International Journal of Mathematics And its Applications
Volume 4, Issue 1–B (2016), 19–26.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Some Class of Sasakian Manifolds with Respect to the Quarter-Symmetric Metric Connection

Research Article

D.G.Prakasha[1]* and B.S.Hadimani[1]

1 Department of Mathematics, Karnatak University, Dharwad-3, India.

Abstract: We consider $\mathcal{M}$-Projective curvature tensor on a Sasakian manifolds with respect to the quarter-symmetric metric connection. Here, we obtain the condition for the Sasakian manifold to be quasi-$\mathcal{M}$-Projectively flat and ϕ-$\mathcal{M}$-projectively flat with respect to the quarter-symmetric metric connection.

MSC: 53C25, 53C35, 53D10.

Keywords: Sasakian manifold, $\mathcal{M}$-projective curvature tensor, quarter-symmetric metric connection, quasi-$\mathcal{M}$-projectively flat, ϕ-$\mathcal{M}$-projectively flat.

1. Introduction

In 1924, A. Friedman and J.A. Schouten ([7, 18]) introduced the notion of a semi-symmetric linear connection on a differentiable manifold. Further, H.A. Hayden [9] introduced the idea of metric connection with torsion on a Riemannian manifold. In 1970, K. Yano [20] studied some curvature and derivational conditions for semi-symmetric connections in Riemannian manifolds. In 1975, S. Golab [8] defined and studied quarter-symmetric linear connection on a differentiable manifold. A linear connection $\widetilde{\nabla}$ in an n-dimensional differentiable manifold is said to be a quarter-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X,Y], \tag{1}$$

$$= \eta(Y)\phi X - \eta(X)\phi Y, \tag{2}$$

where η is a 1-form and ϕ is a tensor of type $(1,1)$. In addition, a quarter-symmetric linear connection $\widetilde{\nabla}$ satisfies the condition $(\widetilde{\nabla}_X g)(Y,Z) = 0$ for all $X,Y,Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields of the manifold M, then $\widetilde{\nabla}$ is said to be a quarter-symmetric metric connection. Studies of various types of quarter-symmetric metric connection and their properties include ([1, 2, 6, 11, 12, 16, 17, 21]) among others.

The $\mathcal{M}$-projective curvature tensor is another important tensor from the differential geometric point of view. The curvature tensor bridges the gap between conformal curvature tensor, conharmonic curvature tensor and concircular curvature tensor on one side and H-projective curvature tensor on the other. This curvature tensor was introduced by pokhariyal and

* *E-mail: prakashadg@mail.com*

Mishra[15]. Some properties of this tensor in Sasakian, Kähler, Kenmotsu and LP-Sasakian manifolds have been studied earlier in the papers ([5, 13, 14, 22]). The $\mathcal{M}$-projective curvature tensor on an n-dimensional $(n > 1)$ Riemannian manifold M was given by

$$\mathcal{M}(X,Y)Z = R(X,Y)Z - \frac{1}{2(n-1)}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\}, \tag{3}$$

for any vector fields $X, Y, Z \in \chi(M)$, where R and S are the curvature tensor and the Ricci tensor respectively on M with respect to the Levi-Civita Connection and Q is the Ricci operator with respect to the Levi-Civita connection and is defines $g(QX,Y) = S(X,Y)$.

The present paper is organized as follows: In section 2, the brief introduction to the Sasakian manifolds were given. In the next section, we give the relation between the Levi-Civita connection and the quarter-symmetric metric connection on a Sasakian manifold. In section 4, we consider two cases of $\mathcal{M}$-projective curvature tensor and give definitions of quasi-$\mathcal{M}$-projectively flat and ϕ-$\mathcal{M}$-projectively flat Sasakian manifold with respect to the quarter-symmetric metric connection. It is proved that, if a Sasakian manifold is quasi-$\mathcal{M}$-projectively flat with repsect to the quarter-symmetric metric connection $\tilde{\tilde{\nabla}}$ then it is an Einstein manifold with respect to connection $\tilde{\tilde{\nabla}}$ and the scalar curvature with respect to the connection $\tilde{\tilde{\nabla}}$ is $2n(n-1)$. Finally, conditions for a Sasakian manifold to be quasi-$\mathcal{M}$-projectively flat and ϕ-$\mathcal{M}$-projectively flat with respect to the connection $\tilde{\tilde{\nabla}}$ are obtained.

2. Sasakian Manifolds

Let M be an almost contact metric manifold of dimension $n(=2m+1)$ equipped with an almost contact metric structure (ϕ, ξ, η, g) consisting of a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g, satisfying

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0, \tag{4}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad X, Y \in TM. \tag{5}$$

From (4) and (5) we easily get

$$g(\phi X, Y) = -g(X, \phi Y), \quad g(X,\xi) = \eta(X), \quad X, Y \in TM. \tag{6}$$

An almost contact metric manifold becomes a contact metric manifold if

$$g(\phi X, Y) = d\eta(X,Y), \quad X, Y \in TM. \tag{7}$$

A contact metric structure is said to be K-contact if ξ is a killing with respect to g. If in such a manifold the relation

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X, \tag{8}$$

holds, where ∇ denotes the Levi-Civita connection of g, then M is called a Sasakian manifold. It is known that every Sasakian manifold is K-contact but converse is not true in general. However, a 3-dimensional K-contact manifold is Sasakian [10].

In a Sasakian manifold M equipped with the structure (ϕ, ξ, η, g), the following relations hold ([3, 19]):

$$\nabla_X \xi = -\phi X, \tag{9}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{10}$$

$$R(\xi, X)Y = (\nabla_X \phi)Y, \tag{11}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{12}$$

$$S(\phi X, \phi Y) = S(X,Y) - (n-1)\eta(X)\eta(Y), \tag{13}$$

for any vector fields $X, Y \in TM$. Where R and S denote the curvature tensor and the Ricci tensor of M, respectively.

Definition 2.1. *An Sasakian manifold M is said to be*

quasi-$\mathcal{M}$-projectively flat if

$$g(M(X,Y)Z, \phi W) = 0, \qquad X, Y, Z, W \in TM,$$

and ϕ-$\mathcal{M}$-projectively flat if

$$g(M(\phi X, \phi Y)\phi Z, \phi W) = 0, \qquad X, Y, Z, W \in TM,$$

3. Quarter-symmetric Metric Connection in a Sasakian Manifold

A quarter-symmetric metric connection $\widetilde{\nabla}$ in a Sasakian manifold is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y. \tag{14}$$

Further, a relation between the curvature tensor $\widetilde{R}$ of the quarter-Symmetric metric connection $\widetilde{\nabla}$ and the curvature tensor R of the Levi-Civita connection ∇ is given by

$$\widetilde{R}(X,Y)Z = R(X,Y)Z - 2d\eta(X,Y)\phi Z + \eta(X)(\nabla_Y \phi)Z - \eta(Y)(\nabla_X \phi)Z. \tag{15}$$

In the view of (8), for a Sasakian manifold the relation between the curvature tensor of M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ and the Levi-Civita connection ∇ is given by

$$\widetilde{R}(X,Y)Z = R(X,Y)Z - 2d\eta(X,Y)\phi Z + \eta(X)g(Y,Z)\xi - \eta(Y)g(X,Z)\xi - \{\eta(X)Y - \eta(Y)X\}\eta(Z), \tag{16}$$

where $R(X,Y)Z$ is the curvature tensor of the connection ∇. From (16) we have

$$\widetilde{R}(X,Y)\xi = 2\{\eta(Y)X - \eta(X)Y\}, \tag{17}$$

$$\widetilde{R}(X,\xi)Y = 2\{\eta(Y)X - g(X,Y)\xi\}. \tag{18}$$

Contracting (16) over X, we obtain

$$\widetilde{S}(Y,Z) = S(Y,Z) - 2d\eta(\phi Z, Y) + g(Y,Z) + (n-2)\eta(Y)\eta(Z), \tag{19}$$

where $\widetilde{S}$ and S are the Ricci tensors of the connections $\widetilde{\nabla}$ and ∇, respectively. So, in a Sasakian manifold, the Ricci tensor of the quarter-symmetric metric connection is symmetric. It follows from (12) and (19) that

$$\begin{aligned} \widetilde{S}(\phi Y, \phi Z) &= \widetilde{S}(Y, Z) - 2(n-1)\eta(Y)\eta(Z), && (20) \\ \widetilde{S}(Y, \xi) &= 2(n-1)\eta(Y). && (21) \end{aligned}$$

Again, contracting (19) over Y and Z, we get

$$\widetilde{r} = r + 2(n-1), \tag{22}$$

where $\widetilde{r}$ and r are the scalar curvatures of the connection $\widetilde{\nabla}$ and ∇, respectively. In an n-dimensional Sasakian manifold M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$, if $\{e_1, ..., e_{n-1}, \xi\}$ is a local orthonormal basis of vector fields in M, then $\{\phi e_1, ..., \phi e_{n-1}, \xi\}$ is also a local orthonormal basis. Therefore, can be verified that

$$\begin{aligned} \sum_{i=1}^{n-1} g(e_i, e_i) &= \sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) = n-1, && (23) \\ \sum_{i=1}^{n-1} g(e_i, Z)\widetilde{S}(Y, e_i) &= \sum_{i=1}^{n-1} g(\phi e_i, Z)\widetilde{S}(Y, \phi e_i) \\ &= \widetilde{S}(Y, Z) - 2(n-1)\eta(Y)\eta(Z), && (24) \\ \sum_{i=1}^{n-1} \widetilde{R}(e_i, Y, Z, e_i) &= \sum_{i=1}^{n-1} \widetilde{R}(\phi e_i, Y, Z, \phi e_i) \\ &= \widetilde{S}(Y, Z) + 2d\eta(\phi Z, Y), && (25) \\ \sum_{i=1}^{n-1} \widetilde{R}(e_i, \phi Y, \phi Z, e_i) &= \sum_{i=1}^{n-1} \widetilde{R}(\phi e_i, \phi Y, \phi Z, \phi e_i) && (26) \\ &= \widetilde{S}(Y, Z) - 2g(Y, Z) - 2(n-2)\eta(Y)\eta(Z), \\ \sum_{i=1}^{n-1} \widetilde{S}(e_i, e_i) &= \sum_{i=1}^{n-1} \widetilde{S}(\phi e_i, \phi e_i) = \widetilde{r} - 2(n-1), && (27) \\ \sum_{i=1}^{n-1} \widetilde{S}(\phi e_i, \phi Z) g(\phi Y, \phi e_i) &= \widetilde{S}(\phi Y, \phi Z). && (28) \end{aligned}$$

These results will be useful in the next section.

3.1. Some Structure Theorems on Sasakian Manifolds with Respect to the Quarter-symmetric Metric Connection

The $\mathcal{M}$-projective curvature tensor $\widetilde{K}$ of an n-dimensional almost contact metric manifold with respect to quarter-symmetric metric connection $\widetilde{\nabla}$ is given by

$$\widetilde{\mathcal{M}}(X, Y)Z = \widetilde{R}(X, Y)Z - \frac{1}{2(n-1)}\{\widetilde{S}(Y, Z)X - \widetilde{S}(X, Z)Y + g(Y, Z)\widetilde{Q}X - g(X, Z)\widetilde{Q}Y\}. \tag{29}$$

$X, Y, Z \in TM$. Analogous to the Definition 2.1, we give the following Definition with respect to the connection $\widetilde{\nabla}$.

Definition 3.1. *A Sasakian manifold M is said to be quasi-$\mathcal{M}$-projectively flat with respect to the quarter-symmetric metric connection if*

$$g(\widetilde{\mathcal{M}}(X, Y)Z, \phi W) = 0, \quad X, Y, Z, W \in TM, \tag{30}$$

and ϕ-$\mathcal{M}$-projectively flat with respect to the quarter-symmetric metric connection if

$$g(\widetilde{\mathcal{M}}(\phi X, \phi Y)\phi Z, \phi W) = 0, \quad X, Y, Z, W \in TM. \tag{31}$$

We begin with the following:

Theorem 3.2. *Let M be an n-dimensional $(n > 3)$ Sasakian manifold with respect to quarter-symmetric metric connection $\widetilde{\nabla}$. If M satisfies*

$$g(\widetilde{\mathcal{M}}(\phi X, Y)Z, \phi W) = 0, \quad X, Y, Z, W \in TM, \tag{32}$$

then the scalar curvature with respect to the connection $\widetilde{\nabla}$ is zero and M is an Einstein manifold with respect to the connection $\widetilde{\nabla}$.

Proof. Let M be an n-dimensional $(n > 3)$ Sasakian manifold with respect to the connection $\widetilde{\nabla}$. From (29) we have

$$\begin{aligned} g(\widetilde{\mathcal{M}}(\phi X, Y)Z, \phi W) &= g(\widetilde{R}(\phi X, Y)Z, \phi W) \\ &- \frac{1}{n-2}\{\widetilde{S}(Y,Z)g(\phi X, \phi W) - \widetilde{S}(\phi X, Z)g(Y, \phi W) \\ &+ g(Y,Z)\widetilde{S}(\phi X, \phi W) - g(\phi X, Z)\widetilde{S}(Y, \phi W)\}, \end{aligned} \tag{33}$$

for $X, Y, Z, W \in TM$. For a local orthonormal basis of vector fields $\{e_1, ..., e_{n-1}, \xi\}$ in M, then (33) gives

$$\begin{aligned} \sum_{i=1}^{n-1} g(\widetilde{\mathcal{M}}(\phi e_i, Y)Z, \phi e_i) &= \sum_{i=1}^{n-1} g(\widetilde{R}(\phi e_i, Y)Z, \phi e_i) \\ &- \frac{1}{2(n-1)} \sum_{i=1}^{n-1} \{\widetilde{S}(Y,Z)g(\phi e_i, \phi e_i) - \widetilde{S}(\phi e_i, Z)g(Y, \phi e_i) \\ &+ g(Y,Z)\widetilde{S}(\phi e_i, \phi e_i) - g(\phi e_i, Z)\widetilde{S}(Y, \phi e_i)\}, \end{aligned} \tag{34}$$

for $Y, Z \in TM$. Using (25), (23), (24) and (27) in above equation, we get

$$\begin{aligned} \sum_{i=1}^{n-1} g(\widetilde{\mathcal{M}}(\phi e_i, Y)Z, \phi e_i) &= \widetilde{S}(Y,Z) + 2d\eta(\phi Z, Y) \\ &- \frac{1}{2(n-1)}\{(n-3)\widetilde{S}(Y,Z) \\ &+ (\widetilde{r} - 2(n-1))g(Y,Z) + 4(n-1)\eta(Y)\eta(Z)\}, \end{aligned} \tag{35}$$

for $Y, Z \in TM$. If M satisfies (32), then from (35) we have

$$\begin{aligned} \widetilde{S}(Y,Z) + 2d\eta(\phi Z, Y) &= \frac{1}{2(n-1)}\{(n-3)\widetilde{S}(Y,Z) \\ &+(\widetilde{r} - 2(n-1))g(Y,Z) + 4(n-1)\eta(Y)\eta(Z)\}, \end{aligned}$$

for $Y, Z \in TM$. This is equivalent to

$$\widetilde{S}(Y,Z) = (\widetilde{r} - 2)g(Y,Z) + 2n\eta(Y)\eta(Z)\}, \tag{36}$$

for $Y, Z \in TM$, where (7) is used. Putting $Y, Z = e_i$ in (36) and using (21) and $\eta(\xi) = 1$, we get $\widetilde{r} = 2n(n-1)$ and consequently (36) reduces to

$$\widetilde{S}(Y,Z) = 2(n-1)g(Y,Z). \tag{37}$$

This means that the manifold is an Einstein with respect to the connection $\widetilde{\nabla}$. □

Theorem 3.3. *An n-dimensional $(n > 3)$ Sasakian manifold M is quasi-$\mathcal{M}$-projectively flat with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ if and only if*

$$\widetilde{R}(X, Y, Z, \phi W) = 2[g(Y, Z)g(X, \phi W) - g(X, Z)g(Y, \phi W)], \tag{38}$$

for all $X, Y, Z, W \in TM$.

Proof. Let M is quasi-$\mathcal{M}$-projectively flat with respect to the connection $\widetilde{\nabla}$, using (37) in

$$\begin{aligned} g(\widetilde{\mathcal{M}}(X,Y)Z, \phi W) &= \widetilde{R}(X, Y, Z, \phi W) \\ &- \frac{1}{2(n-1)}\{\widetilde{S}(Y, Z)g(X, \phi W) - \widetilde{S}(X, Z)g(Y, \phi W) \\ &+ g(Y, Z)\widetilde{S}(X, \phi W) - g(X, Z)\widetilde{S}(Y, \phi W)\}, \end{aligned}$$

we obtain (38). The converse is straightforward. □

Theorem 3.4. *An n-dimensional $(n > 3)$ Sasakian manifold is ϕ-$\mathcal{M}$-projectively flat with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ if and only if M satisfies*

$$\widetilde{R}(\phi X, \phi Y, \phi Z, \phi W) = 2[g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)], \tag{39}$$

for all $X, Y, Z, W \in TM$.

Proof. Let M be an n-dimensional Sasakian manifold. From (29) we have

$$\begin{aligned} g(\widetilde{\mathcal{M}}(\phi X, \phi Y)\phi Z, \phi W) &= g(\widetilde{R}(\phi X, \phi Y)\phi Z, \phi W) \\ &- \frac{1}{2(n-1)}\{\widetilde{S}(\phi Y, \phi Z)g(\phi X, \phi W) - \widetilde{S}(\phi X, \phi Z)g(\phi Y, \phi W) \\ &+ \widetilde{S}(\phi X, \phi W)g(\phi Y, \phi Z) - \widetilde{S}(\phi Y, \phi W)g(\phi X, \phi Z)\}, \end{aligned} \tag{40}$$

for all $X, Y, Z, W \in TM$. For an orthonormal basis of vector fileds $\{e_1, ..., e_{n-1}, \xi\}$ in M, from (40) it follows that

$$\begin{aligned} \sum_{i=1}^{n-1} g(\widetilde{\mathcal{M}}(\phi e_i, \phi Y)\phi Z, \phi e_i) &= \sum_{i=1}^{n-1} g(\widetilde{R}(\phi e_i, \phi Y)\phi Z, \phi e_i) \\ &- \frac{1}{2(n-1)}\sum_{i=1}^{n-1}\{\widetilde{S}(\phi Y, \phi Z)g(\phi e_i, \phi e_i) - \widetilde{S}(\phi e_i, \phi Z)g(\phi Y, \phi e_i) \\ &+ \widetilde{S}(\phi e_i, \phi e_i)g(\phi Y, \phi Z) - \widetilde{S}(\phi Y, \phi e_i)g(\phi e_i, \phi Z)\}, \end{aligned} \tag{41}$$

for all $Y, Z \in TM$. Which in view of (23), (26), (27) and (28) becomes

$$\begin{aligned} \sum_{i=1}^{n-1} g(\widetilde{\mathcal{M}}(\phi e_i, \phi Y)\phi Z, \phi e_i) &= \widetilde{S}(Y, Z) - 2g(Y, Z) - 2(n-2)\eta(Y)\eta(Z) \\ &- \frac{1}{2(n-1)}\{(n-3)\widetilde{S}(\phi Y, \phi Z) + (\widetilde{r} - 2(n-1))g(\phi Y, \phi Z)\}, \end{aligned} \tag{42}$$

for all $Y, Z \in TM$. If M is ϕ-$\mathcal{M}$-projectively flat with respect to the connection $\widetilde{\nabla}$, using (29), (20) and (5) in (42) we get

$$\widetilde{S}(Y, Z) = 2(n-1)g(Y, Z). \tag{43}$$

Putting $Y, Z = e_i$ in (43) and using (21) and $\eta(\xi) = 1$, we get $\widetilde{r} = 2n(n-1)$ and consequently (43) reduces to (37). By replacing X by ϕX and Y by ϕY in (37) one can get $\widetilde{S}(\phi Y, \phi Z) = 2(n-1)g(\phi Y, \phi Z)$ for $Y, Z \in TM$. Now using this value in (40) with (31) we obtain (39). The converse is obvious. □

Theorem 3.5. *Let M^n be an n-dimensional $(n > 3)$ Sasakian manifold. Then the following statements are equivalent:*

(1). M is quasi-$\mathcal{M}$-projectively flat with respect to the connection $\widetilde{\widetilde{\nabla}}$.

(2). M is ϕ-$\mathcal{M}$-projectively flat with respect to the connection $\widetilde{\widetilde{\nabla}}$.

(3). The curvature tensor with respect to the connection $\widetilde{\widetilde{\nabla}}$ of M is given by

$$\widetilde{R}(X, Y)Z = 2[g(Y, Z)X - g(X, Z)Y]. \tag{44}$$

Proof. Let M be an n-dimensional $(n > 3)$ Sasakian manifold. From (30) and (29) it is obvious that (1) implies (2). Now, assume that (2) is true. In a Sasakian manifold, in view of (17) and (18) we can verify

$$\begin{aligned} & \widetilde{R}(\phi^2X, \phi^2Y, \phi^2Z, \phi^2W) \\ = \; & \widetilde{R}(X, Y, Z, W) + 2\{g(X, Z)\eta(Y)\eta(W) \\ - \; & g(Y, Z)\eta(X)\eta(W) + g(Y, W)\eta(X)\eta(Z) - g(X, W)\eta(Y)\eta(Z)\}, \end{aligned} \tag{45}$$

for all $X, Y, Z, W \in TM$. By changing X, Y, Z, W to $\phi X, \phi Y, \phi Z, \phi W$, respectively in (39) and using (45) we get (44). Hence, the statement (2) implies the statement (3). Next, we assume the statement (3) is true. On contracting (44) it follows (37). Using (37) and (44) in (29) we get the statement (1). This completes the proof. □

References

[1] C.S.Bagewadi, D.G.Prakasha and Venkatesha, *A Study of Ricci quarter-symmetric metric connection on a Riemannian manifold*,Indian J. Math., 50(3)(2008), 607-615.

[2] S.C.Biswas and U.C.De, *Quarter-symmetric metric connection in an SP-Sasakian manifold*, Commun. Fac. Sci. Univ. Ank. Series, 46(1997), 49-56.

[3] D.E.Blair, *Contact manifolds in Riemannian geometry. Lecture Notes in Mathematics* Vol. 509, Springer-Verlag, Berlin, New-York, (1976).

[4] D.E.Blair, *Riemannian geometry of contact and symplectic manifolds*, Progress in Mathematics, 203 (Boston, MA; Birkhauser Boston, Inc.,) (2002).

[5] S.K.Chaubey and R.H.Ojha, *On the $\mathcal{M}$-projective curvature tensor of a Kenmotsu manifold*, Different. Geom. Dynam. Syst., 12(2010), 52-60 .

[6] U.C.De and J.Sengupta, *Quarter-symmetric metric connection on a Sasakian manifold*, Commun. Fac. Sci. Univ. Ank. Series,A1., 49(2000), 7-13.

[7] A.Friedmann and J.A.Schouten, *Uber die Geometrie der halbsymmetrischen Ubertragung*, Math. Zeitschr., 21(1924), 211-223.

[8] S.Golab, *On semi-symmetric and quarter-symmetric linear connections*, Tensor. N.S., 29(1975), 293-301.

[9] H.A.Hayden, *Subspaces of a space with torsion*, Proc. London Math. Soc., 34(1932), 27-50.

[10] J.B.Jun and U.K.Kim, *On* 3*-dimensional almost contact metric manifolds*, Kyungpook Math. J., 34(1994), 213-301.

[11] R.S.Mishra and S.N.Pandey, *On quarter-symmetric metric F-connections*, Tensor, N.S., 34(1980), 1-7.

[12] S.Mukhopadhyay, A.K.Roy and B.Barua, *Some properties of a quarter-symmetric metric connection on a Riemannian manifold*, Soochow J. of Math., 17(2) (1991), 205-211.

[13] R.H.Ojha, *A note on the $\mathcal{M}$-projective curvature tensor*, Indian J. Pure Appl. Math., 8(1975), 1531-1534.

[14] R.H.Ojha, *$\mathcal{M}$-projectively flat Sasakian manifolds*, Indian J. Pure Appl. Math., 17(1986), 481-484.

[15] G.P.Pokhariyal and R.S.Mishra, *Curvature tensors and their relativistic signification II*. Yokohama Math. J., 18(1970), 105-108.

[16] D.G.Prakasha and A.Taleshian, *The structure of some classes of Sasakian manifolds with respect to the quarter-Symmetric metric connection*. Int. J. Open problems compt. Math., 3(5)(2010), 1-16.

[17] S.C.Rastogi, *On quarter-symmetric metric connection*, Tensor., 44(2)(1987), 133-141.

[18] J.A.Schouten, *Ricci calculus.*, Springer, (1954).

[19] K.Yano and M.Kon, *Structure on manifolds*, World Scientific, (1984).

[20] K.Yano, *On semi-symmetric metric connections*, Rev. Roumaine Math. Pures Appl., 15(1970), 1579-1586.

[21] K.Yano and T.Imai, *Quarter-symmetric metric connections and their curvature tensors*, Tensor, N.S., 38(1982), 13-18.

[22] F.Ö.Zengin, *On $\mathcal{M}$-projectively flat LP-Sasakian manifolds*, Ukranian Mathematical Journal, 65(2)(2014).