Nano Regular Generalized Star $b$-continuous Functions in Nano Topological Spaces

A. Maheswari$^1$ and M. Sheik John$^1$

1 Department of Mathematics, NGM College, Pollachi, Coimbatore, Tamilnadu, India.

Abstract: In this paper, we introduce and investigate the notions of nano regular generalized star $b$ continuous functions in terms of nano regular generalized star $b$ closed sets in nano topological spaces.

Keywords: Nano topology, $N_r$-continuous, $N_g^*$-continuous, $Nrg^*$-continuous, $Nrg^*b$-continuous, $Nrg^*b$-open and closed function.

1. Introduction

The concept of nano topology was introduced by Lellis Thivagar [7] in the year 2013, which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined a nano continuous functions, nano open mappings, nano closed mappings and nano homeomorphisms and their representations in terms of nano closure and nano interior. In this paper we introduced a new class of nano generalized closed sets and nano regular generalized star $b$ continuous, nano regular generalized star $b$ open and closed function in nano topological spaces.

2. Preliminaries

Definition 2.1 ([7]). Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation. Then $U$ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and it is denoted by $L_R(X)$. i.e., $L_R(X) = \bigcup_{x \in U} \{ R(x) : R(x) \subseteq X \}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.

(2) The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $U_R(X)$. i.e., $U_R(X) = \bigcup_{x \in U} \{ R(x) : R(x) \cap X \neq \emptyset \}$.

(3) The boundary region of $X$ with respect to $R$ is the set of all objects, which can be classified neither as $X$ nor as not-$X$ with respect to $R$ and it is denoted by $B_R(X)$. i.e., $B_R(X) = U_R(X) - L_R(X)$.

* E-mail: vmahes999@gmail.com
Property 2.2 ([7]). If \( (U, R) \) is an approximation space and \( X, Y \subseteq U \), then

1. \( L_R(X) \subseteq X \subseteq U_R(X) \)
2. \( L_R(\phi) = U_R(\phi) = (\phi) \) and \( L_R(U) = U_R(U) = U \)
3. \( U_R(X \cup Y) = U_R(X) \cup U_R(Y) \)
4. \( U_R(X \cap Y) = U_R(X) \cap U_R(Y) \)
5. \( L_R(X \cup Y) = L_R(X) \cup L_R(Y) \)
6. \( L_R(X \cap Y) = L_R(X) \cap L_R(Y) \)
7. \( L_R(X) \subseteq L_R(Y) \) and \( U_R(X) \subseteq U_R(Y) \) whenever \( X \subseteq Y \)
8. \( U_R(X^c) = [L_R(X)]^c \) and \( L_R(X^c) = [U_R(X)]^c \)
9. \( U_R U_R(X) = L_R U_R(X) = U_R(X) \)
10. \( L_R L_R(X) = U_R L_R(X) = L_R(X) \).

Definition 2.3 ([7]). Let \( U \) be the universe, \( R \) be an equivalence relation on \( U \) and \( \tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\} \), where \( X \subseteq U \). Then by Property 2.1, \( \tau_R(X) \) satisfies the following axioms:

1. \( U \) and \( \phi \in \tau_R(X) \).
2. The union of the elements of any subcollection of \( \tau_R(X) \) is in \( \tau_R(X) \).
3. The intersection of the elements of any finite subcollection of \( \tau_R(X) \) is in \( \tau_R(X) \). i.e., \( \tau_R(X) \) is a topological space with nano topology on \( U \) with respect to \( X \). We call \( (U, \tau_R(X)) \) as the nano topological space. The elements of \( \tau_R(X) \) are called as nano open sets.

Remark 2.4 ([8]). If \( \tau_R(X) \) is the nano topology on \( U \) with respect to \( X \), then the set \( B = \{U, L_R(X), B_R(X)\} \) is the basis for \( \tau_R(X) \).

Definition 2.5 ([8]). If \( (U, \tau_R(X)) \) is a nano topological space with respect to \( X \) where \( X \subseteq U \) and if \( A \subseteq U \), Then the nano interior of \( A \) is defined as the union of all nano open subsets of \( A \) and it is denoted by \( NInt(A) \). i.e., \( NInt(A) \) is the largest nano open subset of \( A \). The nano closure of \( A \) is defined as the intersection of all nano closed sets containing \( A \) and it is denoted by \( Ncl(A) \). i.e., \( Ncl(A) \) is the smallest nano closed set containing \( A \).

Definition 2.6 ([8]). Let \( (U, \tau_R(X)) \) be a nano topological space and \( A \subseteq U \). Then \( A \) is said to be

1. Nano semi open if \( A \subseteq Ncl(NInt(A)) \)
2. Nano pre open if \( A \subseteq NInt(Ncl(A)) \)
3. Nano \( \alpha \)-open if \( A \subseteq NInt(Ncl(NInt(A))) \)
4. Nano regular open if \( A = NInt(Ncl(A)) \) \( NSO(U, X), NPO(U, X), NaO(U, X) \) and \( NRO(U, X) \) respectively, denote the families of all Nano semi open, nano pre open and nano \( \alpha \)-open and nano regular open subsets of \( U \).
Let \((U, \tau_R(X))\) be a nano topological space and \(A \subseteq U\), \(A\) is said to be nano semi-closed, nano pre closed, nano \(\alpha\)-closed and nano regular closed if its complement is respectively nano semi open, nano pre open, nano \(\alpha\)-open and nano regular open.

**Definition 2.7** ([4]). A subset \(A\) of \((U, \tau_R(X))\) is called

1. Nano \(b\)-closed set (briefly \(Nb\)-closed) if \(Ncl(NInt(A)) \cap NInt(Ncl(A)) \subseteq A\).
2. Nano generalized closed set (briefly \(Ng\)-closed) if \(Ncl(A) \subseteq V\) whenever \(A \subseteq V\) and \(V\) is nano open in \((U, \tau_R(X))\).
3. Nano generalized \(b\)-closed set (briefly \(Ngb\)-closed) if \(Nbccl(A) \subseteq V\) whenever \(A \subseteq V\) and \(V\) is nano open in \((U, \tau_R(X))\).
4. Nano regular generalized \(b\)-closed set (briefly \(Nrgb\)-closed) if \(Nbccl(A) \subseteq V\) whenever \(A \subseteq V\) and \(V\) is nano regular open in \((U, \tau_R(X))\).

**Definition 2.8** ([12]). Let \((U, \tau_R(X))\) be a nano topological space. A subset \(A\) of \((U, \tau_R(X))\) is called a nano generalized star closed set (briefly \(Ng^*-\) closed), if \(Ncl(A) \subseteq V\) whenever \(A \subseteq V\) and \(V\) is nano \(g\)-open.

**Definition 2.9** ([13]). Let \((U, \tau_R(X))\) be a nano topological space and \(A \subseteq U\). Then \(A\) is said to be nano regular generalized star closed set (briefly \(Nrg^*\)-closed) if \(Ncl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is nano regular open in \((U, \tau_R(X))\). Let \((U, \tau_R(X))\) be a nano topological space and \(A \subseteq U\), \(A\) is said to be nano regular generalized star open set (briefly \(Nrg^*\)-open) if its complement is \(Nrg^*\)-closed.

**Definition 2.10** ([13]). Let \((U, \tau_R(X))\) be a nano topological space and \(A \subseteq U\). Then \(A\) is said to be nano regular generalized star \(b\)-closed set (briefly \(Nrg^*b\)-closed) if \(Nbccl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(Nrg^*\)-open in \((U, \tau_R(X))\). \(Nrg^*O(U, X)\) denotes the family of all nano \(rg^*\)-open subsets of \(U\).

**Definition 2.11** ([7]). Let \((U, \tau_R(X))\) and \((V, \tau_R'(Y))\) be two nano topological spaces. Then a mapping \(f : (U, \tau_R(X)) \rightarrow (V, \tau_R'(Y))\) is nano continuous on \(U\) if the inverse image of every nano open set in \(V\) is nano open in \(U\).

## 3. Nano Regular Generalized Star \(b\)-Continuous in Nano Topological Spaces

**Definition 3.1.** Let \((U, \tau_R(X))\) and \((V, \tau_R'(Y))\) be two nano topological spaces. Then a mapping \(f : (U, \tau_R(X)) \rightarrow (V, \tau_R'(Y))\) is nano regular generalized star continuous (briefly \(Nrg^*\)-continuous) if \(f^{-1}(S)\) is \(Nrg^*\)-open (resp. \(Nrg^*\)-closed) in \((U, \tau_R(X))\) for every nano open set (resp. nano closed set) \(S\) in \((V, \tau_R'(Y))\).

**Definition 3.2.** Let \((U, \tau_R(X))\) and \((V, \tau_R'(Y))\) be two nano topological spaces. Then a mapping \(f : (U, \tau_R(X)) \rightarrow (V, \tau_R'(Y))\) is nano regular generalized star \(b\)-continuous (briefly \(Nrg^*b\)-continuous) if \(f^{-1}(S)\) is \(Nrg^*b\)-open (resp. \(Nrg^*b\)-closed) in \((U, \tau_R(X))\) for every nano open set (resp. nano closed set) \(S\) in \((V, \tau_R'(Y))\).

**Theorem 3.3.** Let \(U\) and \(V\) are any two nano topological spaces. If \(f : (U, \tau_R(X)) \rightarrow (V, \tau_R'(Y))\) \(f\) is nano continuous function, then \(f\) is \(Nrg^*b\)-continuous, but not conversely.

**Proof.** Let \(S\) be any nano closed set in \((V, \tau_R'(Y))\). Since [13] every nano closed set is \(Nrg^*b\)-closed, we have \(f^{-1}(S)\) is \(Nrg^*b\)-closed in \((U, \tau_R(X))\). Therefore, \(f\) is \(Nrg^*b\) continuous.

The converse of the theorem need not be true as seen from the following example.
Example 3.4. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{w\}, \{y, z\}\}$ and $X = \{x, y\}$. Then $\tau_R(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$. Define a mapping $f : U \rightarrow V$ as $f(a) = z; f(b) = w; f(c) = x; f(d) = y$. Then $f$ is Nrg\(^{-}\)-continuous but not nano continuous, as the inverse image of a nano closed set $\{a, b, c\}$ in $V$ is $\{x, y, z\}$ which is not nano closed in $U$.

Theorem 3.5. Let $U$ and $V$ are any two nano topological spaces. If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ $f$ is Nrg\(^{-}\)-continuous function, then $f$ is Ngb-continuous, but not conversely.

Proof. Let $S$ be any nano closed set in $(V, \tau_R(Y))$. Then $f^{-1}(S)$ is Nrg\(^{-}\)-closed in $(U, \tau_R(X))$ as $f$ is Nrg\(^{-}\)-continuous. Since [13] every Nrg\(^{-}\) closed set is Ngb-closed. we have, $f^{-1}(S)$ is Ngb-closed in $(U, \tau_R(X))$. Therefore $f$ is Ngb continuous.

The converse of the theorem need not be true as seen from the following example.

Example 3.6. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{w\}, \{y, z\}\}$ and $X = \{x, y\}$. Then $\tau_R(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$. Define a mapping $f : U \rightarrow V$ as $f(a) = y; f(b) = x; f(c) = z; f(d) = w$. Then $f$ is Ngb-continuous but not Nrg\(^{-}\)-continuous, as the inverse image of a Nrg\(^{-}\)-closed set $\{a, b, c\}$ in $V$ is $\{x, y, z\}$ which is not Nrg\(^{-}\) closed in $U$.

Theorem 3.7. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano g-continuous function, then $f$ is Nrg\(^{-}\)-continuous but not conversely.

Proof. Let $f$ be Ng-continuous function and $S$ be an nano closed set in $(V, \tau_R(Y))$. Then $f^{-1}(S)$ is Nrg\(^{-}\)-closed in $(U, \tau_R(X))$ as $f$ is Nrg\(^{-}\)-continuous. Since [13] every Nrg\(^{-}\) closed set is Ngb-closed, we have, $f^{-1}(S)$ is Ngb-closed in $(U, \tau_R(X))$. Therefore $f$ is Ngb-continuous.

Theorem 3.8. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano g\(^{-}\)-continuous function, then $f$ is Nrg\(^{-}\) continuous but not conversely.

Proof. Let $f$ be Ng\(^{-}\)-continuous function and $S$ be an nano closed set in $(V, \tau_R(Y))$. Then $f^{-1}(S)$ is Nrg\(^{-}\)-closed in $(U, \tau_R(X))$. Since [13] every Nrg\(^{-}\) closed set is Ngb-closed, $f^{-1}(S)$ is Nrg\(^{-}\)closed set. Hence, $f$ is Nrg\(^{-}\)continuous.

Theorem 3.9. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano regular continuous function, then $f$ is Nrg\(^{-}\)continuous but not conversely.

Proof. Let $S$ be any nano closed set in $(V, \tau_R(Y))$. Then $f^{-1}(S)$ is Nr-closed in $(U, \tau_R(X))$ as $f$ is nano regular-continuous. Since [13] every nano r-closed set is Nrg\(^{-}\)closed. we have, $f^{-1}(S)$ is Nrg\(^{-}\)closed in $(U, \tau_R(X))$. Therefore $f$ is Nrg\(^{-}\)continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.10. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{b, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R = \{\{x\}, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define a mapping $f : U \rightarrow V$ as $f(a) = x; f(b) = y; f(c) = z; f(d) = w$. Then $f$ is Nrg\(^{-}\)-continuous but not nano regular continuous.

Theorem 3.11. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a nano continuous function and $U$ and $V$ are any two nano topological spaces. If $f$ is nano g\(^{-}\)-continuous function, then $f$ is Ng continuous but not conversely.

Proof. Let $f$ be Ng\(^{-}\)-continuous function and $S$ be an nano closed set in $(V, \tau_R(Y))$. Then $f^{-1}(S)$ is Ng\(^{-}\)-closed in $(U, \tau_R(X))$. Since [13] every Ng\(^{-}\) closed set is Ng closed, $f^{-1}(S)$ is Ng-closed set. Hence, $f$ is Ng-continuous.
**Theorem 3.12.** A function $f : (U, \tau_U(X)) \to (V, \tau_V(Y))$ is nano $rg^*b$-continuous iff the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed

**Proof.** Let $f$ be nano $rg^*b$-continuous and $F$ be nano closed in $V$. i.e., $V - F$ is nano open in $V$. Since $f$ is nano $rg^*b$-continuous, $f^{-1}(V - F)$ is nano $rg^*b$-open in $U$. i.e., $U - f^{-1}(V - F)$ is nano $rg^*b$-closed in $U$. Thus the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed in $U$, if $f$ is nano $rg^*b$-continuous on $U$.

Conversely, let the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed in $U$. Let $G$ be nano open in $V$, then $V - G$ is nano closed in $V$. Then $f^{-1}(V - G)$ is nano $rg^*b$-closed in $U$, i.e., $U - f^{-1}(G)$ is nano $rg^*b$-closed in $U$. Therefore $f^{-1}(G)$ is nano $rg^*b$-open in $U$. By definition $f$ is nano $rg^*b$-continuous. \(\square\)

**Theorem 3.13.** Let $(U, \tau_U(X))$ and $(V, \tau_V(Y))$ be nano topological spaces and a mapping $f : (U, \tau_U(X)) \to (V, \tau_V(Y))$. Then,

1. Every nano semi continuous function is nano $rg^*b$-continuous.
2. Every nano pre continuous function is nano $rg^*b$-continuous.
3. Every nano $\alpha$ continuous function is nano $rg^*b$-continuous.
4. Every nano regular continuous function is nano $rg^*b$-continuous.

**Proof.**

1. Let $f : (U, \tau_U(X)) \to (V, \tau_V(Y))$ be nano continuous and $S$ be nano semi closed in $V$. Then $f^{-1}(S)$ is nano semi closed in $U$. Since every nano semi closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano semi closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

2. Let $f : (U, \tau_U(X)) \to (V, \tau_V(Y))$ be nano continuous and $S$ be nano pre closed in $V$. Then $f^{-1}(S)$ is nano pre closed in $U$. Since every nano pre closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano pre closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

3. Let $f : (U, \tau_U(X)) \to (V, \tau_V(Y))$ be nano continuous and $S$ be nano $\alpha$ closed in $V$. Then $f^{-1}(S)$ is nano $\alpha$ closed in $U$. Since every nano $\alpha$ closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano $\alpha$ closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous.

4. Let $f : (U, \tau_U(X)) \to (V, \tau_V(Y))$ be nano continuous and $S$ be nano regular closed in $V$. Then $f^{-1}(S)$ is nano regular closed in $U$. Since every nano regular closed set is nano $rg^*b$-closed, $f^{-1}(S)$ is nano $rg^*b$-closed in $U$. Thus, inverse image of every nano regular closed set is nano $rg^*b$-closed. Therefore, $f$ is nano $rg^*b$-continuous. \(\square\)

**Remark 3.14.** The converse of the above theorem need not be true which can be seen from the following examples.

**Example 3.15.** Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \varphi, \{a\}, \{b, c\}, \{a, c\}\}$ and $[\tau_R(X)]^c = \{U, \varphi, \{b, c, d\}, \{d\}, \{a, d\}\}$. Then $V = \{x, y, z, w\}$, with $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$, and $Y = \{x, y\} \subseteq V$. Then, $\tau_{R'}(Y) = \{V, \varphi, \{x\}, \{y, z\}, \{x, y, z\}\}$ and $[\tau_{R'}(Y)]^c = \{V, \varphi, \{y, z, w\}, \{x, w\}, \{w\}\}$.
Define : $f : U \rightarrow V$ as $f(a) = \{z\}, f(b) = \{y\}, f(c) = \{x\}, f(d) = \{w\}$. Then $f^{-1}(\{y, z, w\}) = \{a, b, d\}, f^{-1}(\{x, w\}) = \{c, d\}, f^{-1}(\{w\}) = \{d\}$. Then $f$ is nano $rg^*b$-continuous since the inverse image of every nano closed set in $V$ is nano $rg^*b$-closed in $U$. But

(1). $f$ is not semi continuous, since $f^{-1}(\{y, z, w\}) = \{a, b, d\}$ and $f^{-1}(\{x, w\}) = \{c, d\}$ are not nano semi closed in $U$ where as $\{y, z, w\}$ and $\{x, w\}$ are nano closed in $V$.

(2). $f$ is not pre continuous, since $f^{-1}(\{y, z\}) = \{a, b\}$ and $f^{-1}(\{w\}) = \{d\}$ are not nano pre closed in $U$ where as $\{y, z\}$ and $\{x, w\}$ are nano closed in $V$.

(3). $f$ is not $\alpha$ continuous, since $f^{-1}(\{y, z, w\}) = \{a, b, d\}$ and $f^{-1}(\{y, z\}) = \{a, b\}$ are not nano $\alpha$ closed in $U$ where as $\{y, z, w\}$ and $\{y, z\}$ are nano closed in $V$.

(4). $f$ is not regular continuous, since $f^{-1}(\{y, z, w\}) = \{a, b, d\}$ and $f^{-1}(\{y, z\}) = \{a, b\}$ are not nano regular closed in $U$ where as $\{y, z, w\}$ and $\{y, z\}$ are nano closed in $V$.

4. Nano Regular Generalized Star $b$-Open and Closed Functions

**Definition 4.1.** A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is said to be $Nrg^*b$-open (resp.$Nrg^*b$-closed) function if the image of every nano open (resp.nano closed) set in $(U, \tau_R(X))$ is $Nrg^*b$-open (resp.$Nrg^*b$-closed) in $(V, \tau_R(Y))$.

**Theorem 4.2.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a function, if $f$ is an nano open function, then $f$ is $Nrg^*b$-open function.

**Proof.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a nano open map and $S$ be a nano open set in $U$. Then $f(S)$ is nano open and hence $f(S)$ is $Nrg^*b$-open in $V$. Thus $f$ is $Nrg^*b$-open.

**Theorem 4.3.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a function, if $f$ is an nano closed function, then $f$ is $Nrg^*b$-closed function.

**Proof.** Let $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be a nano closed map and $S$ be a nano closed set in $U$. Then $f(S)$ is nano closed and hence $f(S)$ is $Nrg^*b$-closed in $V$. Thus $f$ is $Nrg^*b$-closed.

**Theorem 4.4.** A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is $Nrg^*b$-closed if and only if for each subset $B$ of $V$ and for each nano open set $G$ containing $f^{-1}(B)$ there exists a $Nrg^*b$-open set $F$ of $V$ such that $B \subseteq F$ and $f^{-1}(B) \subseteq G$.

**Proof.**

**Necessity :** Let $G$ be a nano open subset of $(U, \tau_R(X))$ and $B$ be a subset of $V$ such that $f^{-1}(B) \subseteq G$. Define, $F = V - f(U - G)$. Since, $f$ is $Nrg^*b$-closed, Then $F$ is $Nrg^*b$-open set containing $B$ such that $f^{-1}(F) \subseteq G$.

**Sufficiency :** Let $E$ be a nano closed subset of $(U, \tau_R(X))$. Then $f^{-1}(V - f(E)) \subseteq (U - E)$ and $(U - E)$ is nano open. By hypothesis, there is a $Nrg^*b$-open-set $F$ of $(V, \tau_R(Y))$ such that $V - f(E) \subseteq F$ and $f^{-1}(B) \subseteq U - E$. Therefore $E \subseteq U - f^{-1}(F)$. Hence, $V - F \subseteq f(E) \subseteq f(U - f^{-1}(F)) \subseteq V - F$. Which implies that $f(E) = V - F$ and hence $f(E)$ is $Nrg^*b$-closed in $(V, \tau_R(Y))$. Therefore, $f$ is $Nrg^*b$-closed function.

**Theorem 4.5.** If a function $f : U \rightarrow V$ is nano closed and a map $g : V \rightarrow W$ is $Nrg^*b$-closed then their composition $gof : U \rightarrow W$ is $Nrg^*b$-closed.
Proof. Let $H$ be a nano closed set is $U$. Then $f(H)$ is nano closed in $V$ and $(gof)(H) = g(f(H))$ is $Nrg^*b$-closed, as $g$ is $Nrg^*b$-closed. Hence, $gof$ is $Nrg^*b$-closed.

References


