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New Approach to Solve Fully Fuzzy Transportation Problem

Research Article

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Abstract: In the literature, there are several methods for finding a fuzzy initial basic feasible solution to the fully fuzzy transportation problem. In this paper, we extended the approach [7] into fuzzified form. The Proposed method was illustrated by the numerical examples with a triangular fuzzy number.

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Keywords: Fully fuzzy transportation problem, Triangular fuzzy number, Initial basic feasible solution.

1. Introduction

Transportation Problem is one of the subclasses of Linear Programming Problem in which the objective is to transport various quantities of a single homogeneous commodity that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum. To achives this objective we must know the amount and location of available supplies and the quantities demanded. In addition, we also know the unit transportation cost of the commodity to be transported from various origins to various destinations.

The transportation problem was first presented by Hitchcock [3] and then the systematic solution procedures from the simplex algorithm were further developed, primarily by Dantzig [10] and then by Charnes [11]. In the solution procedure of Transportation Problem, Initial Basic Feasible Solution (IFBS) is known as the fundamental stage for finding an optimal solution. The well recognized classical methods, for finding an IBFS for the transportation problem are North West Corner Rule, Least Cost Method [12, 13] and Vogel's Approximation Method [14]. Again researchers worked and are working on transportation problem to develop new algorithm to find better IBFS for Transportation Problems [15–17], and these methods may be used to solve maximization transportation problems [19–22] and also time minimization Transportation Problems [18, 23–25].

In real life there are many situations where the decision maker is uncertain about the value of transportation cost or supply or demands or transportation amount or all of them. Zimmermann [8] showed that how to solve a fuzzy linear programming. Chanas et al. [1] showed how to solve a transportation problem with crisp cost coefficients and fuzzy supply

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and demand values by transforming it into fuzzy linear programming.

A fully fuzzy transportation means the decision maker is uncertain about the value of transportation cost, supply, demands and also the transportation amount. Kaur and Kumar [4] proposed a method to solve fuzzy transportation problem where the transportation cost is only fuzzy in nature. In the Kumar method (Kaur and Kumar, [4]) generalised fuzzy north-west corner method, generalised fuzzy least cost method and generalised fuzzy Vogel's approximation method is used to find an initial basic feasible solution (IBFS) which is crispy in nature. Then generalised modified distribution method is used to find the optimal solution. The optimal solution is crisp in nature.

In this paper, we extended the approach [7] into fully fuzzified form, where total cost of transportation, total supply and demands has taken fuzzy in nature. To illustrate the proposed approach a numerical example is presented.

2. Preliminaries

2.1. Fuzzy Numbers

A fuzzy set $\widetilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function $\mu_{\widetilde{A}} : \mathbb{R} \to [0,\ 1]$ has the following characteristics.

(a). $\widetilde{A}$ is normal. It means that there exists an $x \in \mathbb{R}$ such that $\mu_{\widetilde{A}} = 1$.

(b). $\widetilde{A}$ is convex. It means that for every $x_1, x_2 \in \mathbb{R}$, $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x - 2) \geq \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$, $\lambda \in [0, 1]$.

(c). $\widetilde{A}$ is upper semi-continuous.

(d). $\sup(\widetilde{A})$ is bounded in $\mathbb{R}$.

2.2. Triangular Fuzzy Number

A fuzzy number $\widetilde{A}$ in $\mathbb{R}$ is said to be triangular fuzzy number if its membership function $\mu_{\widetilde{A}} : \mathbb{R} \to [0,\ 1]$ has the has the following characteristics.

$$\mu_{\widetilde{A}} = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2; \\ 1, & x = a_2; \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise.} \end{cases}$$

It is denoted by $\widetilde{A} = (a_1, a_2, a_3)$, where a_2 is Core (A), a_1 is left width and a_3 is right width. The geometric representation of triangular fuzzy number is shown in figure. The shape of the triangular fuzzy number $\widetilde{A}$ is usually in the form of triangle and hence it is called so.

The parametric form of a triangular fuzzy number is represented by $\widetilde{A} = [a_1 - a_2(1-r), a_1 + a_3(1-r)]$.

Definition 2.1 ([2]). *A positive triangular fuzzy number $\widetilde{A}$ is denoted as $\widetilde{A} = (a_1, a_2, a_3)$ where all $a_1 > 0$.*

Definition 2.2 ([2]). *A negative triangular fuzzy number $\widetilde{A}$ is denoted as $\widetilde{A} = (a_1, a_2, a_3)$ where all $a_3 < 0$.*

Definition 2.3 ([2]). *Two triangular fuzzy numbers $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ are said to be equal iff $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$.*

Definition 2.4 ([2]). *Let $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers then*

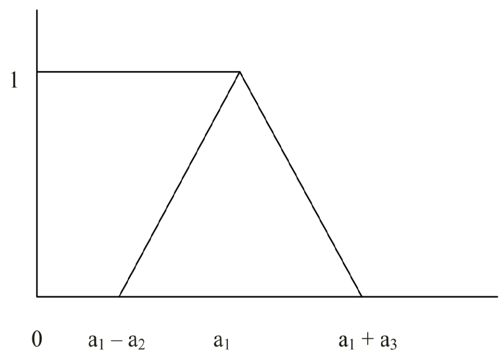

Figure 1. **Membership function of triangular fuzzy number.**

(1). $\widetilde{A} \oplus \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(2). $\widetilde{A} \ominus \widetilde{B} = (a_1 + b_3, a_2 + b_2, a_3 + b_1)$.

(3). If $\widetilde{B} = (b_1, b_2, b_3)$ *be a non-negative triangular fuzzy number then*

$$\widetilde{A} \otimes \widetilde{B} = \begin{cases} (a_1b_1, a_2b_2, a_3b_3), & \text{if } a_1 \geq 0; \\ (a_1b_3, a_2b_2, a_3b_3), & \text{if } a_1 < 0,\ a_3 \geq 0; \\ (a_1b_3, a_2b_2, a_3b_1), & \text{if } a_3 < 0. \end{cases}$$

(4). $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ *if* $k \geq 0$.

(5). $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$ *if* $k \leq 0$.

2.3. Ranking of Triangular Fuzzy Number

Several approach for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for computing the fuzzy numbers is by the use of a ranking function based on their graded means [6]. That is, for every $\widetilde{A} = (a_1, a_2, a_3) \in \mathbb{F}(\mathbb{R})$, the ranking function $\Re : \mathbb{F}(\mathbb{R}) \to \mathbb{R}$ by graded mean is defined as

$$\Re(\widetilde{A}) = \left(\frac{a_1 + 4a_2 + a_3}{6} \right). \tag{1}$$

For any two triangular fuzzy numbers $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ in $\mathbb{F}(\mathbb{R})$, we have the following comparison.

(1). $\widetilde{A} < \widetilde{B}$ if and only iff $\Re(\widetilde{A}) < \Re(\widetilde{B})$.

(2). $\widetilde{A} > \widetilde{B}$ if and only iff $\Re(\widetilde{A}) > \Re(\widetilde{B})$.

(3). $\widetilde{A} \approx \widetilde{B}$ if and only iff $\Re(\widetilde{A}) = \Re(\widetilde{B})$.

(4). $\widetilde{A} \ominus \widetilde{B}$ if and only iff $\Re(\widetilde{A}) \ominus \Re(\widetilde{B})$.

(5). If $\Re(\widetilde{A}) > 0$ then $\widetilde{A} > 0$.

(6). If $\Re(\widetilde{A}) = 0$ then $\widetilde{A} \approx 0$.

3. Fully Fuzzy Transportation Problem

Let us consider the following fully fuzzy transportation problem

$$\begin{aligned} &\text{Minimize} && \sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c_{ij}}\widetilde{x_{ij}} \\ &\text{Subject to} && \sum_{j=1}^{n}\widetilde{x_{ij}} = \widetilde{a_i} \ \text{ for } \ i = 1, 2, \ldots, m \\ &&& \sum_{i=1}^{n}\widetilde{x_{ij}} = \widetilde{b_i} \ \text{ for } \ j = 1, 2, \ldots, n \\ &&& \widetilde{x_{ij}} \geq 0 \ \forall \, i, \, j \end{aligned} \tag{2}$$

where $\widetilde{a_i}$ is the approximate availability of the product at the i^{th} source, $\widetilde{b_i}$ is the approximate demand of the product at the j^{th} destination, $\widetilde{c_{ij}}$ is the approximate cost for transporting one unit of the product from the i^{th} source to the j^{th} destination and $\widetilde{x_{ij}}$ is the approximate amount of units of the product that should be transported from the i^{th} source to j^{th} destination taken as a decision variables. If $\sum_{i=1}^{m}\widetilde{a_i} = \sum_{j=1}^{m}\widetilde{b_j}$ then fully fuzzy transportation problem is said to balanced transportation problem otherwise it is called an unbalanced fully fuzzy transportation problem. For a given basic feasible solution if we associate dual variables $\widetilde{u_i}$ and $\widetilde{v_j}$ with i^{th} row and j^{th} column of the transportation table, respectively. Then u_i and v_j must satisfy the equation.

$$\widetilde{u_i} \oplus \widetilde{v_j} = \widetilde{c_{ij}} \ \text{ for each occupied cell (i, j)}$$

This equation yield $m + n - 1$ equations in $m + n$ dual variables. Dual variables can be calculated by assign any arbitrary dual variables is $\widetilde{0}$. Once the values of u_i and v_j have been determined, evaluation in terms of opportunity cost of each unoccupied cell is done by the equation

$$\widetilde{d_{ij}} = \widetilde{c_{ij}} \ominus (\widetilde{u_i} \oplus \widetilde{v_j}) \ \text{ for each unoccupied cell (i, j).}$$

Hence the dual of the transportation problem (2) can be written as

$$\begin{aligned} &\text{Minimize} \ \sum_{i=1}^{m}\widetilde{a_i} \otimes \widetilde{u_i} \oplus \sum_{j=1}^{n}\widetilde{b_j} \otimes \widetilde{v_j} \\ &\text{subject to} \qquad \widetilde{u_i} \oplus \widetilde{v_j} \leq \widetilde{c_{ij}} \end{aligned}$$

where $\widetilde{u_i}$ and $\widetilde{v_j}$ are unrestricted for all i, j. In fuzzy basic feasible solution of a primal problem $(m + n - 1)$ variables are fuzzy basic and remaining are fuzzy non-basic variables.

4. Proposed Method

In this section new approach is proposed to solve initial fuzzy basic feasible solution for fully fuzzy transportation problem. Here cost, source destination and the variables are all taken as triangular fuzzy numbers.

4.1. Algorithm

Step 1: Construct a fully fuzzy transportation table from the given transportation problem. For unbalanced problem, we do not require to balance the transportation problem.

Step 2: Find the smallest cost cell $\widetilde{c_{ij}}$ in the fully fuzzy transportation table to make the first allocation. Allocate $\widetilde{x_{ij}} = \min\{\widetilde{a_i}, \widetilde{b_j}\}$ in the cell (i, j). In case of ties, select the cell cell where maximum allocation can be allocated. Again in case of some cost cells and allocation select the cell for which sum of fuzzy demand and supply is maximum in the original fully fuzzy transportation table. Finally if all these are same, select the cell randomly.

Step 3: Adjust the supply and demand requirement in the respective rows and columns. Then following cases arise:

Case 1: If the allocation $\widetilde{x_{ij}} = \widetilde{a_i}$, i^{th} row is to be crossed out and $\widetilde{b_j}$ is reduced to $(\widetilde{b_j} \ominus \widetilde{a_i})$. Now complete the allocation along j^{th} column by making the allocation/allocations in the smallest cost cell/cells continuously. Consider that, j^{th} column is exhausted for the allocation $\widetilde{x_{kj}}$ in the cell (k, j). Now, follow the same procedure to complete the allocation along k^{th} row and continue this process until entire rows and columns are exhausted.

Case 2: If the allocation $\widetilde{x_{ij}} = \widetilde{b_j}$, j^{th} column is to be crossed out and $\widetilde{a_i}$ is reduced to $(\widetilde{a_i} \ominus \widetilde{b_j})$. Now by the following the same procedure explained in Case 1, complete the allocation along i^{th} row and continue the process until entire rows and columns are exhausted.

Case 3: If the allocation $\widetilde{x_{ij}} = \widetilde{a_i} = \widetilde{b_j}$, find the next smallest cost cell, (i, k) from the rest of the cost cells along i^{th} row and j^{th} column. Assign a zero in the cell (i, k) and cross out i^{th} row and j^{th} column. After that complete the allocation along k^{th} column following the process described in Case 1 to complete the allocations.

Case 4: For any allocation, other than first allocation made along the row/column satisfies both the row and column. In such case find the smallest cost cell which is along the column/row and assign a zero in that cell and continue the process described in above cases to complete the allocation along the column/row and also to complete entire allocations.

Step 4: Finally calculate the total fuzzy transportation cost which is the sum of the product of cost and corresponding allocated value.

5. Numerical Examples

Example 5.1. *A company manufactures motor cars and it has three factories S_1, S_2, S_3, S_4, S_5 and S_6 whose weekly production capacities are (100, 120, 140), (65, 80, 95), (40, 50, 60), (80, 90, 100), (80, 100, 120) and (55, 60, 65) pieces of cars respectively. The company supplies motor cars to its four showrooms located at D_1, D_2, D_3, D_4, D_5 and D_6 whose weekly demands are (55, 75, 95), (80, 85, 90), (120, 140, 160), (30, 40, 50), (90, 95, 100) and (55, 65, 75) pieces of cars respectively. The transportation costs per piece of motor cars are given in the transportation Table 1. Find out the schedule of shifting of motor cars from factories to showrooms with minimum cost.*

Factories	Destinations						Production Capacity
	D_1	D_2	D_3	D_4	D_5	D_6	
S_1	(10, 12, 14)	(3, 4, 5)	(11, 13, 15)	(16, 18, 20)	(8, 9, 10)	(1, 2, 3)	(100, 120, 140)
S_2	(8, 9, 10)	(14, 16, 18)	(9, 10, 11)	(6, 7, 8)	(13, 15, 16)	(10, 11, 12)	(65, 80, 95)
S_3	(3, 4, 5)	(8, 9, 10)	(9, 10, 11)	(6, 8, 10)	(8, 9, 10)	(6, 7, 8)	(40, 50, 60)
S_4	(8, 9, 10)	(2, 3, 4)	(10, 12, 14)	(3, 6, 9)	(3, 4, 5)	(3, 5, 7)	(80, 90, 100)
S_5	(6, 7, 8)	(10, 11, 12)	(3, 5, 7)	(16, 18, 20)	(1, 2, 3)	(6, 7, 8)	(80, 100, 120)
S_6	(14, 16, 18)	(6, 8, 10)	(3, 4, 5)	(3, 5, 7)	(0, 1, 2)	(9, 10, 11)	(55, 60, 65)
Demand	(55, 75, 95)	(80, 85, 90)	(120, 140, 160)	(30, 40, 50)	(90, 95, 100)	(55, 65, 75)	

Table 1. **Data of the Example 5.1**

Solution: Allocation of various cells in the allocation Table 1 for Example 5.1 is shown in allocation Table 2.

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	(10, 12, 14)	(3, 4, 5)	(11, 13, 15)	(16, 18, 20)	(8, 9, 10)	(1, 2, 3)	(100, 120, 140)
S_2	(8, 9, 10)	(14, 16, 18)	(9, 10, 11)	(6, 7, 8)	(13, 15, 16)	(10, 11, 12)	(65, 80, 95)
S_3	(3, 4, 5)	(8, 9, 10)	(9, 10, 11)	(6, 8, 10)	(8, 9, 10)	(6, 7, 8)	(40, 50, 60)
S_4	(8, 9, 10)	(2, 3, 4)	(10, 12, 14)	(3, 6, 9)	(3, 4, 5)	(3, 5, 7)	(80, 90, 100)
S_5	(6, 7, 8)	(10, 11, 12)	(3, 5, 7)	(16, 18, 20)	(1, 2, 3)	(6, 7, 8)	(80, 100, 120)
S_6	(14, 16, 18)	(6, 8, 10)	(3, 4, 5)	(3, 5, 7)	(0, 1, 2)	(9, 10, 11)	(55, 60, 65)
Demand	(55, 75, 95)	(80, 85, 90)	(120, 140, 160)	(30, 40, 50)	(90, 95, 100)	(55, 65, 75)	

Table 2. **Allocation of various cells are in the allocation table.**

Convert the given fuzzy problem into a crisp value problem by using (1).

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	12	4	13	18	9	2	120
S_2	9	16	10	7	15	11	80
S_3	4	9	10	8	9	7	50
S_4	9	3	12	6	4	5	90
S_5	7	11	5	18	2	7	100
S_6	16	8	4	5	1	10	60
Demand	75	85	140	40	95	65	

Table 3. **Crisp data of the Example 5.1**

Source	D_1	D_2	D_3	D_4	D_5	D_6	Supply
S_1	25 12	30 4	13	18	9	65 2	120
S_2	9	16	75 10	5 7	15	11	80
S_3	50 4	9	10	8	9	7	50
S_4	9	55 3	12	35 6	4	5	90
S_5	7	11	65 5	18	35 2	7	100
S_6	16	8	4	5	60 1	10	60
Demand	75	85	140	40	95	65	

Table 4. **Initial basic feasible solution according to Proposed Method**

Hence the total transportation cost is, $60\times1+35\times2+65\times5+75\times10+5\times7+35\times6+55\times3+30\times4+65\times2+25\times12+50\times4 = 2365$.

6. Conclusion

In this paper, a simple yet effective method was introduced to solve fully fuzzy transportation problem. This method can be used for all kinds of fuzzy transportation problem, whether triangular and trapezoidal fuzzy numbers with normal or abnormal data. The new method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether to maximize or minimize an objective function. The Proposed method was illustrated by the numerical examples with a triangular fuzzy number.

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