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Some Theorems on Anti $T-$Fuzzy Ideal of $\ell-$Ring

Research Article

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Abstract: In this paper, we made an attempt to study the properties of anti $T-$fuzzy ideal of$\ell-$ring and we introduce some definitions and theorems of product of anti $T-$fuzzy ideal of $\ell-$ring.

Keywords: Fuzzy subset, $T-$fuzzy ideal, anti $T-$fuzzy ideal, join of anti $T-$fuzzy ideal, union of anti $T-$fuzzy ideal and product of anti $T-$fuzzy ideal.

1. Introduction

The concept of fuzzy sets was initiated by L.A.Zadeh [9] in 1965. After the introduction of fuzzy sets several researchers explored on the generalization of the concept of fuzzy sets. In this paper we define, characterize and study the anti $T-$fuzzy right and left ideals. Z. D. Wang introduced the basic concepts of TL-ideals. We introduced anti $T-$fuzzy right ideals of $\ell-$ring. We compare fuzzy ideal introduced by Liu to anti $T-$fuzzy ideals. We have shown that ring is regular if and only if union of any anti $T-$ fuzzy right ideal with anti $T-$fuzzy left ideal is equal to its product. We discuss some theorems. We have shown that the product of anti $T-$fuzzy ideal of $\ell-$ring.

2. Main Results

Definition 2.1. *A non-empty set R is called lattice ordered ring or $\ell-$ring if it has four binary operations $+, \cdot, \vee, \wedge$ defined on it and satisfy the following*

(1). $(R, +, \cdot)$ *is a ring*

(2). $(R, \vee, \wedge)$ *is a lattice*

(3). $x + (y \vee z) = (x + y) \vee (x + z)\,;\;\; x + (y \wedge z) = (x + y) \wedge (x + z)$

$(y \vee z) + x = (y + x) \vee (z + x)\,;\;\; (y \wedge z) + x = (y + x) \wedge (z + x)$

(4). $x \cdot (y \vee z) = (xy) \vee (xz)\,;\;\; x \cdot (y \wedge z) = (xy) \wedge (xz)$

$(y \vee z) \cdot x = (yx) \vee (zx)\,;\;\; (y \wedge z) \cdot x = (yx) \wedge (zx)\,,$

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for all x , y, z in R and $x \geq 0$.

Example 2.2. *$(\mathbb{Z}, +, \cdot, \vee, \wedge)$ is a $\ell-$ring, where $\mathbb{Z}$ is the set of all integers.*

Example 2.3. *$(n\mathbb{Z}, +, \cdot, \vee, \wedge)$ is a $\ell-$ring, where $\mathbb{Z}$ is the set of all integers and $n \in \mathbb{Z}$.*

Definition 2.4. *A mapping $T : [0,1] \times [0,1] \to [0,1]$ is called a triangular norm [t-norm] if and only if it satisfies the following conditions:*

(1). $T(x,1) = T(1,x) = x$, for all $x \in [0,1]$.

(2). $T(x,y) = T(y,x)$, for all $x, y \in [0,1]$.

(3). $T(x, T(y,z)) = T(T(x,y), z)$ for all $x, y, z \in [0,1]$.

(4). $T(x,y) \leq T(x,z)$, whenever $y \leq z$, for all $x, y, z \in [0,1]$.

Definition 2.5. *A mapping from a nonempty set X to $[0,1]$; $\mu : X \to [0,1]$ is called a fuzzy subset of X.*

Definition 2.6. *A fuzzy subset μ of a ring R is called anti $T-$fuzzy right ideal if*

(1). $\mu(x-y) \leq T(\mu(x), \mu(y))$

(2). $\mu(xy) \leq \mu(x)$, for all x , y in R

Definition 2.7. *A fuzzy subset μ of a ring R is called anti $T-$fuzzy left ideal if*

(1). $\mu(x-y) \leq T(\mu(x), \mu(y))$

(2). $\mu(xy) \leq \mu(y)$, for all x , y in R

Theorem 2.8. *Every fuzzy right ideal of a ring R is an anti $T-$fuzzy right ideal.*

Proof. Let μ be a fuzzy right ideal of R. Then $\mu(x-y) \leq T(\mu(x), \mu(y))$ and $\mu(xy) \leq \mu(x)$, for all $x, y \in R$. Hence μ is an anti $T-$fuzzy ideal. □

Definition 2.9. *A fuzzy subset μ of a lattice ordered ring (or $\ell-$ring) R is called an anti fuzzy sub $\ell-$ring of R, if the following conditions are satisfied*

(1). $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$

(2). $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$

(3). $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

(4). $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$, for all x , y in R

Example 2.10. *Consider an anti-fuzzy subset μ of the $\ell-$ring $(Z, +, \cdot, \vee, \wedge)$*

$$\mu_1(x) = \begin{cases} 0.4, & \text{if } x \in \langle 2 \rangle; \\ 0.7, & \text{if } Z - \langle 2 \rangle. \end{cases}$$

Then μ_1 is an anti-fuzzy sub $\ell-$ring.

Definition 2.11. *A fuzzy subset μ of an $\ell-$ring R is called an anti fuzzy $\ell-$ring ideal (or) fuzzy $\ell-$ideal of R, if for all x , y in R the following conditions are satisfied*

(1). $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$

(2). $\mu(xy) \leq \min\{\mu(x), \mu(y)\}$

(3). $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

(4). $\mu(x \wedge y) \leq \min\{\mu(x), \mu(y)\}$, *for all x , y in R*

Definition 2.12. *A fuzzy subset μ of a ring R is called an anti $T-$fuzzy ideal, if the following conditions are satisfied,*

(1). $\mu(x-y) \leq T(\mu(x), \mu(y))$

(2). $\mu(xy) \leq \mu(x);\ \ \mu(xy) \leq \mu(y)$, *for all $x, y \in R$.*

Definition 2.13. *A fuzzy subset μ of a $\ell-$ring R is called an anti $T-$fuzzy ideal, if the following conditions are satisfied,*

(1). $\mu(x-y) \leq T(\mu(x), \mu(y))$

(2). $\mu(xy) \leq \mu(x);\ \ \mu(xy) \leq \mu(y)$

(3). $mu(x \vee y) \leq T(\mu(x), \mu(y)$

(4). $\mu(x \wedge y) \leq T(\mu(x), \mu(y))$, *for all x , y in R.*

Definition 2.14. *Now $(R = \{a, b, c\}, +, \cdot, \vee, \wedge)$ is a $\ell-$ring. The operations $+, \cdot, \vee$ and $\wedge$ defined by the following. Consider an anti-fuzzy subset μ_A of the $\ell-$ring R.*

$$\mu(x) = \begin{cases} 0.2, & \text{if } x = a; \\ 0.5, & \text{if } x = b; \\ 0.8, & \text{if } x = c. \end{cases}$$

Then μ is an anti $T-$fuzzy ideal of $\ell-$ring R.

Theorem 2.15. *If μ and λ are any two anti $T-$fuzzy ideal of $\ell-$rings R_1 and R_2 then the product of $\mu \times \lambda$ is also anti $T-$fuzzy ideal of $\ell-$ring $R_1 \times R_2$.*

Proof. Given μ and λ are any two anti $T-$fuzzy ideal of $\ell-$rings R_1 and R_2 respectively. Let $x, y \in R$.

(1). $$\begin{aligned}(\mu \times \lambda)(x-y) &= T(\mu(x-y), \lambda(x-y)) \\ &\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\ &= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\ &= T(T(T(\mu \times \lambda)(x)), \mu(y)), \lambda(y)) \\ &= T(T(\mu \times \lambda)(x)), T(\mu \times \lambda)(y))) \\ &= T((\mu \times \lambda)(x), (\mu \times \lambda)(y))\end{aligned}$$

Therefore, $(\mu \times \lambda)(x-y) \leq T((\mu \times \lambda)(x), (\mu \times \lambda)(y))$ for all $x, y \in R$.

(2). Since $\mu(xy) \leq \mu(x)$ and $\lambda(xy) \leq (x)$. Now $(\mu \times \lambda)(xy) \leq T(\mu(xy), \lambda(xy)) \leq T(\mu(x), \lambda(x)) \leq (\mu \times \lambda)(x)$. Therefore $(\mu \times \lambda)(xy) \leq (\mu \times \lambda)(x)$, for all $x, y \in R$.

(3). $(\mu \times \lambda)(x \vee y) = T(\mu(x \vee y), \lambda(x \vee y))$

$$\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))$$

$$= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))$$

$$= T(T(T(\mu \times \lambda)(x)), \mu(y)), \lambda(y))$$

$$= T(T(\mu \times \lambda)(x)), T(\mu \times \lambda)(y)))$$

$$= T((\mu \times \lambda)(x), (\mu \times \lambda)(y))$$

Therefore, $(\mu \times \lambda)(x \vee y) \leq T((\mu \times \lambda)(x), (\mu \times \lambda)(y))$ for all $x, y \in R$.

(4). $(\mu \times \lambda)(x \wedge y) = T(\mu(x \wedge y), \lambda(x \wedge y))$

$$\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))$$

$$= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))$$

$$= T(T(T(\mu \times \lambda)(x)), \mu(y)), \lambda(y))$$

$$= T(T(\mu \times \lambda)(x)), T(\mu \times \lambda)(y)))$$

$$= T((\mu \times \lambda)(x), (\mu \times \lambda)(y))$$

Therefore, $(\mu \times \lambda)(x \wedge y) \leq T((\mu \times \lambda)(x), (\mu \times \lambda)(y))$ for all $x, y \in R$. Thus, $\mu \times \lambda$ is an anti $T-$fuzzy right ideal of $\ell-$ring $R_1 \times R_2$.

□

Theorem 2.16. *If μ_i are anti $T-$fuzzy ideal of $\ell-$rings R_i, then $\prod \mu_i$ is an anti $T-$fuzzy ideal of $\ell-$ring ΠR_i.*

Proof. If μ_i are anti T-fuzzy ideal of $\ell-$rings R_i. Let $x, y \in R$ and let $\mu_i = \mu_1 \times \mu_2 \times \ldots \times \mu_n$

(1). $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x - y) = T(\mu_1(x - y), \mu_2(x - y), \ldots, \mu_n(x - y))$

$$\leq T(T(\mu_1(x), \mu_1(y)), T(\mu_2(x), \mu_2(y)), \ldots, T(\mu_n(x), \mu_n(y)))$$

$$= T(T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x)), T((\mu_1 \times \mu_2 \times \ldots.. \times \mu_n)(y)))$$

$$= T((\mu_1 \times \mu_2 \times \ldots.. \times \mu_n)(x), (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y))$$

Therefore, $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x - y) \leq T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x), (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y))$, for all $x, y \in R$.

(2). Since $\mu_i(xy) \leq \mu_i(x)$ and $\lambda_i(xy) \leq \lambda_i(x)$. Now,

$$(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(xy) = T(\mu_1(xy), \mu_2(xy), \ldots, \mu_n(xy))$$

$$\leq T(\mu_1(x), \mu_2(x), \ldots, \mu_n(x))$$

$$\leq (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x)$$

Therefore, $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(xy) \leq (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x)$, for all $x, y \in R$.

(3). $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x \vee y) = T(\mu_1(x \vee y), \mu_2(x \vee y), \ldots, \mu_n(x \vee y))$

$$\leq T(T(\mu_1(x), \mu_1(y)), T(\mu_2(x), \mu_2(y)), \ldots, T(\mu_n(x), \mu_n(y)))$$

$$= T(T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x)), T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y)))$$

$$= T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x), (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y))$$

Therefore, $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x \vee y) \leq T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x), (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y))$ for all $x, y \in R$.

(4). $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x \wedge y) = T(\mu_1(x \wedge y), \mu_2(x \wedge y), \ldots, \mu_n(x \wedge y))$

$$\leq T(T(\mu_1(x), \mu_1(y)), T(\mu_2(x), \mu_2(y)), \ldots, T(\mu_n(x), \mu_n(y)))$$

$$= T(T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x)), T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y)))$$

$$= T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x), (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y))$$

Therefore, $(\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x \wedge y) \leq T((\mu_1 \times \mu_2 \times \ldots \times \mu_n)(x), (\mu_1 \times \mu_2 \times \ldots \times \mu_n)(y))$, for all $x, y \in R$.

Thus $\mu_1 \times \mu_2 \times \cdots \times \mu_n$ is an anti T-fuzzy ideal of $\ell-$ring R_i. Hence $\prod \mu_i$ is an anti $T-$fuzzy ideal of $\ell-$ring R_i. □

Theorem 2.17. *Let R_1 and R_2 be $\ell-$rings. If μ_1 and μ_2 are any two anti $T-$fuzzy ideal of $\ell-$ring R_1 and R_2 respectively, then $\mu = \mu_{1\times}\mu_2$ is an anti $T-$fuzzy ideal of the direct product of $R_1 \times R_2$.*

Proof. Let μ_1 and μ_2, are any two anti $T-$fuzzy ideal of $\ell-$rings R_1 and R_2 respectively. Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in R_1 \times R_2$,

(1). $\mu((x_1, x_2) - (y_1, y_2)) = \mu(x_1 - y_1, x_2 - y_2)$

$$= (\mu_{1\times}\mu_2)(x_1 - y_1, x_2 - y_2)$$

$$= T(\mu_1(x_1 - y_1), \mu_1(x_2 - y_2))$$

$$\leq T(T(\mu_1(x_1), \mu_1(y_1)), T(\mu_1(x_2), \mu_1(y_2)))$$

$$\geq T(T(\mu_1(x_1), \mu_1(x_2)), T(\mu_1(y_1), \mu_1(y_2)))$$

$$= T((\mu_{1\times}\mu_2)(x_1, x_2), (\mu_{1\times}\mu_2)(y_1, y_2))$$

$$= T(\mu(x_1, x_2), \mu(y_1, y_2))$$

Therefore, $\mu((x_1, x_2) - (y_1, y_2)) \leq T(\mu(x_1, x_2), \mu(y_1, y_2))$, for all $(x_1, x_2), (y_1, y_2) \in R_1 \times R_2$.

(2). Since $\mu_i(xy) \leq \mu_i(x)$ and $\lambda_i(xy) \leq \lambda_i(x)$

$$\mu((x_1, x_2)(y_1, y_2)) = \mu(x_1 y_1, x_2 y_2)$$

$$= (\mu_{1\times}\mu_2)(x_1 y_1, x_2 y_2)$$

$$\leq T(\mu_1(x_1, y_1), \mu_2(x_2, y_2))$$

$$= (\mu_{1\times}\mu_2)(x_1, x_2)$$

Therefore, $\mu((x_1, x_2)(y_1, y_2)) \leq (\mu_{1\times}\mu_2)(x_1, x_2)$, for all $x, y \in R$.

(3). $\mu((x_1, x_2) \vee (y_1, y_2)) = \mu(x_1 \vee y_1, x_2 \vee y_2)$

$$= (\mu_{1\times}\mu_2)(x_1 \vee y_1, x_2 \vee y_2)$$

$$= T(\mu_1(x_1 \vee y_1), \mu_1(x_2 \vee y_2))$$

$$\leq T(T(\mu_1(x_1), \mu_1(y_1)), T(\mu_1(x_2), \mu_1(y_2)))$$

$$\geq T(T(\mu_1(x_1), \mu_1(x_2)), T(\mu_1(y_1), \mu_1(y_2)))$$

$$= T((\mu_{1\times}\mu_2)(x_1, x_2), (\mu_{1\times}\mu_2)(y_1, y_2))$$

$$= T(\mu(x_1, x_2), \mu(y_1, y_2))$$

Therefore $\mu((x_1, x_2) \vee (y_1, y_2)) \leq T(\mu(x_1, x_2), \mu(y_1, y_2))$, for all $(x_1, x_2), (y_1, y_2) \in R_1 \times R_2$.

(4). $\mu((x_1,x_2)\wedge(y_1,y_2)) = \mu(x_1\wedge y_1, x_2\wedge y_2)$

$$= (\mu_{1\times}\mu_2)(x_1\wedge y_1, x_2\wedge y_2)$$

$$= T(\mu_1(x_1\wedge y_1), \mu_1(x_2\wedge y_2))$$

$$\leq T(T(\mu_1(x_1), \mu_1(y_1)), T(\mu_1(x_2), \mu_1(y_2)))$$

$$\geq T(T(\mu_1(x_1), \mu_1(x_2)), T(\mu_1(y_1), \mu_1(y_2)))$$

$$= T((\mu_{1\times}\mu_2)(x_1,x_2), (\mu_{1\times}\mu_2)(y_1,y_2))$$

$$= T(\mu(x_1,x_2), \mu(y_1,y_2))$$

Therefore $\mu_A((x_1,x_2)\wedge(y_1,y_2)) \leq T(\mu(x_1,x_2), \mu(y_1,y_2))$, for all $(x_1,x_2), (y_1,y_2) \in R_1\times R_2$.

Thus $\mu = \mu_1\times\mu_2$ is an anti $T-$fuzzy ideal of the direct product of $R_1\times R_2$. □

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