Super Lehmer-3 Mean Labeling of Tree Related Graphs

Research Article

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Abstract: Let $f : V(G) \to \{1, 2, \ldots, p + q\}$ be an injective function. The induced edge labeling $f^*(e = uv)$ is defined by

\[
 f^*(e) = \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil \quad \text{or} \quad \left\lfloor \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rfloor,
\]

then $f$ is called Super Lehmer-3 mean labeling, if \( \{f(V(G)) \cup \{f(e)/e \in E(G)\} = \{1, 2, 3, \ldots, p + q\} \). A graph which admits Super Lehmer-3 Mean labeling is called Super Lehmer-3 Mean graph. In this paper we prove that $P_m \Theta K_{1,n}$, $(P_m, S_n)$.

Keywords: Super Lehmer-3 mean graph, $P_m \Theta K_{1,n}$, $(P_m, S_n)$.

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1. Introduction

A graph considered here are finite, undirected and simple. The vertex set and the edge set of a graph is denoted by $V(G)$ and $E(G)$ respectively. Lehmer mean is another type of generalized mean. For standard terminology and notations, we follow [2] and for the detailed survey of graph labeling we follow [1]. [3, 4] introduced the concept of Harmonic Mean Labeling of Graph and its basic results was proved. We will provide a brief summary of other in formations which are necessary for our present investigation.

Definition 1.1. A graph $G = (V, E)$ with $p$ vertices and $q$ edges is called Lehmer-3 mean graph. If it is possible to label vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, 3, \ldots, q + 1$ in such a way that when each edge $e = uv$ is labeled with

\[
 f^*(e) = \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil \quad \text{or} \quad \left\lfloor \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rfloor,
\]

then the edge labels are distinct. In this case $f$ is called Lehmer-3 mean labeling of $G$.

2. Main Results

Theorem 2.1. The graph $P_m \Theta K_{1,n}$ is a super lehmer 3-mean graph.

Proof. Let $\{u_i, 1 \leq i \leq m, u_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertices and $\{e_i, 1 \leq i \leq m - 1, e_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the edges which are denoted as in Figure 1

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First we label the vertices as follows: Define $f : V \rightarrow \{1, 2, \ldots, p + q\}$ by

For $1 \leq i \leq m$, $f(u_i) = 2i - 1$; For $1 \leq i \leq m$, $1 \leq j \leq n$, $f(u_{ij}) = 2m + 2n(i - 1) + 2j - 1$. Then the induced edge labels are:

For $1 \leq i \leq m - 1$, $f^*(e_i) = 2i$; For $1 \leq i \leq m$, $1 \leq j \leq n$, $f^*(e_{ij}) = 2m + 2n(i - 1) + 2(j - 1)$. Therefore, the graph $P_m \Theta K_{1,n}$ is a super lehmer 3-mean graph. Super lehmer 3-mean labeling of $P_4 \Theta K_{1,4}$ is shown in Figure 2.

**Theorem 2.2.** The graph $(P_m, S_n)$ is a super lehmer-3 mean graph.

**Proof.** Let $\{u_i, v_i, 1 \leq i \leq m, v_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertices and $\{a_i, 1 \leq i \leq m - 1, b_i, 1 \leq i \leq m, e_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the edges which are denoted as in Figure 3.

First we label the vertices as follows: Define $f : V \rightarrow \{1, 2, \ldots, p + q\}$ by

For $1 \leq i \leq m$

$f(u_i) = 2i - 1$
\[ f(v_i) = 2n + 2(m + 1)(i - 1) + 1 \]

For \( 1 \leq i \leq m \), \( 1 \leq j \leq n \), \( f(v_{ij}) = 2n + 2(m + 1)(i - 1) + 2j + 1 \). Then the induced edge labels are:

For \( 1 \leq i \leq m - 1 \), \( f^*(a_i) = 2i \)

For \( 1 \leq i \leq m \), \( f^*(b_i) = 2n + 2(m + 1)(i - 1) \)

For \( 1 \leq i \leq m \), \( 1 \leq i \leq n \), \( f^*(e_{ij}) = 2n + 2(m + 1)(i - 1) + 2j \). Therefore, the graph \((P_m, S_n)\) is a super lehmer 3-mean graph. Super lehmer 3-mean labeling of \((P_m, S_n)\) is shown in Figure 4

![Figure 4. Super Lehmer-3 mean graph \((P_m, S_n)\)](image)

References


