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Non Existence of Skolem Mean Labeling for Eight Star Graph

Research Article

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Abstract: In this paper, we prove if $\ell \leq m < n$, the eight star $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m - n| > 4 + 6\ell$ for $\ell = 2, 3, \ldots$; $m = 2, 3, \ldots$.

MSC: 05C78.

Keywords: Skolem Mean graph, Star, Eight Star Graph.

1. Introduction

In [2], we proved the following theorems to study the existence of skolem mean graphs. We proved the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $\ell \leq m < n$. The three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m - n| > 4 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $\ell \leq m < n$. The four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + 2\ell$ for $\ell = 2, 3, \ldots$; $m = 2, 3, \ldots$ and $\ell \leq m < n$. The four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m - n| > 4 + 2\ell$ for $\ell = 2, 3, \ldots$; $m = 2, 3, \ldots$ and $\ell \leq m < n$. In [3]. The five star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 4 + 3\ell$ for $\ell = 2, 3, \ldots$; $m = 2, 3, \ldots$ and $\ell \leq m < n$. Further, we prove the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 7$ for $m = 1, 2, 3, \ldots$ and $1 \leq m < n$; The four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m - n| > 7$ for $m = 1, 2, 3, \ldots$ and $1 \leq m < n$; The five star $K_{1,1} \cup K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m - n| = 8$ for $m = 1, 2, 3, \ldots$ and $1 \leq m < n$.

Definition 1.1. *The eight star is the disjoint union of $K_{1,a}, K_{1,b}, K_{1,c}, K_{1,d}, K_{1,e}, K_{1,f}, K_{1,g}, K_{1,h}$ and is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e} \cup K_{1,f} \cup K_{1,g} \cup K_{1,h}$.*

2. Main Section

Theorem 2.1. *The eight star $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m - n| > 4 + 6\ell$ for $\ell = 2, 3, \ldots$; $m = 2, 3, \ldots$*

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Proof. : Let $G = 7K_{1,2} \cup K_{1,19}$ where,

$$V(G) = \{v_{i,j} : 1 \le i \le 7; 0 \le j \le 2\} \cup \{v_{8,j} : 0 \le j \le 19\}$$
$$E(G) = \{v_{i,0} : v_{i,j} : 1 \le i \le 7; 1 \le j \le 2\} \cup \{v_{8,0}v_{8,j} : 1 \le j \le 19\}.$$

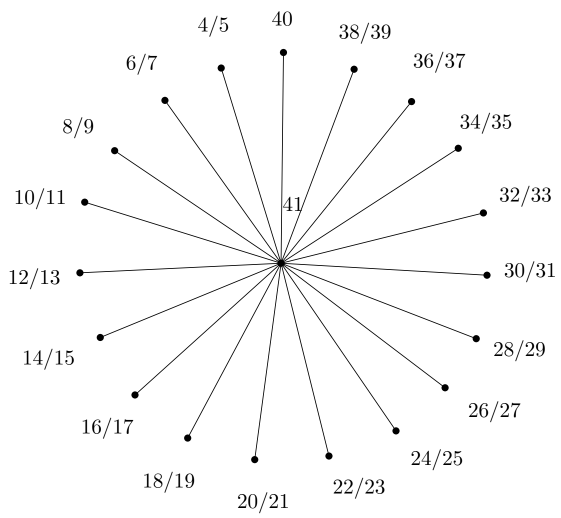
Then, $p = 41$ and $q = 33$. Suppose G is a skolem mean graph. Then there exists a function f from the vertex set of G to $\{1, 2, 3, \ldots p\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, \ldots p\}$ defined by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if} f(u) + f(v) \text{is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if} f(u) + f(v) \text{is odd} \end{cases}$$

then the resulting edges get distinct lables from the set $\{2, 3, \ldots p\}$. Let $t_{i,j}$ be the label given to the vertex $v_{i,j}$ for $1 \le i \le 7$; $0 \le j \le 2$ and $v_{8,j}$ for $0 \le j \le 19$ and $x_{i,j}$ be the corresponding edge label of the edge $v_{i,0}v_{i,j}$ for $1 \le i \le 7$; $0 \le j \le 2$ and $v_{8,0}v_{8,j}$ for $1 \le j \le 19$. Let us first consider the case that $t_{8,0} = 41$. If $v_{8,j} = 2n$ and $t_{8,k} = 2n + 1$ for some n and for some j and k then

$$f^*(v_{8,0}v_{8,j}) = \frac{41 + 2n}{2} = 21 + n = \frac{41 + 2n + 1}{2} = f^*(v_{8,0}v_{8,k}).$$

This is not possible as f^* is a bijection. Therefore the nineteen vertices $t_{8,j}$ for $1 \le j \le 19$ are among the 21 numbers 1, (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or 15), (16 or 17),(18 or 19), (20 or 21), (22 or 23), (24 or 25),(26 or 27), (28 or 29), (30 or 31), (32 or 33), (34 or 35), (36 or 37), (38 or 39) and 40. Since $t_{8,0} = 41$, first let us consider all the biggest edge labels possible for $K_{1,19}$. That is for nineteen vertices, $t_{8,j}$ for $1 \le j \le 19$ consider the nineteen choices that may induce the larger edge values. Orales, If 1 and 2 or 3 belongs to $t_{8,j}, 1 \le j \le 19$, then $x_{8,j}, 1 \le j \le 19$ will be $21, 22, \cdots, 39$ and 40 and (38 or 39) does not belong to $t_{8,j}, 1 \le j \le 19$, that is they should be allotted to $t_{i,j}$ where $1 \le i \le 7$, $0 \le j \le 2$, then $x_{i,j}$ will be greater than 21, which is not possible. Therefore the 19 choices are, (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or 15), (16 or 17), (18 or 19), (20 or 21), (22 or 23), (24 or 25),(26 or 27), (28 or 29), (30 or 31), (32 or 33), (34 or 35), (36 or 37), (38 or 39) and 40.



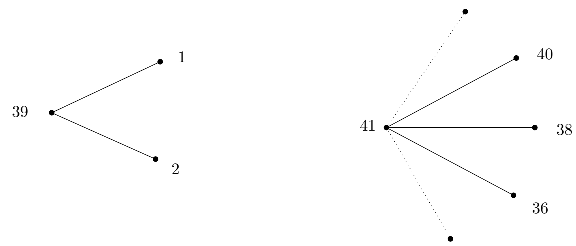
The corresponding edge labels are $23, 24, \cdots, 41$. Primarily, $t_{8,1}$ is 40. Next, $t_{8,2}$ is 38 or 39. First we consider the case that $t_{8,2} = 38$.

Case A: $t_{8,2} = 38$ (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{1,0} = 39$).

Now 39 is a label of either $t_{i,0}$ for $1 \le i \le 7$ or $t_{i,j}$ for $1 \le i \le 7; 1 \le j \le 2$. That is 39 is a label of pendent or non pendent vertex in a $k_{1,2}$ component of G. Let us assume that $t_{1,0} = 39$.

Case A1: $t_{1,0} = 39$ (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{1,0} = 39$).

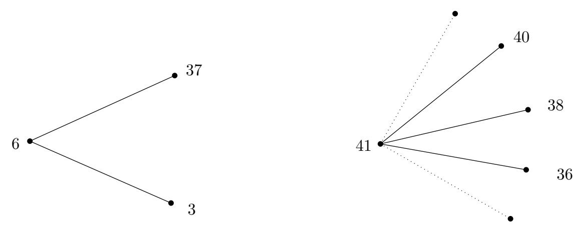
If $t_{1,0} = 39$ then $t_{1,1}$ take the values one among $1, 2, \cdots, 5$. (As $t_{1,1} \geq 6$ would imply that $x_{1,1} \geq 23$ this is not possible). Let $t_{1,1} = 1$ and $t_{1,2} = 2 or 3$, suppose $t_{1,2} = 2$. Then the corresponding edge labels are $x_{1,1} = 20$ and $x_{1,2} = 21$. Next $t_{8,3}$ is either 36 or 37.



Case B: $t_{8,3} = 36$ (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{8,3} = 36; t_{1,0} = 39; t_{1,1} = 1, t_{1,2} = 2; x_{8,1} = 41; x_{8,2} = 40; x_{8,3} = 39; x_{1,1} = 20; x_{1,2} = 21$).

If $t_{8,3} = 36$ then let $t_{2,0} = 37$. If $t_{2,1} \geq 8$ then $x_{2,1} \geq 23$ this is not possible. Hence, $t_{2,1}$ should be among $3, 4 \cdots, 7$, let $t_{2,1} = 3$, then $x_{2,1} = 20$ but $x_{1,1} = 20$, therefore $t_{2,1} \neq 3$. Also $t_{2,1} \neq 4$ and 5, due to the same reason, so let $t_{2,1} = 6$, then $x_{2,1} = 22$. $t_{2,2}$ should be labeled as such $x_{2,2} \leq 19$. 1 is the only choice for $t_{2,2}$ but 1 is already allotted to $t_{1,1}$ which implies $t_{2,0} \neq 37$. Suppose that $t_{2,0} = 6$ and $t_{2,1} = 37$, then $t_{2,2} = 3$ implies $x_{2,2} = 5$

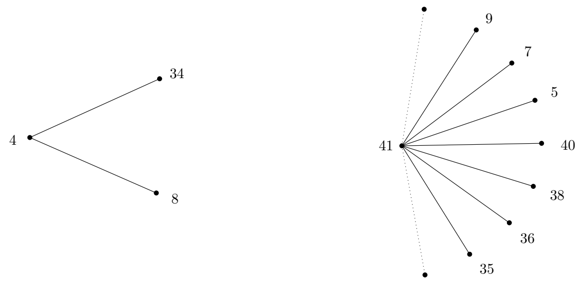
Case C: $t_{8,4} = 34$ or 35 (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{8,3} = 36; t_{1,0} = 39; t_{1,1} = 1, t_{1,2} = 2; t_{2,0} = 6; t_{2,1} = 37; t_{2,2} = 3; x_{8,1} = 41; x_{8,2} = 40; x_{8,3} = 39; x_{1,1} = 20; x_{1,2} = 21; x_{2,1} = 22; x_{2,2} = 5$).

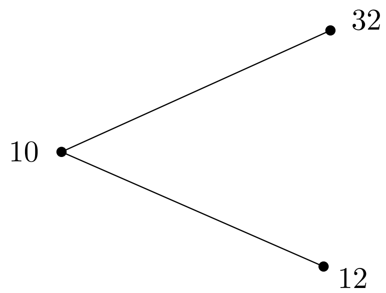


Now, let $t_{8,4} = 34$, so 35 should be a label of unlabeled vertex. To avoid the complications let us allot 35 to a pendent vertex. Without loss of generality, let it be $t_{3,1}$, that is $t_{3,1} = 35$. $x_{3,i}, 1 \leq i \leq 2$ should be less than or equal to 19 as well as not equal to 5. The smallest of the available vertex label is 4, if 4 allotted to $t_{3,0}$, then $x_{2,i}$ will be greater than 20. Hence let $t_{8,4} = 35$ and $t_{3,1} = 34$, now if $t_{3,0} = 4$, then $x_{3,1} = 19$. $t_{3,2} \neq 5$ as it may imply $x_{3,2} = 5$, which is a contradiction. Therefore let $t_{3,2} = 8$, this leads to conclude the following label, $t_{8,19} = 5$, $t_{8,18} = 7$ and $t_{8,17} = 9$ implying the following edge labels $x_{8,19} = 23; x_{8,18} = 24 x_{8,17} = 25$ and $x_{3,2} = 6$.

Case D: $t_{8,5} = 32$ or 33 (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{8,3} = 36; t_{8,4} = 35; t_{8,19} = 5; t_{8,18} = 7; t_{8,17} = 9; t_{1,0} = 39; t_{1,1} = 1, t_{1,2} = 2; t_{2,0} = 6; t_{2,1} = 37; t_{2,2} = 3; t_{3,0} = 4; t_{3,1} = 34; t_{3,2} = 8; x_{8,1} = 41; x_{8,2} = 40; x_{8,3} = 39; x_{8,4} = 38; x_{8,19} = 23; x_{8,18} = 24; x_{8,17} = 25; x_{1,1} = 20; x_{1,2} = 21; x_{2,1} = 22; x_{2,2} = 5; x_{3,1} = 19; x_{3,2} = 6$).

Let $t_{8,5} = 32$, then $t_{4,1} = 33$. So $x_{8,5} = 37$ and $x_{4,1}$ should be exclusive. All the remaining possibilities of $t_{4,0}$ are greater than or equal to 10, which implies $x_{4,1}$ will be greater than or equal to 22; but we see that all the edge labels greater than or equal to 22 are already allotted. $t_{4,0}$ seem to be left without choice of label. Now let us switch $t_{1,0}$ and $t_{1,1}$, therefore $t_{1,0} = 1$ and $t_{1,1} = 39$ which implies $x_{1,1} = 20$. We've $t_{1,2} = 2$ implying $x_{1,2} = 2$, now we have the edge label 21 free to be allotted, $x_{4,1}$ to be 21, we shall change $t_{8,5} = 33$ and $t_{4,1} = 32$, now $t_{4,0} = 10$ will imply $x_{4,1} = 21$ and $t_{8,16} = 11$. Also let $t_{4,2} = 12$ which in turn implies, $x_{4,2} = 11$ and $t_{8,15} = 13$





Case E: $t_{8,6} = 30$ or 31 (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{8,3} = 36; t_{8,4} = 35; t_{8,5} = 33; t_{8,19} = 5; t_{8,18} = 7; t_{8,17} = 9; t_{8,16} = 11; t_{8,15} = 13; t_{1,0} = 1; t_{1,1} = 39, t_{1,2} = 2; t_{2,0} = 6; t_{2,1} = 37; t_{2,2} = 3; t_{3,0} = 4; t_{3,1} = 34; t_{3,2} = 8; t_{4,0} = 10; t_{4,1} = 32; t_{4,2} = 12; x_{8,1} = 41; x_{8,2} = 40; x_{8,3} = 39; x_{8,4} = 38; x_{8,5} = 37; x_{8,19} = 23; x_{8,18} = 24; x_{8,17} = 25; x_{8,16} = 26; x_{8,15} = 27x_{1,1} = 20; x_{1,2} = 2; x_{2,1} = 22; x_{2,2} = 5; x_{3,1} = 19; x_{3,2} = 6; x_{4,1} = 21; x_{4,2} = 11$).

Let us first suppose, $t_{8,6} = 30$. And let $t_{5,1} = 31$, for $x_{5,1}$ and $x_{5,2}$ to be exclusive, they shouldn't be greater than 19 and $x_{5,1}$ can't be smaller than 16. Also, note that all the vertex label less than 13 are already labeled. Which asserts that $x_{5,1}$can not be exclusive at present. So now let us remove the vertex label 2 from $t_{1,2}$ and fix it in $t_{5,0}$ since it is most needed here. Let $t_{1,2} = 14$, implies $x_{1,2} = 8$ and $t_{8,14} = 15$. Therefore $t_{5,0} = 2$ implies $x_{5,1} = 17$.

Case F: $t_{8,7} = 28$ or 29 (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{8,3} = 36; t_{8,4} = 35; t_{8,5} = 33; t_{8,6} = 30; t_{8,19} = 5; t_{8,18} = 7; t_{8,17} = 9; t_{8,16} = 11; t_{8,15} = 13; t_{8,14} = 15; t_{1,0} = 1; t_{1,1} = 39, t_{1,2} = 2; t_{2,0} = 6; t_{2,1} = 37; t_{2,2} = 3; t_{3,0} = 4; t_{3,1} = 34; t_{3,2} = 8; t_{4,0} = 10; t_{4,1} = 32; t_{4,2} = 12; t_{5,0} = 2; t_{5,1} = 31; x_{8,1} = 41; x_{8,2} = 40; x_{8,3} = 39; x_{8,4} = 38; x_{8,5} = 37; x_{8,6} = 36x_{8,19} = 23; x_{8,18} = 24; x_{8,17} = 25; x_{8,16} = 26; x_{8,15} = 27; x_{8,14} = 28; x_{1,1} = 20; x_{1,2} = 8; x_{2,1} = 22; x_{2,2} = 5; x_{3,1} = 19; x_{3,2} = 6; x_{4,1} = 21; x_{4,2} = 11; x_{5,1} = 17$). Let $t_{8,7} = 28$ and yet unlabeled vertex $t_{5,2} = 29$ implies $x_{5,2} = 16$. Note that $x_{5,2}$ is exclusive from all the other edge labels.

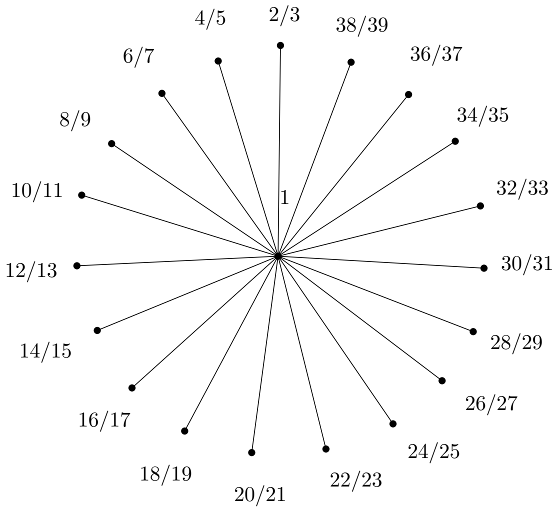
Case G: $t_{8,8} = 26$ or 27 (we've $t_{8,0} = 41; t_{8,1} = 40; t_{8,2} = 38; t_{8,3} = 36; t_{8,4} = 35; t_{8,5} = 33; t_{8,6} = 30; t_{8,7} = 28; t_{8,19} = 5; t_{8,18} = 7; t_{8,17} = 9; t_{8,16} = 11; t_{8,15} = 13; t_{8,14} = 15; t_{1,0} = 1; t_{1,1} = 39; t_{1,2} = 14; t_{2,0} = 6; t_{2,1} = 37; t_{2,2} = 3; t_{3,0} = 4; t_{3,1} = 34; t_{3,2} = 8; t_{4,0} = 10; t_{4,1} = 32; t_{4,2} = 12; t_{5,0} = 2; t_{5,1} = 31; t_{5,2} = 29; x_{8,1} = 41; x_{8,2} = 40; x_{8,3} = 39; x_{8,4} = 38; x_{8,5} = 37; x_{8,6} = 36; x_{8,7} = 35; x_{8,19} = 23; x_{8,18} = 24; x_{8,17} = 25; x_{8,16} = 26; x_{8,15} = 27; x_{8,14} = 28; x_{1,1} = 20; x_{1,2} = 8; x_{2,1} = 22; x_{2,2} = 5; x_{3,1} = 19; x_{3,2} = 6; x_{4,1} = 21; x_{4,2} = 11; x_{5,1} = 17; x_{5,2} = 16$).

Suppose that $t_{8,8} = 26$ and one of the unlabeled vertex should be 27, we know that all the vertex label smaller than 15 are allotted to the vertices, so giving label greater than 15 to the adjacent vertex of the unknown vertex labeled 27, they will induce an edge label 21, but 21 is already the edge label of $x_{4,1}$. Which fails the bijection of the labeling defined.

Obviously, $G = 7K_{1,2} \cup K_{1,19}$ is not a skolem mean graph for $t_{8,0} = 41$. A similar argument can prove that G is not a skolem mean graph when $t_{8,0}$ takes other values as such the edges $x_{8,j}$ gets the higher values. Now let us consider the interrogation when the vertices of $K_{1,19}$ are labeled as such its edges receives the smaller labels. That is consider the labeling when

$t_{8,0} = 1$, then for some j and k we see that, if $t_{8,j} = 2n$ and $t_{8,k} = 2n+1$, then $x_{8,j} = \frac{1+2n}{2} = n+1 = \frac{1+2n+1}{2} = x_{8,k}$. This is not possible as f^* is a bijection. Therefore the vertex labels to label the nineteen pendent vertices of $K_{1,19}$ as such those labels induce the smaller edge labels, are (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or 15), (16 or 17), (18 or 19), (20 or 21), (22 or 23), (24 or 25), (26 or 27), (28 or 29), (30 or 31), (32 or 33), (34 or 35), (36 or 37) and (38 or 39) and the corresponding edge labels are, $\{2, 3, \cdots, 20\}$.

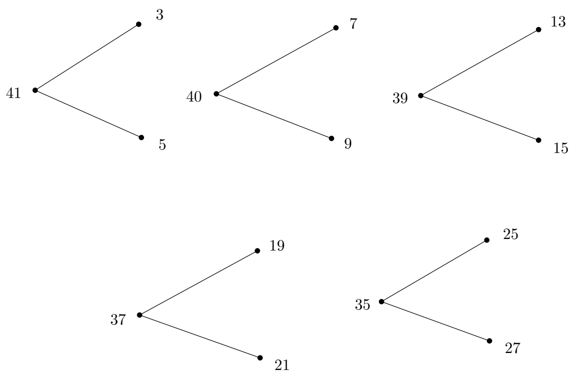
If 40 and 41 belong to $t_{8,j}$ then $x_{8,j} = 21, 20, \cdots, 3$; 2 and 3 does not belong to $t_{8,j}$, then they must be assigned to other $t_{i,j}$, then the incident $x_{i,j}$ will be greater than 2 and less than 21, which is not supposed to happen. Hence 40 and 41 does not belong to $t_{8,j}$. Therefore the other $t_{i,j}$ should be assigned with labels as such the incident edges $x_{i,j}$ gets labels greater than 20. So we shall choose the smaller number among the available choices in $t_{8,j}$.



Therefore the labels $t_{8,j}, 1 \leq j \leq 19$ be $2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38$. The remaining vertex labels are $3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 40, 41$ with this we should label the remaining vertices as such they never induce edge label less than 21. The possibilities of the labels to induce edge label less than 21 are when the labels 3, 5, 7, 9, 11, 13, 15, 17, 19, 21 mend among themselves. Therefore let us allot the possible biggest numbers to the non-pendant vertex of $K_{1,2}$ components of G so that they will generate big edge labels, also the smaller labels will not mend among themselves.

Therefore, $t_{1,0} = 41, t_{2,0} = 40, t_{3,0} = 39, t_{4,0} = 37, t_{5,0} = 35, t_{6,0} = 33$ and $t_{7,0} = 31$. The remaining vertex labels are, $3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29$ and the remaining edge labels are $21, 22, \cdots, 41$. Now the remaining vertex labels should be allotted to the vertices, $t_{i,j}, 1 \leq i \leq 7, 1 \leq j \leq 2$, such that they induce distinct edge labels greater than 20. To obtain this let us allot the least of the possible numbers to the vertex adjacent to the biggest possible non-pendant vertex. Let, $t_{1,1} = 3 \Rightarrow x_{1,1} = 22$; $t_{1,2} = 5 \Rightarrow x_{1,2} = 23$. $t_{2,1} = 7 \Rightarrow x_{2,1} = 24$; $t_{2,2} = 9 \Rightarrow x_{2,2} = 25$. $t_{3,1} = 11 \Rightarrow x_{3,1} = 25$ but $x_{2,2} = 25$, therefore $t_{3,1} \neq 11$. If $t_{3,1} = 13 \Rightarrow x_{3,1} = 26$; $t_{3,2} = 15 \Rightarrow x_{3,2} = 27$. $t_{4,1} = 17 \Rightarrow x_{4,1} = 27$ but $x_{3,2} = 27$, therefore $t_{4,1} \neq 17$. If $t_{4,1} = 19 \Rightarrow x_{4,1} = 28$; $t_{4,2} = 21 \Rightarrow x_{4,2} = 29$. $t_{5,1} = 23 \Rightarrow x_{5,1} = 29$ but $x_{4,2} = 29$, therefore $t_{5,1} \neq 23$. If $t_{5,1} = 25 \Rightarrow x_{5,1} = 30$; $t_{5,2} = 27 \Rightarrow x_{5,2} = 31$.

The vertices, $t_{6,1}, t_{6,2}, t_{7,1}, t_{7,2}$ are yet to be labeled. The remaining vertex labels are $11, 17, 23, 29$ and the remaining edge labels are $21, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41$. We have to label the remaining vertices with the remaining vertex labels such that they induce labels from the remaining edge labels exclusively.



Among the remaining edge labels, the possibilities to get the edge label 41 is its ends to be 40 and 41 or vice versa. We have allotted 40 and 41 to the non-pendant vertex of two different components of G. Therefore getting the edge label 41 is not possible. Also, the possibilities to induce the edge label 40 are the edge with ends 41 and 38 or 40 and 39, which is also not possible cause all the four labels have been allotted to the non-pendant vertices of different components of G. From the remaining labels the biggest edge label possible is 31 [cause, the biggest non-pendant vertex label with unlabeled pendant vertices is 35 and the biggest label among the remaining vertex labels is 29, together they may induce the edge label 31, which is already the label of $x_{5,2}$, so inducing any other edge label bigger than 31 is not possible]. We know that, $t_{6,1}, t_{6,2}, t_{7,1}, t_{7,2}$ are yet to be labeled and $11, 17, 23, 29$. If $t_{6,1} = 11, t_{6,2} = 17, t_{7,1} = 23, t_{7,2} = 29$, then $x_{6,1} = 22, x_{6,2} = 25, x_{7,1} = 27$, $x_{7,2} = 30$, we see that all the four edge labels already exists. Therefore $t_{6,j} \neq 11$ and 17 and $t_{7,j} \neq 23$ and 27 for $j = 1, 2$.

Now let $t_{6,1} = 23, t_{6,2} = 29, t_{7,1} = 11$ and $t_{7,2} = 17$, implies $x_{6,1} = 28, x_{6,2} = 31, x_{7,1} = 21$ and $x_{7,2} = 24$. Here 21 is the only exclusive edge label and all the other edge labels already exists. Therefore, $t_{6,j} \neq 23$ and 29, $t_{7,2} \neq 17$ and $t_{7,1} = 11$, implying $x_{7,1} = 21$. And $23, 27, 17$ are yet to be labeled. Let us try to label them exclusively. Suppose if the vertices of $t_{1,j}, j = 0, 1, 2$ are replaced. If $t_{1,0}$ is replaced by any one of the remaining vertex labels, we see that it induces the edge label less than 20, which fails the bijection property of f, since all the edge labels less than 20 are the edge labels of $K_{1,19}$ component of G. If pendant vertices of $t_{1,j}, j = 1, 2$ is replaced by the remaining vertex labels, $23, 37, 17$ we see that they induce the edge labels $32, 34, 29$ respectively. But $x_{4,2} = 29$, so let us neglect the vertex label inducing the edge label 29 and replace 3 and 5 of $t_{1,1}$ and $t_{1,2}$ by 23 and 27. So the edge label, $x_{1,1} = 32$ and $x_{1,2} = 34$, they are exclusive. Now the remaining vertex labels are $3, 5, 17$.

If $t_{2,0}$ is replaced by any one of the remaining vertex, then it induce the edge labels less than 20, which is not preferred. If the pendant vertices are replaced, then it induce the edge labels, $22, 23, 29$ respectively. Here the edge label 22 and 23 are exclusive, but $x_{4,2} = 29$. Replacing 7 and 9 by 3 and 5, we get $x_{2,1} = 22$ and $x_{2,2} = 23$. Now the remaining vertex labels are $7, 9, 17$. If $t_{3,0}$ is replaced by any one of the remaining vertex, then it induce the edge labels less than 20, which is not preferred. If the pendant vertices are replaced, then it induce the edge labels, $23, 24, 28$ respectively. Here the edge label 24 is exclusive, $x_{4,1} = 28$and $x_{2,2} = 22$. So let us replace 13 by 9, hence $t_{3,1} = 9$ implies $x_{3,1} = 24$. Now the remaining vertex labels are $7, 13, 17$. We know the vertices yet to be labeled are $t_{6,2}, t_{7,1}, t_{7,2}$, on labeling 7 or 13 or 17 to $t_{6,2}$, we get the following edge labels, (remember that $t_{6,0} = 33$ and $t_{7,0} = 31$) $x_{6,2}$ will be 20 or 23 or 25 respectively, having $x_{8,19} = 20$ and $x_{2,2} = 23$,the edge label 25 is only exclusive. Therefore let us fix $t_{6,2} = 17$.

The remaining vertex labels are 7 and 13; the vertices yet to be labeled are $t_{7,1}$ and $t_{7,2}$, on allotting them we get $x_{7,1} = 19$and $t_{7,2} = 22$, acknowledge that both the edge labels already exists, ($x_{8,18} = 19$ and $x_{2,1} = 22$) even on switching the values of $t_{7,1}$ and $t_{7,2}$ we will get the same edge labels, that is we have two vertex labels and two vertices yet to label and labeling them in all the ways induces the edge labels that already exists. Hence, we have failed to generate a skolem mean labeling for $G = 7K_{1,2} \cup K_{1,19}$, even when the $K_{1,19}$ component of G takes smaller of the values. Hence, $G = 7K_{1,2} \cup K_{1,19}$ is not a skolem mean graph when G assumes smaller as well as greater values. Hence $G = 7K_{1,2} \cup K_{1,19}$, ia not a skolem mean graph. That is G is not a skolem mean graph when $|m - n| = 5 + 6\ell$. In a similar way we shall prove that $G = 7K_{1,2} \cup K_{1,20}$ is also not a skolem mean graph. Argumentally we may assert that graph with bigger differences between m and n will never make a skolem mean graph. Hence, the eight star $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m - n| > 4 + 6\ell$ for $\ell = 2, 3, ...$; $m = 2, 3, ...$ □

3. Applications

The skolem mean labeling is applied on a graph (network) in order to enhance fastness, efficient communication and various issues,

(1). A protocol, with secured communication can be achieved, provided the graph (network) is sufficiently connected.

(2). To find an efficient way for safer transmissions in areas such as Cellular telephony, Wi-Fi, Security systems and many more.

(3). Channel labeling can be used to determine the time at which sensor communicate.

Researchers may get the use of skolem mean labeling in their research concerned with the above discussed issues.

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