Supra $\alpha$-Locally Closed Sets and Supra $\alpha$-Locally Continuous Functions in Supra Topological Spaces

Research Article

S.Dayana Mary$^1*$ and N.Nagaveni$^1$

1 Department of Mathematics, Coimbatore Institute of Technology, Tamilnadu, India.

Abstract: The aim of this paper is to introduce a new type of sets called supra $\alpha$-locally closed sets and new type of functions called supra $\alpha$-locally continuous functions. Furthermore, we obtain some of their properties.

Keywords: S-$\alpha$-LC sets, S-$\alpha$-LC* sets, S-$\alpha$-LC** sets, S-$\alpha$-L-continuous and S-$\alpha$-L-irresolute.

1. Introduction

In 1965, $\alpha$-sets and $\beta$-sets were defined and studied in topological spaces by Njastad [7]. In topological spaces, Gnanambalet. al. [3] introduced $\alpha$-locally closed sets and discussed its properties. Gnanambal and Balachandran [4] defined the notion of $\beta$-locally closed sets in topological spaces. The supra topological spaces, S-continuous functions and S*-continuous functions were introduced by Mashhouret. al. [6]. In 2008, Devi et. al. [2] defined and investigated the concept of supra $\alpha$-open sets and so-continuous maps in supra topological spaces. Ravi et.al. [8] introduced and studied supra $\beta$-open sets and supra $\beta$-continuous maps. Dayana Mary and Nagaveni [1] defined and discussed supra $\beta$-locally closed sets and their functions. In this paper we introduce the concept of supra $\alpha$-locally closed sets and study its basic properties. Also we introduce the concepts of supra $\alpha$-locally continuous maps and investigate several properties for these classes of maps.

2. Preliminaries

Throughout this paper, $(X, \tau)$, $(Y, \sigma)$ and $(Z, \eta)$ (or simply, X, Y and Z) represent topological space on which no separation axioms are assumed, unless explicitly stated. For a subset A of $(X, \tau)$, cl(A) and int(A) represent the closure of A with respect to $\tau$ and the interior of A with respect to $\tau$, respectively. Let P(X) be the power set of X. The complement of A is denoted by $X-A$ or $A^c$. Now we recall some Definitions and results which are useful in the sequel.

**Definition 2.1** ([6, 9]). Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ is said to a supra topology on X if $X \in \mu$ and $\mu$ is closed under arbitrary unions. The pair $(X, \mu)$ is called a supra topological space. The elements of $\mu$ are said to be supra open in $(X, \mu)$. Complement of supra open sets are called supra closed sets.

**Definition 2.2** ([9]). Let $A$ be a subset $(X, \mu)$. Then

---

* E-mail: dayanamaryv@gmail.com
Definition 2.3 ([6]). A set $(X, \tau)$ be a topological space and $\mu$ be a supra topology of $X$. We call $\mu$ is a supra topology associated with $\tau$ if $\tau \subseteq \mu$.

Definition 2.4 ([2]). Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\tau \subseteq \mu$. A function $f : (X, \tau) \to (Y, \sigma)$ is called supra continuous, if the inverse image of each open set of $Y$ is a supra open set in $X$.

Definition 2.5 ([6, 9]). Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\mu$ and $\lambda$ be supra topologies associated with $\tau$ and $\sigma$ respectively. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be supra irresolute, if $f^{-1}(A)$ is a supra open set of $X$ for every supra open set $A$ in $Y$.

Definition 2.6 ([2]). Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $\alpha$-open if $A \subseteq \text{int}^{\alpha}(\text{cl}^{\alpha}(A))$. The complement of supra $\alpha$-open set is called supra $\alpha$-closed. The class of all supra $\alpha$-open sets is denoted by $S_{\alpha}O(X)$.

Definition 2.7 ([2]). Let $A$ be a subset $(X, \mu)$. Then

(1). The supra $\alpha$-closure of a set $A$ is, denoted by $\text{cl}^{\alpha}_\lambda (A)$, defined as $\text{cl}^{\alpha}_\lambda (A) = \cap \{B : B$ is a supra $\alpha$-closed and $A \subseteq B\}$.

(2). The supra $\alpha$-interior of a set $A$ is, denoted by $\text{int}^{\alpha}_\lambda (A)$, defined as $\text{int}^{\alpha}_\lambda (A) = \cup \{B : B$ is a supra $\alpha$-open and $B \subseteq A\}$.

Definition 2.8 ([8]). Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $\beta$-open if $A \subseteq \text{cl}^{\beta}(\text{int}^{\beta}(A))$. The complement of supra $\beta$-open set is called supra $\beta$-closed. The class of all supra $\beta$-open sets is denoted by $S_{\beta}O(X)$.

Definition 2.9 ([8]). Let $A$ be a subset $(X, \mu)$. Then

(1). The supra $\beta$-closure of a set $A$ is, denoted by $\text{cl}^{\beta}_\lambda (A)$, defined as $\text{cl}^{\beta}_\lambda (A) = \cap \{B : B$ is a supra $\beta$-closed and $A \subseteq B\}$.

(2). The supra $\beta$-interior of a set $A$ is, denoted by $\text{int}^{\beta}_\lambda (A)$, defined as $\text{int}^{\beta}_\lambda (A) = \cup \{B : B$ is a supra $\beta$-open and $B \subseteq A\}$.

Definition 2.10 ([1]). Let $(X, \mu)$ be a supra topological space. A subset $A$ of $(X, \mu)$ is called supra $\beta$-locally closed set (briefly supra $\beta$-LC set), if $A = U \cap V$, where $U$ is supra $\beta$-open in $(X, \mu)$ and $V$ is supra $\beta$-closed in $(X, \mu)$. The collection of all supra $\beta$-locally closed sets of $X$ will be denoted by $S_{\beta}LC(X)$.

Definition 2.11 ([1]). Let $(X, \mu)$ be a supra topological space. A subset $A$ of $(X, \mu)$ is called supra $\beta$-dense, if $\text{cl}^{\beta}_\mu (A) = X$.

Definition 2.12 ([1]). A supra topological space $(X, \mu)$ is called supra $\beta$-submaximal, if every supra dense subset is supra $\beta$-open in $X$.

3. Supra $\alpha$-Locally Closed Sets

In this section, we introduce the notions of supra $\alpha$-locally closed sets and discuss some of their properties.

Definition 3.1. Let $(X, \mu)$ be a supra topological space. A subset $A$ of $(X, \mu)$ is called supra $\alpha$-locally closed set (briefly supra $\alpha$-LC set), if $A = U \cap V$, where $U$ is supra $\alpha$-open in $(X, \mu)$ and $V$ is supra $\alpha$-closed in $(X, \mu)$. The collection of all supra $\alpha$-locally closed sets of $X$ will be denoted by $S_{\alpha}LC(X)$.

Remark 3.2. Every supra $\alpha$-closed set (resp. supra $\alpha$-open set) is $S_{\alpha}LC$. 
Definition 3.3. Let \((X, \mu)\) be a supra topological space. The collection of all subsets \(A\) in \((X, \mu)\) given by \(A = U \cap V\), where \(U\) is a supra \(\alpha\)-open set and \(V\) is a supra \(\alpha\)-closed set of \((X, \mu)\), is denoted by \(S-\alpha-LC*(X, \mu)\).

Definition 3.4. Let \((X, \mu)\) be a supra topological space. The collection of all subsets \(A\) in \((X, \mu)\) given by \(A = U \cap V\), where \(U\) is a supra open set and \(V\) is a supra \(\alpha\)-closed set of \((X, \mu)\), is denoted by \(S-\alpha-LC**(X, \mu)\).

Definition 3.5. Let \(A, B \subseteq (X, \mu)\). Then \(A\) and \(B\) are said to be supra \(\alpha\)-separated if \(A \cap \text{cl}^\alpha_b(B) = B \cap \text{cl}^\alpha_a(A) = \phi\).

Theorem 3.6. Let \(A\) be a subset of \((X, \mu)\). If \(A \in S-\alpha-LC*(X, \mu)\), then \(A\) is \(S-\alpha-LC\).

Proof. Let \(A \in S-\alpha-LC*(X, \mu)\), then \(A = U \cap V\), where \(U\) is supra \(\alpha\)-open set and \(V\) is supra closed. Since every supra closed set is supra \(\alpha\)-closed, \(A \in S-\alpha-LC(X, \mu)\).

Theorem 3.7. Let \(A\) be a subset of \((X, \mu)\). If \(A \in S-\alpha-LC**(X, \mu)\), then \(A\) is \(S-\alpha-LC\).

Proof. The proof follows from the fact that, every supra open set is supra \(\alpha\)-open set.

Example 3.8. Let \(X = \{a, b, c, d\}\) and \(\mu = \{\phi, X, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}\). Then \(S-\alpha-LC(X, \mu) = S-\alpha-LC*(X, \mu) = P(X)-\{\{a, b\}, \{c, d\}\}\). \(S-\alpha-LC**(X, \mu) = P(X)-\{\{a, b\}, \{c, d\}, \{a, c, d\}\}\).

Theorem 3.9. For a subset \(A\) of \((X, \mu)\), the following are equivalent:

(i) \(A \in S-\alpha-LC*(X, \mu)\).

(ii) \(A = U \cap \text{cl}^\mu(A)\), for some supra \(\alpha\)-open set \(U\).

(iii) \(\text{cl}^\mu(A) - A\) is supra \(\alpha\)-closed.

(iv) \(A \cup [X-\text{cl}^\mu(A)]\) is supra \(\alpha\)-open.

Proof. (i) \(\Rightarrow\) (ii): Given \(A \in S-\alpha-LC*(X, \mu)\). Then there exist a supra \(\alpha\)-open subset \(U\) and a supra closed subset \(V\) such that \(A = U \cap V\). Since \(A \subset U\) and \(A \subset \text{cl}^\mu(A)\), \(A \subset U \cap \text{cl}^\mu(A)\).

Conversely, \(\text{cl}^\mu(A) \subset V\) and hence \(A = U \cap V \supset A \cap \text{cl}^\mu(A)\). Therefore, \(A = U \cap \text{cl}^\mu(A)\).

(ii) \(\Rightarrow\) (i): Let \(A = U \cap \text{cl}^\mu(A)\), for some supra \(\alpha\)-open set \(U\). Then, \(\text{cl}^\mu(A)\) is supra closed and hence \(A = U \cap \text{cl}^\mu(A)\in S-\alpha-LC*(X, \mu)\).

(iii) \(\Rightarrow\) (iii): Let \(A = U \cap \text{cl}^\mu(A)\), for some supra \(\alpha\)-open set \(U\). Then \(A \in S-\alpha-LC*(X, \mu)\). This implies \(U\) is supra \(\alpha\)-open and \(\text{cl}^\mu(A)\) is supra closed. Therefore, \(\text{cl}^\mu(A) - A\) is supra \(\alpha\)-closed.

(iii) \(\Rightarrow\) (ii): Let \(U = X - [\text{cl}^\mu(A) - A]\). By (iii), \(U\) is supra \(\alpha\)-open in \(X\). Then \(A = U \cap \text{cl}^\mu(A)\) holds.

(iii) \(\Rightarrow\) (iv): Let \(P = \text{cl}^\mu(A) - A\) be supra \(\alpha\)-closed. Then \(X - P = X - [\text{cl}^\mu(A) - A] = A \cup [X - \text{cl}^\mu(A)]\). Since \(X - P\) is supra \(\alpha\)-open, \(A \cup [X - \text{cl}^\mu(A)]\) is supra \(\alpha\)-open.

(iv) \(\Rightarrow\) (iii): Let \(A = U \cup [X - \text{cl}^\mu(A)]\). Since \(X - U\) is supra \(\alpha\)-closed and \(X - U = \text{cl}^\mu(A) - A\) is supra \(\alpha\)-closed.

Theorem 3.10. For a subset \(A\) of \((X, \mu)\), the following are equivalent:

(i) \(A \in S-\alpha-LC(X, \mu)\).

(ii) \(A = U \cap \text{cl}^\mu(A)\), for some supra \(\alpha\)-open set \(U\).

(iii) \(\text{cl}^\mu(A) - A\) is supra \(\alpha\)-closed.

(iv) \(A \cup [X - \text{cl}^\mu(A)]\) is supra \(\alpha\)-open.
(v). \( A \subseteq \text{int}_\alpha (A \cup [X - cl^\mu_\alpha (A)]) \).

Proof. (i) \( \Rightarrow \) (ii): Given \( A \in S\alpha\text{-LC}(X, \mu) \). Then there exist a supra \( \alpha \)-open subset \( U \) and a supra \( \alpha \)-closed subset \( V \) such that \( A = U \cap V \). Since \( A \subseteq U \) and \( A \subseteq cl^\mu_\alpha (A) \), \( A \subseteq U \cap cl^\mu_\alpha (A) \).

Conversely, \( cl^\mu_\alpha (A) \subseteq V \) and hence \( A = U \cap V \supseteq U \cap cl^\mu_\alpha (A) \). Therefore \( A = U \cap cl^\mu_\alpha (A) \).

(ii) \( \Rightarrow \) (i): Let \( A = U \cap cl^\mu_\alpha (A) \), for some supra \( \alpha \)-open set \( U \). Then, \( cl^\mu_\alpha (A) \) is supra \( \alpha \)-closed and hence \( A = U \cap cl^\mu_\alpha (A) \in S\alpha\text{-LC}^*(X, \mu) \).

(iii) \( \Rightarrow \) (ii): Let \( A = U \cap cl^\mu_\alpha (A) \), for some supra \( \alpha \)-open set \( U \). This implies \( U \) is supra \( \alpha \)-open and \( cl^\mu_\alpha (A) \) is supra \( \alpha \)-closed. Therefore, \( cl^\mu_\alpha (A) - A \) is supra \( \alpha \)-closed.

(iv) \( \Rightarrow \) (vi): Let \( P = cl^\mu_\alpha (A) - A \). Then, \( X - P = X - [cl^\mu_\alpha (A) - A] = A \cup ([X - cl^\mu_\alpha (A)]) \). Since \( X - P \) is supra \( \alpha \)-open, \( A \cup [X - cl^\mu_\alpha (A)] \) is supra \( \alpha \)-open.

(vi) \( \Rightarrow \) (v): Let \( U = A \cup ([X - cl^\mu_\alpha (A)]) \). Since \( X - U \) is supra \( \alpha \)-closed and \( X - U = cl^\mu_\alpha (A) - A \) is supra \( \alpha \)-closed.

(v) \( \Rightarrow \) (iv): It is obvious.

Theorem 3.11. If \( P \subseteq Q \subseteq X \) and \( Q \) is \( S\alpha\text{-LC} \), then there exists a \( S\alpha\text{-LC} \) set \( R \) such that \( P \subseteq R \subseteq Q \).

Theorem 3.12. For a subset \( A \) of \((X, \mu)\), if \( A \in S\alpha\text{-LC}^*(X, \mu) \), then there exist a supra open set \( P \) such that \( A = P \cap cl^\mu_\alpha (A) \).

Proof. Let \( A \in S\alpha\text{-LC}^*(X, \mu) \). Then \( A = P \cap V \), where \( P \) is supra open set and \( V \) is supra \( \alpha \)-closed set. Then \( A = P \cap V \Rightarrow A \subseteq P \). Obviously, \( A \subseteq cl^\mu_\alpha (A) \). Therefore

\[ A \subseteq P \cap cl^\mu_\alpha (A) \quad (1) \]

Also we have \( cl^\mu_\alpha (A) \subseteq V \). This implies

\[ A = P \cap V \supseteq P \cap cl^\mu_\alpha (A) \Rightarrow A \supseteq P \cap cl^\mu_\alpha (A) \quad (2) \]

From (1) and (2), we have \( A = P \cap cl^\mu_\alpha (A) \).

Theorem 3.13. For a subset \( A \) of \((X, \mu)\), if \( A \in S\alpha\text{-LC}^*(X, \mu) \), then there exist an supra open set \( P \) such that \( A = P \cap cl^\mu_\alpha (A) \).

Proof. Let \( A \in S\alpha\text{-LC}^*(X, \mu) \). Then \( A = P \cap V \), where \( P \) is supra open set and \( V \) is supra \( \alpha \)-closed set. Then \( A = P \cap V \Rightarrow A \subseteq P \). Then \( A \subseteq cl^\mu_\alpha (A) \). Therefore

\[ A \subseteq P \cap cl^\mu_\alpha (A) \quad (3) \]

Also we have \( cl^\mu_\alpha (A) \subseteq V \). This implies

\[ A = P \cap V \supseteq P \cap cl^\mu_\alpha (A) \Rightarrow A \supseteq P \cap cl^\mu_\alpha (A) \quad (4) \]

From (3) and (4), we get \( A = P \cap cl^\mu_\alpha (A) \).

Theorem 3.14. Let \( A \) be a subset of \((X, \mu)\). If \( A \in S\alpha\text{-LC}^*(X, \mu) \), then \( cl^\mu_\alpha (A) \)-A supra \( \alpha \)-closed and \( A \cup ([X - cl^\mu_\alpha (A)]) \) is supra \( \alpha \)-open.
Remark 3.15. The converse of the above theorem need not be true as seen from the following example.

Example 3.16. Let $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, c, d\}\}$. Then $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$ is the set of all supra $\alpha$-closed sets in $X$ and $\text{S-}\alpha\text{-LC}^*(X, \mu) = P(X) - \{\{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. If $A = \{a, b, c\}$, then $\text{cl}^\alpha(A) - A = \{d\}$ is supra $\alpha$-closed and $A \cup [(X - \text{cl}^\alpha(A)) = \{a, b, c\}$ is supra $\alpha$-open but $A \notin \text{S-}\alpha\text{-LC}^*(X, \mu)$.

Example 3.18. Let $X=\{a, b, c, d\}$ with supra topological space $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c, d\}\}$. Let $A=\{a\} \in \text{S-}\alpha\text{-LC}(X, \mu)$ (respectively, $\text{S-}\alpha\text{-LC}^*(X, \mu)$ and $\text{S-}\alpha\text{-LC}^**(X, \mu)$) and $B=\{d\} \in \text{S-}\alpha\text{-LC}(X, \mu)$ (respectively, $\text{S-}\alpha\text{-LC}^*(X, \mu)$ and $\text{S-}\alpha\text{-LC}^**(X, \mu)$). Here $A$ and $B$ are supra $\alpha$-separated, because $A \cap \text{cl}^\alpha(B) = B \cap \text{cl}^\alpha(A) = \phi$. Then $A \cup B = \{a, d\} \notin \text{S-}\alpha\text{-LC}(X, \mu)$ (respectively, $\text{S-}\alpha\text{-LC}^*(X, \mu)$ and $\text{S-}\alpha\text{-LC}^**(X, \mu)$).

Definition 3.19. Let $(X, \mu)$ be a supra topological space. A subset $A$ of $(X, \mu)$ is called supra dense, if $\text{cl}^\alpha(A) = X$.

Definition 3.20. A supra topological space $(X, \mu)$ is called supra submaximal, if every supra dense subset is supra open in $X$.

Definition 3.21. Let $(X, \mu)$ be a supra topological space. A subset $A$ of $(X, \mu)$ is called supra $\alpha$-dense, if $\text{cl}^\alpha(A) = X$.

Definition 3.22. A supra topological space $(X, \mu)$ is called supra $\alpha$-submaximal, if every supra $\alpha$-dense subset is supra $\alpha$-open in $X$.

Example 3.23. Consider the supra topological space $(X, \mu)$ with $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. Here $X$ and $(a, b, c)$ are the supra $\alpha$-dense sets and also supra $\alpha$-open sets in $X$. Therefore $X$ is supra $\alpha$-submaximal.

Remark 3.24.

(1). Every supra submaximal space is supra $\alpha$-submaximal.

(2). Every supra submaximal space is supra $\beta$-submaximal.

(3). Every supra $\alpha$-submaximal space is supra $\beta$-submaximal.

Remark 3.25. The converses of the above statements are not true. The following diagram and examples illustrates this fact.
Example 3.26. Consider the supra topological space $(X, \mu)$ with $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. In this supra topological space, the subsets $X$ and $\{a, b, c\}$ are supra dense (resp., supra $\alpha$-dense and resp., supra $\beta$-dense). Thus the supra topological space $(X, \mu)$ is supra submaximal (resp., supra $\alpha$-submaximal space and resp., supra $\beta$-submaximal).

Example 3.27. Consider the supra topological space $(X, \mu)$ with $X = \{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b, c\}\}$. In this supra topological space, the supra $\alpha$-open sets are $\phi, X, \{a\}, \{b\}, \{c\}, \{a, b, c\}$ and $\{a, b, d\}$. The supra $\beta$-open sets are $\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}$ and $\{a, c, d\}$. Since all supra $\beta$-dense sets are supra $\beta$-open sets, $(X, \mu)$ is supra $\beta$-submaximal. Here the subset $\{a, c\}$ is supra $\alpha$-dense and not supra $\alpha$-open set. Thus the supra topological space $(X, \mu)$ is not supra $\alpha$-submaximal space. Also the subset $\{a, b, d\}$ is supra dense and not supra open set. Therefore the supra topological space $(X, \mu)$ is not supra submaximal space.

Theorem 3.28. A supra topological space $(X, \mu)$ is supra $\alpha$-submaximal if and only if $P(X) = S-\alpha-LC(X)$ holds.

Proof. Necessity: Let $A \in P(X)$ and $G = A \cup [X- cl^n(X)]$. Then $cl^n(X) = X$ and so $G$ is supra $\alpha$-dense and hence supra $\alpha$-open by assumption. By Theorem 3.10, $A \in S-\alpha-LC(X)$. Hence $P(X) = S-\alpha-LC(X)$.

Sufficiency: Let every subset of $X$ be supra $\alpha$-locally closed. Let $A$ be supra $\alpha$-dense in $X$. Then $cl^n(X) = X$. Now $A = A \cup [X- cl^n(A)]$. By Theorem: 3.10, $A$ is supra $\alpha$-open. Hence $X$ is supra $\alpha$-submaximal.

Theorem 3.29. Let $(X, \mu)$ and $(Y, \lambda)$ be the supra topological spaces.

(1) If $M \in S-\alpha-LC(X, \mu)$ and $N \in S-\alpha-LC(Y, \lambda)$, then $M \times N \in S-\alpha-LC(X \times Y, \mu \times \lambda)$.

(2) If $M \in S-\alpha-LC^*(X, \mu)$ and $N \in S-\alpha-LC^*(Y, \lambda)$, then $M \times N \in S-\alpha-LC^*(X \times Y, \mu \times \lambda)$.

(3) If $M \in S-\alpha-LC^{**}(X, \mu)$ and $N \in S-\alpha-LC^{**}(Y, \lambda)$, then $M \times N \in S-\alpha-LC^{**}(X \times Y, \mu \times \lambda)$.

Proof. Let $M \in S-\alpha-LC(X, \mu)$ and $N \in S-\alpha-LC(Y, \lambda)$. Then there exist a supra $\alpha$-open sets $P$ and $P'$ of $(X, \mu)$ and $(Y, \lambda)$ and supra semi-closed sets $Q$ and $Q'$ of $(X, \mu)$ and $(Y, \lambda)$ respectively such that $M = P \cap Q$ and $N = P' \cap Q'$. Then $M \times N = (P \times P') \cap (Q \times Q')$ holds. Hence $M \times N \in S-\alpha-LC(X \times Y, \mu \times \lambda)$.

The proofs of (2) and (3) are similar to that of (1).

Theorem 3.30. If $A$ is supra $\alpha$-locally closed set in $(X, \mu)$, Then $A$ is supra $\beta$-locally closed set in $(X, \mu)$.

Proof. Since every supra $\alpha$-open set is supra $\beta$-open, $S-\alpha-LC(X, \mu) \subseteq S-\beta-LC(X, \mu)$, for any supra topological space $(X, \mu)$.

Remark 3.31. A supra $\beta$-locally closed set need not be a supra $\alpha$-locally closed set. The following example supports this fact.

Example 3.32. Consider the supra topological space in Example 3.8, the subset $\{a, b\}$ is a supra $\beta$-locally closed set and not a supra $\alpha$-locally closed set.

4. Supra $\alpha$-Locally Continuous Functions

In this section we define a new type of functions called Supra $\alpha$-locally continuous functions ($S-\alpha$-L-continuous functions), supra $\alpha$-locally irresolute functions ($S-\alpha$-L-irresolve functions) and study some of their properties.
Definition 4.1. Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\tau \subseteq \mu\). A function \(f : (X, \tau) \to (Y, \sigma)\) is called S-\(\alpha\)-L-continuous (resp., S-\(\alpha\)-L*-continuous, resp., S-\(\alpha\)-L**-continuous), if \(f^{-1}(A) \in S\alpha\text{-LC}(X, \mu)\), (resp., \(f^{-1}(A) \in S\alpha\text{-LC}^*(X, \mu)\), resp., \(f^{-1}(A) \in S\alpha\text{-LC}**(X, \mu)\)) for each \(A \in \sigma\).

Definition 4.2. Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\mu\) and \(\lambda\) be the supra topologies associated with \(\tau\) and \(\sigma\) respectively. A function \(f : (X, \tau) \to (Y, \sigma)\) is said to be S-\(\alpha\)-L-irresolute (resp., S-\(\alpha\)-L* - irresolute, resp., S-\(\alpha\)-L**-irresolute) if \(f^{-1}(A) \in S\alpha\text{-LC}(X, \mu)\), (resp., \(f^{-1}(A) \in S\alpha\text{-LC}^*(X, \mu)\), resp., \(f^{-1}(A) \in S\alpha\text{-LC}**(X, \mu)\)) for each \(A \in S\alpha\text{-LC}(Y, \lambda)\) (resp., \(A \in S\alpha\text{-LC}^*(Y, \lambda)\), resp., \(A \in S\alpha\text{-LC}**(Y, \lambda)\)).

Theorem 4.3. Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\mu\) be a supra topology associated with \(\tau\). Let \(f : (X, \tau) \to (Y, \sigma)\) be a function. If \(f\) is S-\(\alpha\)-L-continuous or S-\(\alpha\)-L*-continuous, then it is S-\(\alpha\)-L-continuous.

Theorem 4.4. Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\mu\) and \(\lambda\) be the supra topologies associated with \(\tau\) and \(\sigma\) respectively. Let \(f : (X, \mu) \to (Y, \sigma)\) be a function. If \(f\) is S-\(\alpha\)-L-irresolute (respectively S-\(\alpha\)-L* - irresolute, respectively S-\(\alpha\)-L**-irresolute), then it is S-\(\alpha\)-L-continuous. (respectively S-\(\alpha\)-L* - continuous, respectively S-\(\alpha\)-L**-continuous).

Remark 4.5. Converse of Theorem 4.3 need not be true as seen from the following example.

Example 4.6. Let \(X = Y = \{a, b, c, d\}\) with \(\tau = \{\emptyset, X, \{a, c, d\}\}\), \(\sigma = \{\emptyset, Y, \{b, c, d\}\}\) and \(\mu = \{\emptyset, X, \{a, b\}\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}\). Define \(f : (X, \mu) \to (Y, \sigma)\) by \(f(a) = b, f(b) = a, f(c) = d\) and \(f(d) = c\). Here \(f\) is S-\(\alpha\)-L**-continuous and it is not S-\(\alpha\)-L**-irresolute.

Remark 4.7. The following example provides a function which is S-\(\alpha\)-L**- continuous function but not S-\(\alpha\)-L**- irresolute function.

Example 4.8. Let \(X = Y = \{a, b, c, d\}\) with \(\tau = \{\emptyset, X, \{a, b\}, \{a, b, d\}, \{a, b, c\}, \{a, c, d\}\}\), \(\sigma = \{\emptyset, Y, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}\), and \(\lambda = \{\emptyset, Y, \{b, c\}\}\). Define \(f : (X, \mu) \to (Y, \sigma)\) by \(f(a) = b, f(b) = a, f(c) = d\) and \(f(d) = c\). Here \(f\) is S-\(\alpha\)-L**-continuous and it is not S-\(\alpha\)-L**-irresolute.

Theorem 4.9. Let \(f : (X, \tau) \to (Y, \sigma)\) be supra \(\alpha\)-LC-continuous and \(A\) be supra \(\alpha\)-open in \(X\). Then the restriction \(f|A : A \to Y\) is S-\(\alpha\)-L-continuous.

Proof. Let \(U\) be supra open in \(Y\). Then \(f^{-1}(U)\) in supra \(\alpha\)-LC in \(X\). So \(f^{-1}(U) = G \cap H\) where \(G\) is supra \(\alpha\)-open and \(H\) is supra \(\alpha\)-closed in \(X\). Now \((f^{-1}(A))^{-1}(U) = (G \cap H) \cap A = G \cap (H \cap A)\) (resp. \((G \cap A) \cap H\) where \(H \cap A\) is supra \(\alpha\)-closed (resp. \(G \cap A\) is supra \(\alpha\)-open)) in \(X\). Therefore \((f^{-1}(A))^{-1}(U)\) is supra \(\alpha\)-LC in \(X\). Hence \(f|A\) is supra \(\alpha\)-L-continuous.

Theorem 4.10. A supra topological space \((X, \mu)\) is supra \(\alpha\)-submaximal if and only if every function having \((X, \mu)\) as domain is supra \(\alpha\)-L-continuous.

Proof. Necessity: Let \((X, \mu)\) be supra \(\alpha\)-submaximal. Then \(\alpha\)-LC(X) = P(X) by Theorem: 3.28. Let \(f : (X, \mu) \to (Y, \lambda)\) be a function and \(A \subseteq \sigma\). Then \(f^{-1}(A) \subseteq \alpha\text{-LC}(X)\) and so \(f\) is S-\(\alpha\)-L-continuous.

Sufficiency: Let every function having \((X, \mu)\) as domain be supra \(\alpha\)-L-continuous. Let \(Y = \{0, 1\}\) and \(\sigma = \{\emptyset, Y, \{0\}\}\). Let \(A \subseteq (X, \mu)\) and \(f : (X, \mu) \to (Y, \lambda)\) be defined by \(f(x) = 0\) if \(x \in A\) and \(f(x) = 1\) if \(x \notin A\). Since \(f\) is supra \(\alpha\)-L-continuous, \(A \subseteq \alpha\text{-LC}(X, \mu)\). Hence \(P(X) = \alpha\text{-LC}(X)\). Therefore \(X\) is supra \(\alpha\)-submaximal by Theorem: 3.28.

Theorem 4.11. If \(g : X \to Y\) is S-\(\alpha\)-L-continuous and \(h : Y \to Z\) is supra continuous, then \(h \circ g : X \to Z\) is S-\(\alpha\)-L-continuous.

Proof. Let \(g : X \to Y\) is S-\(\alpha\)-L-continuous and \(h : Y \to Z\) is supra continuous. By the Definitions, \(g^{-1}(V) \in \alpha\text{-LC}(X), V \in Y\) and \(h^{-1}(W) \in Y, W \subseteq Z\). Let \(W \subseteq Z\). Then \((h \circ g)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)\), for \(V \subseteq Y\). From this, \((h \circ g)^{-1}(W) = g^{-1}(V) \in \alpha\text{-LC}(X), W \subseteq Z\). Therefore \(h \circ g\) is S-\(\alpha\)-L- continuous.
Remark 4.12. If \( g: X \to Y \) is \( S\alpha\) -irresolute and \( h: Y \to Z \) is \( S\alpha\) -continuous, then \( h \circ g: X \to Z \) is \( S\alpha\) -continuous.

Proof. Let \( g: X \to Y \) is \( S\alpha\) -irresolute and \( h: Y \to Z \) is \( S\alpha\) -continuous. By the Definitions, \( g^{-1}(V) \in S\alpha\text{-LC}(X) \), for \( V \in S\alpha\text{-LC}(Y) \) and \( h^{-1}(W) \in S\alpha\text{-LC}(Y) \), for \( W \in Z \). Let \( W \in Z \). Then \((h \circ g)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)\), for \( V \in S\alpha\text{-LC}(Y) \). This implies, \((h \circ g)^{-1}(W) = g^{-1}(V) \in S\alpha\text{-LC}(X) \), \( W \in Z \). Hence \( h \circ g \) is \( S\alpha\) -continuous.

Theorem 4.13. If \( g: X \to Y \) and \( h: Y \to Z \) are \( S\alpha\) -irresolute, then \( h \circ g: X \to Z \) is also \( S\alpha\) -irresolute.

Proof. By the hypothesis and the Definitions, we have \( g^{-1}(V) \in S\alpha\text{-LC}(X) \), for \( V \in S\alpha\text{-LC}(Y) \) and \( h^{-1}(W) \in S\alpha\text{-LC}(Y) \), for \( W \in S\alpha\text{-LC}(Z) \). Let \( W \in S\alpha\text{-LC}(Z) \). Then \((h \circ g)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V) \), for \( V \in S\alpha\text{-LC}(Y) \). Therefore, \((h \circ g)^{-1}(W) = g^{-1}(V) \in S\alpha\text{-LC}(X) \), \( W \in S\alpha\text{-LC}(Z) \). Thus \( h \circ g \) is \( S\alpha\) -irresolute.

References