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Properties of Anti $T-$Fuzzy Ideal of $\ell-$Ring

Research Article

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Abstract: In this paper, we made an attempt to study the properties of anti $T-$fuzzy ideal of $\ell-$ring and we introduce some definitions and theorems in join, union, join of a family and the union of a family of anti $T-$fuzzy ideal of $\ell-$ring.

Keywords: Fuzzy subset, $T-$fuzzy ideal, anti $T-$ fuzzy ideal, join of anti $T-$fuzzy ideal, union of anti $T-$fuzzy ideal, join of a family anti $T-$fuzzy ideal and the union of a family of anti $T-$fuzzy ideal.

1. Introduction

The concept of fuzzy sets was initiated by L.A.Zadeh [9] in 1965. After the introduction of fuzzy sets several researchers explored on the generalization of the concept of fuzzy sets. In this paper we define, characterize and study the anti $T-$fuzzy right and left ideals. Z. D. Wang introduced the basic concepts of TL-ideals. We introduced anti $T-$fuzzy right ideals of $\ell-$ ring. We compare fuzzy ideal introduced by Liu to anti $T-$fuzzy ideals. We have shown that ring is regular if and only if union of any anti $T-$fuzzy right ideal with anti $T-$fuzzy left ideal is equal to its product. We discuss some of its properties. We have shown that the join of anti $T-$fuzzy ideal of $\ell-$ring, union of anti $T-$fuzzy ideal of $\ell-$ring, join of a family anti $T-$fuzzy ideal of $\ell-$ring and the union of a family of anti $T-$fuzzy ideal of $\ell-$ring are anti T- fuzzy ideals.

Definition 1.1. *A non-empty set R is called lattice ordered ring or $\ell-$ring if it has four binary operations $+, \cdot, \vee, \wedge$ defined on it and satisfy the following*

(1). $(R, +, \cdot)$ *is a ring*

(2). $(R, \vee, \wedge)$ *is a lattice*

(3). $x + (y \vee z) = (x + y) \vee (x + z)\,;\;\; x + (y \wedge z) = (x + y) \wedge (x + z)$

$(y \vee z) + x = (y + x) \vee (z + x)\,;\;\; (y \wedge z) + x = (y + x) \wedge (z + x)$

(4). $x \cdot (y \vee z) = (xy) \vee (xz)\,;\;\; x \cdot (y \wedge z) = (xy) \wedge (xz)$

$(y \vee z) \cdot x = (yx) \vee (zx)\,;\;\; (y \wedge z) \cdot x = (yx) \wedge (zx)\,,$

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for all x, y, z in R and $x \geq 0$.

Example 1.2. $(\mathbb{Z}, +, \cdot, \vee, \wedge)$ *is a $\ell-$ring, where $\mathbb{Z}$ is the set of all integers.*

Example 1.3. $(n\mathbb{Z}, +, \cdot, \vee, \wedge)$ *is a $\ell-$ring, where $\mathbb{Z}$ is the set of all integers and $n \in \mathbb{Z}$.*

Definition 1.4. *A mapping $T : [0,1] \times [0,1] \to [0,1]$ is called a triangular norm [t−norm] if and only if it satisfies the following conditions:*

(1). $T(x,1) = T(1,x) = x$, *for all* $x \in [0,1]$

(2). $T(x,y) = T(y,x)$ *for all* $x, y \in [0,1]$.

(3). $T(x, T(y,z)) = T(T(x,y)z)$ *for all* $x, y, z \in [0,1]$

(4). $T(x,y) \leq T(x,z)$, *whenever* $y \leq z$, *for all* $x, y, z \in [0,1]$

Definition 1.5. *A mapping from a nonempty set X to [0, 1], $\mu : X \to [0,1]$ is called a fuzzy subset of X.*

Definition 1.6. *Let μ and λ be an fuzzy subsets of a set X. An fuzzy subset $\mu \cup \lambda$ is defined as $(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}$.*

Example 1.7. *Let $\mu = \{(a, 0.4), (b, 0.7), (c, 0.3)\}$ and $\lambda = \{(a, 0.5), (b, 0.3), (c, 0.4)\}$ be an fuzzy subsets of $X = \{a, b, c\}$. The union of two fuzzy subsets of μ and λ is $\mu \cup \lambda = \{(a, 0.5), (b, 0.7), (c, 0.4)\}$.*

Definition 1.8. *Let μ and λ be the fuzzy subsets of a set X. An fuzzy subset $\mu \vee \lambda$ is defined as $(\mu \vee \lambda)(x) = T(\mu(x), \lambda(x))$.*

Definition 1.9. *A fuzzy subset μ of a ring R is called anti T-fuzzy right ideal if*

(1). $\mu(x - y) \leq T(\mu(x), \mu(y))$, *for all* $x, y \in R$

(2). $\mu(xy) \leq \mu(x)$, *for all* $x, y \in R$.

Definition 1.10. *A fuzzy subset μ of a ring R is called anti T-fuzzy left ideal if*

(1). $\mu(x - y) \leq T(\mu(x), \mu(y))$, *for all* $x, y \in R$

(2). $\mu(xy) \leq \mu(y)$, *for all* $x, y \in R$.

Definition 1.11. *A fuzzy subset μ of a lattice ordered ring (or $\ell-$ring) R is called an anti fuzzy sub $\ell-$ring of R, if the following conditions are satisfied*

(1). $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

(2). $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$

(3). $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$

(4). $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$, *for all* $x, y \in R$.

Example 1.12. *Consider an fuzzy subset μ_1 of the $\ell-$ring $(Z, +, \cdot, \vee, \wedge)$*

$$\mu_1(x) = \begin{cases} 0.4, & \text{if } x \in \langle 2 \rangle \\ 0.7, & \text{if } Z - \langle 2 \rangle. \end{cases}$$

Then μ_1 is an anti-fuzzy $\ell-$sub ring.

Definition 1.13. *A fuzzy subset μ of an $\ell-$ring R is called an anti fuzzy $\ell-$ring ideal (or) fuzzy $\ell-$ideal of R, if for all $x, y \in R$ the following conditions are satisfied*

(1). $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

(2). $\mu(x \wedge y) \leq \min\{\mu(x), \mu(y)\}$

(3). $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$

(4). $\mu(xy) \leq \min\{\mu(x), \mu(y)\}$*, for all* $x, y \in R$.

Definition 1.14. *A fuzzy subset μ of a ring R is called an anti T-fuzzy ideal, if the following conditions are satisfied,*

(1). $\mu(x - y) \leq T(\mu(x), \mu(y))$

(2). $\mu(xy) \leq \mu(x)$; $\mu(xy) \leq \mu(y)$ *for all* $x, y \in R$.

Definition 1.15. *A fuzzy subset μ of a $\ell-$ring R is called an anti T-fuzzy ideal, if the following conditions are satisfied,*

(1). $\mu(x - y) \leq T(\mu(x), \mu(y))$

(2). $\mu(xy) \leq \mu(x)$; $\mu(xy) \leq \mu(y)$

(3). $\mu(x \vee y) \leq T(\mu(x), \mu(y))$

(4). $\mu(x \wedge y) \leq T(\mu(x), \mu(y))$*, for all* $x, y \in R$.

Example 1.16. *Now $(R = \{a, b, c\}, +, \cdot, \vee, \wedge)$ is a $\ell-$ring. The operations $+, \cdot, \vee$ and $\wedge$ defined by the following. Consider any fuzzy subset μ of the $\ell-$ring R*

$$\mu(x) = \begin{cases} 0.2 & \textit{if } x = a \\ 0.5 & \textit{if } x = b \\ 0.8 & \textit{if } x = c \end{cases}$$

Then μ is an anti T-fuzzy ideal of $\ell-$ring R.

2. Main Results

Theorem 2.1. *If μ and λ are anti T-fuzzy ideals of a $\ell-$ring R, then $\mu \vee \lambda$ is an anti T-fuzzy ideal of a $\lambda \vee \ell-$ring R.*

Proof. Given μ and λ are anti T-fuzzy ideals of a $\ell-$ring R. Let $x, y \in R$.

(1).
$$\begin{aligned}(\mu \vee \lambda)(x - y) &= T(\mu(x - y), \lambda(x - y)) \\ &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\ &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\ &= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y)) \\ &= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y))) \\ &= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))\end{aligned}$$

Therefore, $(\mu \vee \lambda)(x - y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$ for all $x, y \in R$.

(2). Since $\mu(xy) \leq \mu(x)$ and $\lambda(xy) \leq \lambda(x)$. Now

$$\begin{aligned}(\mu \vee \lambda)(xy) &\leq T(\mu(xy), \lambda(xy))\\ &\leq T(\mu(x), \lambda(x))\\ &\leq (\mu \vee \lambda)(x)\end{aligned}$$

Therefore $(\mu \vee \lambda)(xy) \leq (\mu \vee \lambda)(x)$, for all $x, y \in R$.

(3). $$\begin{aligned}(\mu \vee \lambda)(x \vee y) &= T(\mu(x \vee y), \lambda(x \vee y))\\ &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))\\ &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))\\ &= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y))\\ &= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y)))\\ &= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))\end{aligned}$$

Therefore, $(\mu \vee \lambda)(x \vee y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$ for all $x, y \in R$.

(4). $$\begin{aligned}(\mu \vee \lambda)(x \wedge y) &= T(\mu(x \wedge y), \lambda(x \wedge y))\\ &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))\\ &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))\\ &= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y))\\ &= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y)))\\ &= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))\end{aligned}$$

Therefore, $(\mu \vee \lambda)(x^y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$ for all $x, y \in R$.

Thus $\mu \vee \lambda$, is an anti T–Fuzzy right ideal of a ℓ–ring R. □

Theorem 2.2. *If μ and λ are anti T-fuzzy ideals of a ℓ–ring R, then $\mu \cup \lambda$, is an anti T-fuzzy ideal of a ℓ–ring R.*

Proof. Let μ and λ are anti T-fuzzy ideals of a ℓ–ring R. Let $x, y \in R$.

(1). $$\begin{aligned}(\mu \cup \lambda)(x - y) &= \max\{\mu(x - y), \lambda(x - y))\\ &\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\lambda(x), \lambda(y)\}\}\\ &= \max\{\max\{\max\{\mu(x), \mu(y)\}, \lambda(x)\}, \lambda(y)\}\\ &= \max\{\max\{\max\{\mu(x), \lambda(x)\}, \mu(y)\}, \lambda(y)\}\\ &= \max\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}\\ &= \max\{(\mu \cup \lambda)(x), (\mu \cup \lambda)(y)\}\end{aligned}$$

Therefore, $(\mu \cup \lambda)(x - y) \leq \max\{(\mu \cup \lambda)(x), (\mu \cup \lambda)(y)\}$ for all $x, y \in R$.

(2). Since $\mu(xy) \leq \mu(x)$ and $\lambda(xy) \leq \lambda(x)$. Now

$$\begin{aligned}(\mu \cup \lambda)(xy) &\leq \max\{\mu(xy), \lambda(xy)\}\\ &\leq \max\{\mu(x), \lambda(x)\}\\ &\leq (\mu \cup \lambda)(x)\end{aligned}$$

Therefore $(\mu \cup \lambda)(xy) \leq (\mu \cup \lambda)(x)$, for all $x, y \in R$.

(3). $(\mu \cup \lambda)(x \vee y) = \max\{\mu(x \vee y), \lambda(x \vee y))$

$$\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\lambda(x), \lambda(y)\}\}$$

$$= \max\{\max\{\max\{\mu(x), \mu(y)\}, \lambda(x)\}, \lambda(y)\}$$

$$= \max\{\max\{\max\{\mu(x), \lambda(x)\}, \mu(y)\}, \lambda(y)\}$$

$$= \max\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}$$

$$= \max\{(\mu \cup \lambda)(x), (\mu \cup \lambda)(y)\}$$

Therefore, $(\mu \cup \lambda)(x \vee y) \leq \max\{(\mu \cup \lambda)(x), (\mu \cup \lambda)(y)\}$ for all $x, y \in R$.

(4). $(\mu \cup \lambda)(x \wedge y) = \max\{\mu(x \wedge y), \lambda(x \wedge y))$

$$\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\lambda(x), \lambda(y)\}\}$$

$$= \max\{\max\{\max\{\mu(x), \mu(y)\}, \lambda(x)\}, \lambda(y)\}$$

$$= \max\{\max\{\max\{\mu(x), \lambda(x)\}, \mu(y)\}, \lambda(y)\}$$

$$= \max\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}$$

$$= \max\{(\mu \cup \lambda)(x), (\mu \cup \lambda)(y)\}$$

Therefore, $(\mu \cup \lambda)(x \wedge y) \leq \max\{(\mu \cup \lambda)(x), (\mu \cup \lambda)(y)\}$ for all $x, y \in R$.

Thus, $\mu \cup \lambda$ is an anti T–fuzzy ideal of a $\ell-$ring R. □

Theorem 2.3. *The join of a family of an anti T–fuzzy ideal of $\ell-$ring R is an anti T–fuzzy ideal of a $\ell-$ring R.*

Proof. Let $\{u_\alpha : \alpha \in I\}$ be a family of anti T–fuzzy ideal of $\ell-$ring R. Let $A = \bigvee_{\alpha \in I} u_\alpha$ and Let x and y in R. Then

(1). $\mu(x - y) = T(\mu(x - y), \mu(x - y))$

$$\leq T(T(\mu(x), \mu(y)), T(\mu(x), \mu(y))) \text{ (by definition)}$$

$$= T(T(\mu(x), \mu(y))$$

$$= T(\mu(x), \mu(y))$$

Therefore $\mu(x - y) \leq T(\mu(x), \mu(y))$, for all $x, y \in R$.

(2). Since $\mu(xy) \leq \mu(x)$and $\mu(xy) \leq \mu(y)$. Now

$$\mu(xy) \leq T(\mu(xy), \mu(xy))$$

$$= T(\mu(x), \mu(x)), \text{ (by definition)}$$

$$= \mu(x)$$

Therefore, $\mu(xy) \leq \mu(x)$ for all $x, y \in R$.

(3). $\mu(x \vee y) = T(\mu(x \vee y), \mu(x \vee y))$

$$\leq T(T(\mu(x), \mu(y)), T(\mu(x), \mu(y))) \text{ (by definition)}$$

$$= T(T(\mu(x), \mu(y))$$

$$= T(\mu(x), \mu(y))$$

Therefore $\mu(x \vee y) \leq T(\mu(x), \mu(y))$, for all $x, y \in R$.

(4). $\mu(x^y) = T(\mu(x^y), \mu(x^y))$

$\leq T(T(\mu(x), \mu(y)), T(\mu(x), \mu(y)))$ (by definition)

$= T(T(\mu(x), \mu(y))$

$= T(\mu(x), \mu(y))$

Therefore $\mu(x^y) \leq T(\mu(x), \mu(y))$, for all $x, y \in R$.

Thus the join of a family of an anti T–fuzzy ideal of ℓ–ring R is an anti T–fuzzy ideal of a ℓ–ring R. □

Theorem 2.4. *The union of a family of an anti T–fuzzy ideal of ℓ–ring R is an anti T–fuzzy ideal of a ℓ–ring R.*

Proof. Let $\{U_\alpha : \alpha \in I\}$ be a family of T-fuzzy ideal of R and let $A = \bigcup_{\alpha \in I} U_\alpha$. Let x and y in R. Then

(1). $\mu(x - y) = \max\{\mu(x - y), \mu(x - y)\}$

$\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\mu(x), \mu(y)\}\}$

$= \max\{\max\{\mu(x), \mu(y)\}\}$

$= \max\{\mu(x), \mu(y)\}$

Therefore, $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$ for all $x, y \in R$.

(2). Since $\mu(xy) \leq \mu(x)$and $\mu(xy) \leq \mu(y)$. Now

$\mu(xy) \leq \max\{\mu(xy), \mu(xy)\}$

$\leq \max\{\mu(x), \mu(x)\}$

$\leq \mu(x)$

Therefore, $\mu(xy) \leq \mu(x)$ for all $x, y \in R$.

(3). $\mu(x \vee y) = \max\{\mu(x \vee y), \mu(x \vee y)\}$

$\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\mu(x), \mu(y)\}\}$

$= \max\{\max\{\mu(x), \mu(y)\}\}$

$= \max\{\mu(x), \mu(y)\}$

Therefore, $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$ for all $x, y \in R$.

(4). $\mu(x \wedge y) = \max\{\mu(x \wedge y), \mu(x \wedge y)\}$

$\leq \max\{\max\{\mu(x), \mu(y)\}, \max\{\mu(x), \mu(y)\}\}$

$= \max\{\max\{\mu(x), \mu(y)\}\}$

$= \max\{\mu(x), \mu(y)\}$

Therefore, $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$ for all $x, y \in R$.

Thus union of a family of anti T–fuzzy ideal of ℓ–ring R is an anti T–fuzzy ideal of a ℓ–ring R. □

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