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Slightly (b, μ)-Continuous Functions

Research Article

O.Ravi[1]*, N.Navaneethakrishnan[2] and P.Jeyalakshmi[1]

1 Department of Mathematics, P.M.Thevar College, Usilampatti, Madurai Dt, Tamil Nadu, India.

2 Department of Mathematics, Kamaraj College, Tuticorin, Tamil Nadu, India.

Abstract: We study the concepts related to λ-b-open sets, λ-b-closed sets and (b, μ)-continuity. Also some basic properties of slightly (b, μ)-continuous functions and connectedness and covering properties of slightly (b, μ)-continuous functions are investigated.

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1. Introduction

In [2, 3, 4], Csaszar founded the theory of generalized topological spaces and studied the extremely elementary character of these classes. Especially he introduced the notion of continuous functions on generalized topological spaces and investigated characterizations of generalized continuous functions (= (λ, μ)-continuous functions in [3]). In [6, 7, 8], Min introduced the notions of weak (λ, μ)-continuity, almost (λ, μ)-continuity, (α, μ)-continuity, (σ, μ)-continuity, (π, μ)-continuity and (β, μ)-continuity on generalized topological spaces.

In this paper, we study the concepts related to λ-b-open sets, λ-b-closed sets and (b, μ)-continuity. Also some basic properties of slightly (b, μ)-continuous functions and connectedness and covering properties of slightly (b, μ)-continuous functions are investigated.

2. Preliminaries

Definition 2.1. *Let X be a nonempty set and λ be a collection of subsets of X. Then λ is called a generalized topology (briefly GT) on X iff $\phi \in \lambda$ and $G_1 \in \lambda$ for $i \in I \neq \phi$ implies $G = \bigcup_{i \in I} G_i \in \lambda$. We call the pair (X, λ) a generalized topological space (briefly GTS) on X.*

The elements of λ are called λ-open sets and the complements are called λ-closed sets. The generalized closure of a subset S of X, denoted by c_λ(S), is the intersection of λ-closed sets including S. And the interior of S, denoted by i_λ(S), is the union of λ-open sets contained in S.

* E-mail: siingam@yahoo.com

Definition 2.2. *Let (X, λ) be a generalized topological space and $A \subseteq X$. Then A is said to be*

(1). λ-semi-open [3] if $A \subseteq c_\lambda(i_\lambda(A))$,

(2). λ-preopen [3] if $A \subseteq i_\lambda(c_\lambda(A))$,

(3). λ-α-open [3] if $A \subseteq i_\lambda(c_\lambda(i_\lambda(A)))$,

(4). λ-β-open [3] if $A \subseteq c_\lambda(i_\lambda(c_\lambda(A)))$,

(5). λ-b-open [12] if $A \subseteq c_\lambda(i_\lambda(A)) \cup i_\lambda(c_\lambda(A))$.

The complement of λ-semi-open (resp. λ-preopen, λ-α-open, λ-β-open, λ-b-open) is said to be λ-semi-closed (resp. λ-preclosed, λ-α-closed, λ-β-closed, λ-b-closed).

Let us denote by $\sigma(\lambda_X)$ (briefly σ_X or σ) the class of all λ-semi-open sets on X, by $\pi(\lambda_X)$ (briefly π_X or π) the class of all λ-preopen sets on X, by $\alpha(\lambda_X)$ (briefly α_X or α) the class of all λ-α-open sets on X, by $\beta(\lambda_X)$ (briefly β_X or β) the class of all λ-β-open sets on X, by $\mathrm{b}(\lambda_X)$ (briefly b_X or b) the class of all λ-b-open sets on X. Obviously [9] $\lambda_X \subseteq \alpha(\lambda_X) \subseteq \sigma(\lambda_X) \subseteq \beta(\lambda_X)$ and $\lambda_X \subseteq \alpha(\lambda_X) \subseteq \pi(\lambda_X) \subseteq \beta(\lambda_X)$.

Lemma 2.3 ([3])**.** *Let (X, λ) be a generalized topological space and $A \subseteq X$. Then A is λ-α-open if and only if it is λ-semi-open and λ-preopen.*

Definition 2.4 ([10])**.** *A GTS (X, λ) is said to be λ-extremally disconnected if the λ-closure of every λ-open set of X is λ-open in X.*

Definition 2.5 ([10])**.** *Let (X, λ) be a generalized topological space and $B \subseteq X$. Let $\phi \in B$. Then B is called a Base for λ if $\{\cup B' : B' \subseteq B\} = \lambda$. We also say that λ is generated by B.*

Definition 2.6. *Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be*

(1). (λ, μ)-continuous [4] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-open in X,

(2). (α, μ)-continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-α-open in X,

(3). (σ, μ)-continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-semiopen in X,

(4). (π, μ)-continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-preopenmin X,

(5). (β, μ)-continuous [7] if for each μ-open set U in Y, $f^{-1}(U)$ is λ-β-open in X.

Remark 2.7 ([7])**.** *Let f be a function between GTS's (X, λ) and (Y, μ). Then we have the following implications.*

$$\begin{array}{ccccc} (\lambda,\ \mu)\text{-continuous} & \longrightarrow & (\alpha,\ \mu)\text{-continuous} & \longrightarrow & (\sigma,\ \mu)\text{-continuous} \\ & & \downarrow & & \downarrow \\ & & (\pi,\ \mu)\text{-continuous} & \longrightarrow & (\beta,\ \mu)\text{-continuous} \end{array}$$

Remark 2.8 ([3])**.** *Let (X, λ) be a generalized topological space and $A \subseteq X$. The λ-closure (resp. λ-α-closure, λ-semi-closure, λ-preclosure, λ-β-closure, λ-b-closure) of a subset A of X, denoted by $c_\lambda(A)$ (resp. $c_\alpha(A)$, $c_\sigma(A)$, $c_\pi(A)$, $c_\beta(A)$, $c_b(A)$), is the intersection of all λ-closed (resp. λ-α-closed, λ-semi-closed, λ-preclosed, λ-β-closed, λ-b-closed) sets containing A.*

The λ-interior (resp. λ-α-interior, λ-semi-interior, λ-preinterior, λ-β-interior, λ-b-interior) of a subset A of X, denoted by $i_\lambda(A)$ (resp. $i_\alpha(A)$, $i_\sigma(A)$, $i_\pi(A)$, $i_\beta(A)$, $i_b(A)$), is the union of all λ-open (resp. λ-α-open, λ-semi-open, λ-preopen, λ-β-open, λ-b-open) sets contained in A.

Lemma 2.9 ([4]). *For a subset A of a GTS (X, λ), we have*

(1). $X - i_\lambda(X - A) = c_\lambda(A)$,

(2). $X - c_\lambda(X - A) = i_\lambda(A)$.

Lemma 2.10 ([4]). *Let A be a subset in a GTS (X, λ). Then*

(1). $i_\lambda(c_\lambda(i_\lambda(c_\lambda(A)))) = i_\lambda(c_\lambda(A))$ and $c_\lambda(i_\lambda(c_\lambda(i_\lambda(A)))) = c_\lambda(i_\lambda(A))$.

(2). $(i_\lambda(c_\lambda(A)))' = c_\lambda(i_\lambda(A'))$ and $(c_\lambda(i_\lambda(A)))' = i_\lambda(c_\lambda(A'))$.

Theorem 2.11 ([3]). *For any subset A in a GTS (X, λ), the following statements are true.*

(1). $c_\sigma(A) = A \cup i_\lambda(c_\lambda(A))$ and $i_\sigma(A) = A \cap c_\lambda(i_\lambda(A))$.

(2). $c_\pi(A) \supseteq A \cup c_\lambda(i_\lambda(A))$ and $i_\pi(A) \subseteq A \cap i_\lambda(c_\lambda(A))$.

Lemma 2.12 ([4]). *Let $f : (X, \lambda) \to (Y, \mu)$ be a function. For the sets A and B of (X, λ) and (Y, μ) respectively, the following statements hold:*

(1). $f(f^{-1}(B)) \subseteq B$;

(2). $f^{-1}(f(A)) \supseteq A$;

(3). $f(A') \supseteq (f(A))'$;

(4). $f^{-1}(B') = (f^{-1}(B))'$;

(5). if f is injective, then $f^{-1}(f(A)) = A$;

(6). if f is surjective, then $f(f^{-1}(B)) = B$;

(7). if f is bijective, then $f(A') = (f(A))'$.

Definition 2.13 ([5]). *A set A in a GTS (X, λ) is called*

(1). λ-clopen if it is both λ-open and λ-closed.

(2). λ-b-clopen if it is both λ-b-open and λ-b-closed.

Definition 2.14 ([5]). *A subset A in a GTS (X, λ) is said to be λ-locally λ-closed set (briefly λ-LC set) if $A = M \cap N$, where M is a λ-open set and N is a λ-closed set.*

Definition 2.15 ([11]). *A GTS (X, λ) is said to be λ-connected if it cannot be expressed as the union of two nonempty disjoint λ-open sets.*

Definition 2.16 ([1]). *Let $f : (X, \lambda) \to (Y, \mu)$ be a function from a GTS (X, λ) to a GTS (Y, μ). Then the function $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ is called the λ-graph function of f. Recall that for a function $f : (X, \lambda) \to (Y, \mu)$, the subset $\{(x, f(x)) : x \in X\} \subseteq X \times Y$ is called the λ-graph of f and is denoted by $G(f)$.*

3. λ-b-closed Sets and (b, μ)-continuity

In this section, we obtain some new results which are related to λ-b-open sets, λ-b-closed sets and (b, μ)-continuity.

Definition 3.1. *Let (X, λ) and (Y, μ) be GTS's. Then a function f : (X, λ) $\to$ (Y, μ) is said to be*

(1). (b, μ)-continuous if for each μ-open set U in Y, f^{-1}(U) is λ-b-open in X,

(2). (λ-LC, μ)-continuous if for each μ-open set U in Y, f^{-1}(U) is λ-locally λ-closed in X.

Remark 3.2. *Any union of λ-b-open sets is a λ-b-open set.*

Theorem 3.3. *For a GTS (X, λ), the following properties are equivalent.*

(1). (X, λ) is λ-extremally disconnected.

(2). $i_\lambda(A)$ is λ-closed for every λ-closed subset A of X.

(3). $c_\lambda(i_\lambda(A)) \subseteq i_\lambda(c_\lambda(A))$ for every subset A of X.

(4). Every λ-semi-open set is λ-preopen.

(5). The λ-closure of every λ-β-open subset of X is λ-open.

(6). Every λ-β-open set is λ-preopen.

(7). For every subset A of X, A is λ-α-open if and only if it is λ-semi-open.

Proof.

(1) $\Rightarrow$ (2): Let A$\subseteq$ X be a λ-closed set. Then A$'$ is λ-open in X. By (1), $c_\lambda(A')=(i_\lambda(A)')$ is λ-open. Thus, $i_\lambda(A)$ is λ-closed.

(2) $\Rightarrow$ (3): Let A be any set of X. Then $(i_\lambda(A))'$ is λ-closed in X. By (2), $i_\lambda(i_\lambda(A)')$ is λ-closed in X. Therefore $i_\lambda((i_\lambda(A))')$ $= (c_\lambda(i_\lambda(A)))'$ is λ-closed in X and $c_\lambda(i_\lambda(A))$ is λ-open in X and hence $c_\lambda(i_\lambda(A)) \subseteq i_\lambda(c_\lambda(A))$.

(3) $\Rightarrow$ (4): Let A be λ-semi-open. By (3), we have $A \subseteq c_\lambda(i_\lambda(A)) \subseteq i_\lambda(c_\lambda(A))$. Thus A is λ-preopen.

(4) $\Rightarrow$ (5): Let A be a λ-β-open set. Then $c_\lambda(A)$ is λ-semi-open. By (4), $c_\lambda(A)$ is λ-preopen. Thus, $c_\lambda(A) \subseteq i_\lambda(c_\lambda(A))$ and hence $c_\lambda(A)$ is λ-open.

(5) $\Rightarrow$ (6): Let A be λ-β-open set. By (5), $c_\lambda(A) = i_\lambda(c_\lambda(A))$. Thus, $A \subseteq c_\lambda(A) = i_\lambda(c_\lambda(A))$ and hence A is λ-preopen.

(6) $\Rightarrow$ (7): Let A be a λ-semi-open set. Since a λ-semi-open set is λ-β-open, then by (6), it is λ-preopen. Since A is λ-semi-open and λ-preopen, A is λ-α-open.

(7) $\Rightarrow$ (1): Let A be a λ-open set of X. Then $c_\lambda(A)$ is λ-semi-open and by (7) $c_\lambda(A)$ is λ-α-open. Therefore $c_\lambda(A) \subseteq$ $i_\lambda(c_\lambda(i_\lambda(c_\lambda(A)))) = i_\lambda(c_\lambda(A))$ and hence $c_\lambda(A) = i_\lambda(c_\lambda(A))$. Hence $c_\lambda(A)$ is λ-open and (X, λ) is λ-extremally disconnected.

□

Theorem 3.4. *For a subset A of a GTS (X, λ), then the following are equivalent.*

(1). A is a λ-locally λ-closed set.

(2). $A = M \cap c_\lambda(A)$ for some λ-open set M.

Proof.

(1) $\Rightarrow$ (2): Since A is a λ-locally λ-closed set. Then $A = M \cap N$ where M is λ-open and N is λ-closed. So $A \subseteq M$ and A$\subseteq$N. Hence $c_\lambda(A) \subseteq c_\lambda(N)$. Therefore A$\subseteq$ $M \cap c_\lambda(A) \subseteq M \cap c_\lambda(N) = M \cap N = A$. Thus $A = M \cap c_\lambda(A)$.

(2) $\Rightarrow$ (1): Since $A = M \cap c_\lambda(A)$ for some λ-open set M. Let $c_\lambda(A) = N$ be a λ-closed. We obtain $A = M \cap N$ where M is λ-open and N is λ-closed. Hence A is a λ-locally λ-closed set. □

Theorem 3.5. *Let (X, λ) be a λ-extremally disconnected space and $A \subseteq X$, the following properties are equivalent.*

(1). A is a λ-open set.

(2). A is λ-α-open and a λ-locally λ-closed set.

(3). A is λ-preopen and a λ-locally λ-closed set.

(4). A is λ-semi-open and a λ-locally λ-closed set.

(5). A is λ-b-open and a λ-locally λ-closed set.

Proof.

(1) $\Rightarrow$ (2): It follows from the fact that every λ-open set is λ-α-open and λ-locally λ-closed set.

(2) $\Rightarrow$ (3), (2) $\Rightarrow$ (4), (3) $\Rightarrow$ (5) and (4) $\Rightarrow$ (5): Obvious.

(5) $\Rightarrow$ (1): Suppose that A is a λ-b-open set and a λ-locally λ-closed set. It follows that A$\subseteq$ $c_\lambda(i_\lambda(A)) \cup i_\lambda(c_\lambda(A))$. Since A is a λ-locally λ-closed set, then there exists a λ-open set M such that A = M $\cap$ $c_\lambda(A)$. It follows that A$\subseteq$ M $\cap$ [$c_\lambda(i_\lambda(A))$ $\cup$ $i_\lambda(c_\lambda(A))$] = [M$\cap$ $c_\lambda(i_\lambda(A))$] $\cup$ [M$\cap$ $i_\lambda(c_\lambda(A))$] $\subseteq$ [M$\cap$ $i_\lambda(c_\lambda(A))$] $\cup$ [M$\cap$ $i_\lambda(c_\lambda(A))$] (by Theorem 3.3) = $i_\lambda(M \cap c_\lambda(A))$ $\cup$ $i_\lambda(M\cap c_\lambda(A))$ = $i_\lambda(A) \cup i_\lambda(A) = i_\lambda(A)$. Thus, A$\subseteq$ $i_\lambda(A)$ and hence A is a λ-open set in X. □

Theorem 3.6. *For a subset A of a GTS (X, λ), if $A = c_\pi(A) \cap c_\sigma(A)$, then A is a λ-b-closed set.*

Proof. Let A = $c_\pi(A) \cap c_\sigma(A) \supseteq (A\cup c_\lambda(i_\lambda(A))) \cap (A\cup i_\lambda(c_\lambda(A))) \supseteq c_\lambda(i_\lambda(A)) \cap i_\lambda(c_\lambda(A))$. This implies $c_\lambda(i_\lambda(A)) \cap i_\lambda(c_\lambda(A)) \subseteq A$. Thus A is λ-b-closed set in X. □

Theorem 3.7. *Let f : (X, λ) $\to$ (Y, μ) where (X, λ) is a λ-extremally disconnected space and (Y, μ) is a GTS. Then the following properties are equivalent.*

(1). f is a (λ, μ)-continuous.

(2). f is (α, μ)-continuous and a (λ-LC, μ)-continuous.

(3). f is (π, μ)-continuous and a (λ-LC, μ)-continuous.

(4). f is (σ, μ)-continuous and a (λ-LC, μ)-continuous.

(5). f is (b, μ)-continuous and a (λ-LC, μ)-continuous.

Proof. It is obvious from Theorem 3.5. □

4. Slightly (b, μ)-Continuous Functions

In this section, basic properties of slightly (b, μ)-continuous functions and connectedness and covering properties of slightly (b, μ)-continuous functions are investigated.

Definition 4.1. *Let (X, λ) and (Y, μ) be generalized topological spaces. A function f : (X, λ) $\to$ (Y, μ) is said to be slightly (b, μ)-continuous if for each point $x \in X$ and each μ-clopen set M in Y containing f(x), there exists a λ-b-open set N in X containing x such that f(N) $\subseteq$ M.*

Theorem 4.2. *For a function f : (X, λ) $\to$ (Y, μ), the following statements are equivalent:*

(1). f is slightly (b, μ)-continuous;

(2). for every μ-clopen set A in Y, $f^{-1}(A)$ is λ-b-open;

(3). for every μ-clopen set A in Y, $f^{-1}(A)$ is λ-b-closed;

(4). for every μ-clopen set A in Y, $f^{-1}(A)$ is λ-b-clopen.

Proof.
(1) $\Rightarrow$ (2): Let A be a μ-clopen set in Y and let x $\in$ f^{-1}(A). Since f(x) $\in$ A, by (1), there exists a λ-b-open set M in X containing x such that M$\subseteq$ f^{-1}(A). We obtain that f^{-1}(A) = $\cup$ M (x $\in$ f^{-1}(A)). Thus, f^{-1}(A) is λ-b-open.
(2) $\Rightarrow$ (3): Let A be a λ-clopen set in Y. Then, A$'$ is λ-clopen. By (2), $f^{-1}(A') = (f^{-1}(A))'$ is λ-b-open. Thus, f^{-1}(A) is λ-b-closed.
(3) $\Rightarrow$ (4): It can be shown easily.
(4) $\Rightarrow$ (1): Let A be a λ-clopen set in Y containing f(x). By (4), f^{-1}(A) is λ-b-clopen. Take M = f^{-1}(A). Then, f(M)$\subseteq$A. Hence, f is slightly (b, μ)-continuous. □

Remark 4.3. *Obviously (b, μ)-continuity implies slightly (b, μ)-continuity. The following example shows that this implication is not reversible.*

Example 4.4. *Let $X = \{a,\ b\}$, $Y = \{p,\ q\}$ and λ, μ are considered as follows: $\lambda = \{\phi,\ X,\ \{a\}\}$ and $\mu = \{\phi,\ Y,\ \{q\}\}$. Then the function $f : (X,\ \lambda) \to (Y,\ \mu)$ defined by $f(a) = p$, $f(b) = q$ is slightly (b, μ)-continuous but not (b, μ)-continuous.*

Theorem 4.5. *Suppose that Y has a base consisting of λ-clopen sets. If $f : (X,\ \lambda) \to (Y,\ \mu)$ is slightly (b, μ)-continuous, then f is (b, μ)-continuous.*

Proof. Let x $\in$ X and let A be a μ-open set in Y containing f(x). Since Y has a base consisting of λ-clopen sets, there exists a λ-clopen set M containing f(x) such that M $\subseteq$ A. Since f is slightly (b, μ)-continuous, then there exists a λ-b-open set N in X containing x such that f(N) $\subseteq$ M $\subseteq$ A. Thus, f is (b, μ)-continuous. □

Theorem 4.6. *Let $f : (X,\ \lambda) \to (Y,\ \mu)$ be a function and let $g : X \to X \times\ Y$ be the λ-graph function of f, defined by g(x) = (x, f(x)) for every $x \in X$. If g is slightly (b, μ)-continuous, then f is slightly (b, μ)-continuous.*

Proof. Let M be μ-clopen set in Y, then X $\times$ M is μ-clopen set in X $\times$ Y. Since g is slightly (b, μ)-continuous, then f^{-1}(M) = g^{-1}(X $\times$ M) is λ-b-open in X. Thus, f is slightly (b, μ)-continuous. □

Theorem 4.7. *A filter base Λ is said to be λ-b-convergent to a point x in X if for any λ-b-open set M in X containing x, there exists a set $N \in \Lambda$ such that N$\subseteq$ M.*

Definition 4.8. *A filter base Λ is said to be λ-co-convergent to a point x in X if for any λ-clopen set M in X containing x, there exists a set $N \in \Lambda$ such that N$\subseteq$M.*

Theorem 4.9. *If a function $f : (X,\ \lambda) \to (Y,\ \mu)$ is slightly (b, μ)-continuous, then for each point $x \in X$ and each filter base Λ in X λ-b-converging to x, the filter base $f(\Lambda)$ is λ-co-convergent to f(x).*

Proof. Let x $\in$ X and Λ be any filter base in X λ-b-converging to x. Since f is slightly (b, μ)-continuous, then for any μ-clopen set M in Y containing f(x), there exists a λ-b-open set N in X containing x such that f(N) $\subseteq$ M. Since Λ is λ-b-converging to x, there exists a P $\in$ Λ such that P$\subseteq$N. This means that f(P) $\subseteq$ M and therefore the filter base f(Λ) is λ-co-convergent to f(x). □

Definition 4.10. *A GTS (X, λ) is said to be λ-b-connected if it cannot be expressed as the union of two nonempty disjoint λ-b-open sets.*

Theorem 4.11. *If f : (X, λ) $\rightarrow$ (Y, μ) is slightly (b, μ)-continuous surjective function and X is λ-b-connected space, then Y is μ-connected space.*

Proof. Suppose that Y is not μ-connected space. Then there exists nonempty disjoint μ-open sets M and N such that Y = M $\cup$ N. Therefore, M and N are μ-clopen sets in Y. Since f is slightly (b, μ)-continuous, then f^{-1}(M) and f^{-1}(N) are λ-b-closed and λ-b-open in X. Moreover, f^{-1}(M) and f^{-1}(N) are nonempty disjoint and X = f^{-1}(M) $\cup$ f^{-1}(N). This shows that X is not λ-b-connected. This is a contradiction. By contradiction, Y is μ-connected. □

Definition 4.12. *A GTS (X, λ) is said to be*

(1). λ-b-compact if every λ-b-open cover of X has a finite subcover.

(2). countably λ-b-compact if every λ-b-open countably cover of X has a finite subcover.

(3). λ-b-Lindelof if every λ-b-open cover of X has a countable subcover.

(4). mildly λ-compact if every λ-clopen cover of X has a finite subcover.

(5). mildly λ-countably λ-compact if every λ-clopen countably cover of X has a finite subcover.

(6). mildly λ-Lindelof if every cover of X by λ-clopen sets has a countable subcover.

Theorem 4.13. *Let f : (X, λ) $\rightarrow$ (Y, μ) be a slightly (b, μ)-continuous surjection. Then the following statements hold:*

(1). if X is λ-b-compact, then Y is mildly μ-compact.

(2). if X is λ-b-Lindelof, then Y is mildly μ-Lindelof.

(3). if X is countably λ-b-compact, then Y is mildly μ-countably μ-compact.

Proof. Let $\{A_\alpha : \alpha \in I\}$ be any μ-clopen cover of Y. Since f is slightly (b, μ)-continuous, then $\{f^{-1}(A_\alpha) : \alpha \in I\}$ is a λ-b-open cover of X. Since X is λ-b-compact, there exists a finite subset I_0 of I such that X = $\cup\{f^{-1}(A_\alpha) : \alpha \in I_0\}$. Thus, we have Y = $\cup\{A_\alpha : \alpha_0 \in I\}$ and Y is mildly μ-compact. The other proofs are similar. □

Definition 4.14. *A GTS (X, λ) is said to be*

(1). λ-b-closed-compact if every λ-b-closed cover of X has a finite subcover.

(2). countably λ-b-closed-compact if every countable cover of X by λ-b-closed sets has a finite subcover.

(3). λ-b-closed-Lindelof if every cover of X by λ-b-closed sets has a countable subcover.

Theorem 4.15. *Let f : (X, λ) $\rightarrow$ (Y, μ) be a slightly (b, μ)-continuous surjection. Then the following statements hold:*

(1). if X is λ-b-closed-compact, then Y is mildly μ-compact.

(2). if X is λ-b-closed-Lindelof, then Y is mildly μ-Lindelof.

(3). if X is countably λ-b-closed-compact, then Y is mildly μ-countably μ-compact.

Proof. It can be obtained similarly as Theorem 4.13. □

5. μ-separation Axioms

In this section, we investigate the relationships between slightly (b, μ)-continuous functions and μ-separation axioms.

Definition 5.1. *A GTS (X, λ) is said to be λ-b-T_1 if for each pair of distinct points x and y of X, there exist λ-b-open sets M and N containing x and y respectively such that $y \notin M$ and $x \notin N$.*

Definition 5.2. *A GTS (X, λ) is said to be λ-co-T_1 if for each pair of distinct points x and y of X, there exist λ-clopen sets M and N containing x and y respectively such that $y \notin M$ and $x \notin N$.*

Theorem 5.3. *If $f : (X, \lambda) \to (Y, \mu)$ is a slightly (b, μ)-continuous injection and Y is μ-co-T_1, then X is λ-b-T_1.*

Proof. Suppose that Y is μ-co-T_1. For any distinct points x and y in X, there exist λ-clopen sets M and N in Y such that $f(x) \in M$, $f(y) \notin M$, $f(x) \notin N$ and $f(y) \in N$. Since f is slightly (b, μ)-continuous, $f^{-1}(M)$ and $f^{-1}(N)$ are λ-b-open sets in X such that $x \in f^{-1}(M)$, $y \in f^{-1}(M)$, $x \notin f^{-1}(N)$ and $y \notin f^{-1}(N)$. This shows that X is λ-b-T_1. □

Definition 5.4. *A GTS (X, λ) is said to be λ-b-T_2 (λ-b-Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint λ-b-open sets M and N in X such that $x \in M$ and $y \in N$.*

Definition 5.5. *A GTS (X, λ) is said to be λ-co-T_2 (λ-co-Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint λ-clopen sets M and N in X such that $x \in M$ and $y \in N$.*

Theorem 5.6. *If $f : (X, \lambda) \to (Y, \mu)$ is a slightly (b, μ)-continuous injection and Y is μ-co-T_2, then X is λ-b-T_2.*

Proof. For any pair of distinct points x and y in X, there exist disjoint μ-clopen sets M and N in Y such that $f(x) \in M$ and $f(y) \in N$. Since f is slightly (b, μ)-continuous, $f^{-1}(M)$ and $f^{-1}(N)$ are λ-b-open in X containing x and y respectively. We have $f^{-1}(M) \cap f^{-1}(N) = \phi$. This shows that X is λ-b-T_2. □

Definition 5.7. *A GTS (X, λ) is called λ-co-regular (respectively λ-b-regular) if for each λ-clopen (respectively λ-b-closed) set A and each point $x \notin A$, there exist disjoint λ-open sets M and N such that $A \subseteq M$ and $x \in N$.*

Definition 5.8. *A GTS (X, λ) is said to be λ-co-normal (respectively λ-b-normal) if for every pair of disjoint λ-clopen (respectively λ-b-closed) sets A and B in X, there exist disjoint λ-open sets M and N such that $A \subseteq M$ and $B \subseteq N$.*

Theorem 5.9. *If f is slightly (b, μ)-continuous injective λ-open function from a λ-b-regular space X onto a GTS (Y, μ) then Y is μ-co-regular.*

Proof. Let A be μ-clopen set in Y and $y \notin A$. Take $y = f(x)$. Since f is slightly (b, μ)-continuous, $f^{-1}(A)$ is a λ-b-closed set. Take $B = f^{-1}(A)$. We have $x \notin B$. Since X is λ-b-regular, there exist disjoint λ-open sets M and N in X such that $B \subseteq M$ and $x \in N$. We obtain that $A = f(B) \subseteq f(M)$ and $y = f(x) \in f(N)$ such that f(M) and f(N) are disjoint μ-open sets. This shows that Y is μ-co-regular. □

Theorem 5.10. *If f is slightly (b, μ)-continuous injective λ-open function from a strongly λ-b-normal space (X, λ) onto a GTS (Y, μ) then Y is μ-co-normal.*

Proof. Let A and B be disjoint μ-clopen sets in Y. Since f is slightly (b, μ)-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are λ-b-closed sets. Take $M = f^{-1}(A)$ and $N = f^{-1}(B)$. We have $M \cap N = \phi$. Since X is strongly λ-b-normal, there exist disjoint λ-open sets P and Q such that $M \subseteq P$ and $N \subseteq Q$. We obtain that $A = f(M) \subseteq f(P)$ and $B = f(N) \subseteq f(Q)$ such that f(P) and f(Q) are disjoint μ-open sets. Thus, Y is λ-co-normal. □

Definition 5.11. *A λ-graph G(f) of a function f : (X, λ) $\rightarrow$ (Y, μ) is said to be λ-b-co-closed if for each (x, y) $\in$ (X $\times$ Y) $\setminus$ G(f), there exist a λ-b-open set M in X containing x and a μ-clopen set N in Y containing y such that (M $\times$ N) $\cap$ G(f) = ϕ.*

Lemma 5.12. *A λ-graph G(f) of a function f : (X, λ) $\rightarrow$ (Y, μ) is λ-b-co-closed in X $\times$ Y if and only if for each (x, y) $\in$ (X $\times$ Y) $\setminus$ G(f), there exist a λ-b-open set M in X containing x and a μ-clopen set N in Y containing y such that f(M) $\cap$ N = ϕ.*

Theorem 5.13. *If f : (X, λ) $\rightarrow$ (Y, μ) is slightly (b, μ)-continuous and Y is μ-co-Hausdorff, then G(f) is λ-b-co-closed in X $\times$ Y.*

Proof. Let (x, y) $\in$ (X $\times$ Y) $\setminus$ G(f), then f(x) $\neq$ y. Since Y is μ-co-Hausdorff, there exist μ-clopen sets M and N in Y with f(x) $\in$ M and y $\in$ N such that M $\cap$ N = ϕ. Since f is slightly (b, μ)-continuous, there exists a λ-b-open set P in X containing x such that f(P) $\subseteq$ M. Therefore, we obtain y $\in$ N and f(P) $\cap$ N = ϕ. This shows that G(f) is λ-b-co-closed. □

Theorem 5.14. *If f : (X, λ) $\rightarrow$ (Y, μ) is (b, μ)-continuous and Y is μ-co-T_1, then G(f) is λ-b-co-closed in X $\times$ Y.*

Proof. Let (x, y) $\in$ (X $\times$ Y) $\setminus$ G(f), then f(x) $\neq$ y. Since Y is μ-co-T_1, there exist μ-clopen sets M and N in Y with f(x) $\in$ M and y $\in$ N such that there exists a λ-clopen set N in Y such that f(x) $\notin$ N and y $\notin$ N. Since f is (b, μ)-continuous, there exists a λ-b-open set f^{-1}(A) in X containing x such that f(M) $\subseteq$ N. Therefore, we obtain that f(M) $\cap$ (N$'$) = ϕ and N$'$ is μ-clopen set containing y. This shows that G(f) is λ-b-co-closed in X $\times$ Y. □

Theorem 5.15. *Let f : (X, λ) $\rightarrow$ (Y, μ) has a λ-b-co-closed graph G(f). If f is injective, then X is λ-b-T_1.*

Proof. Let x and y be any two distinct points of X. Then, we have (x, f(y)) $\in$ (X $\times$ Y) $\setminus$ G(f). By definition of λ-b-co-closed graph, there exist a λ-b-open set M in X and a λ-clopen set N in Y such that x $\in$ M, f(y) $\in$ N and f(M) $\cap$ N = ϕ; hence M $\cap$ f^{-1}(N) = ϕ. Therefore, we have y $\notin$ M. This implies that X is λ-b-T_1. □

Definition 5.16. *A function f : (X, λ) $\rightarrow$ (Y, μ) is called always λ-b-open if the image of each λ-b-open set in X is μ-b-open set in Y.*

Theorem 5.17. *Let f : (X, λ) $\rightarrow$ (Y, μ) has a λ-b-co-closed graph G(f). If f is surjective always λ-b-open function, then Y is μ-b-T_2.*

Proof. Let y and z be any distinct points of Y. Since f is surjective, f(x) = y for some x $\in$ X and (x, z) $\in$ (X $\times$ Y) $\setminus$ G(f). By λ-b-co-closedness of graph G(f), there exists a λ-b-open set M in X and a μ-clopen set N in Y such that x $\in$ M, z $\in$ N and (M $\times$ N) $\cap$ G(f) = ϕ. Then, we have f(M) $\cap$ N = ϕ. Since f is always λ-b-open, then f(M) is μ-b-open such that f(x) = y $\in$ f(M). This implies that Y is μ-b-T_2. □

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