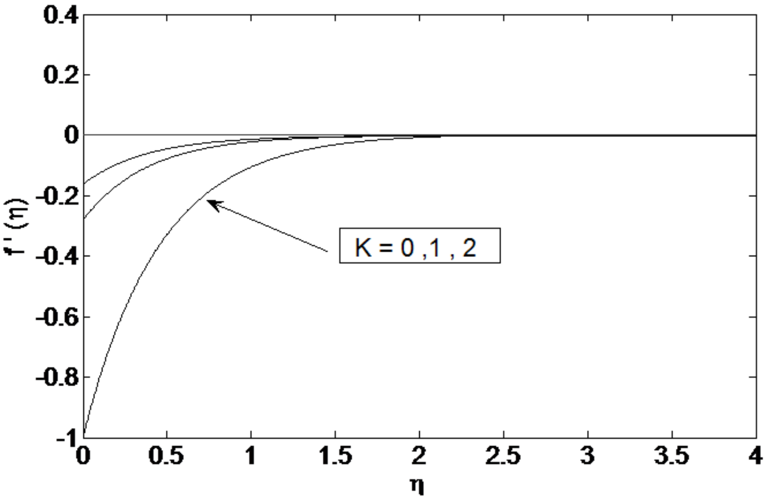

Figure 2. **Velocity profile for various values of K when M=1 and S=3.**

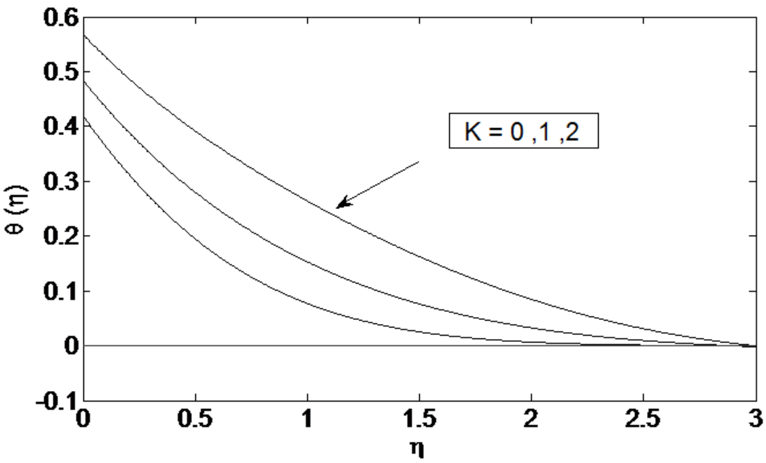

Figure 3. **Temperature profile for various values of K when M=1, Pr=1, A=1 and S=1.**

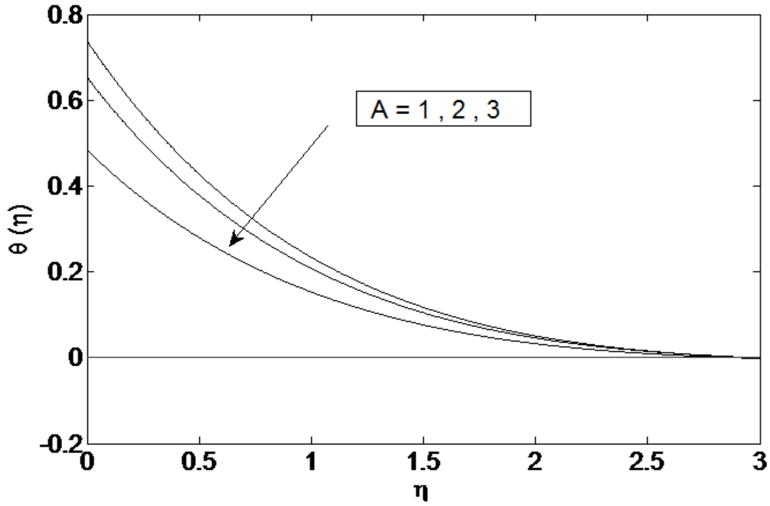

Figure 4. **Temperature profile for various values of A when M=1, Pr=1, K=1 and S=1.**

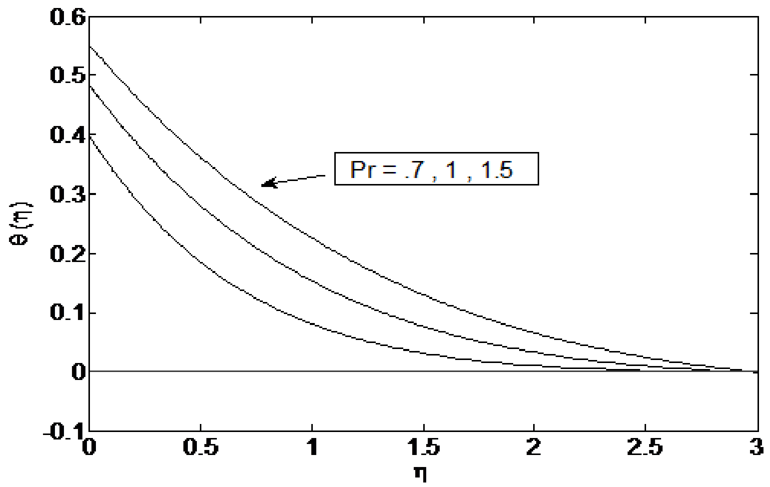

Figure 5. **Temperature profile for various values of Pr when M=1, A=1, K=1 and S=1.**

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