International Journal of Mathematics And its Applications
Volume 5, Issue 4–A (2017), 19–30.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Contra Semi*δ-Continuous Functions in Topological Spaces

Research Article

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Abstract: In this paper we define contra-semi*δ-continuous, contra-semi*δ-irresolute, semi*δ-open and semi*δ-closed functions and investigate their properties.

MSC: 54C05.

Keywords: Contra-semi*δ-continuous, contra-semi*δ-irresolute, semi*δ-open map, semi*δ-closed map.

1. Introduction

In 1996, Dontchev introduced and investigated the notions of contra-continuity. Later Zainab Aodia Athbanaih introduced and investigated the concept of contra $(\delta, g\delta)$-continuous functions. Quite recently the authors have introduced the concept of semi*δ-open sets and studied their properties. The aim of this paper is to introduce and investigate a new class of functions called contra-semi*δ-continuous, contra-semi*δ-irresolute, semi*δ-open and semi*δ-closed.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. We recall some known definitions needed in this paper.

Definition 2.1. *Let (X, τ) be a topological space. A subset A of the space X is said to be*

(1). Semi-open if $A \subseteq Cl(Int(A))$ and semi-open if $A \subseteq Cl^*(Int(A))$.*

*(2). Pre open if $A \subseteq Int(Cl(A))$ and pre*open if $A \subseteq Int^*(Cl(A))$.*

(3). Semi-pre open if $A \subseteq Cl(Int(Cl(A)))$ and semi-pre open if $A \subseteq Cl^*(pInt(A)))$.*

(4). α-open if $A \subseteq Int(Cl(Int(A)))$ and α^*-open if $A \subseteq Int^*(Cl(Int^*(A)))$.

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(5). Regular-open if $A = Int(Cl(A))$ *and* $\delta-$*open if* $A = \delta Int(A)$.

(6). semi α*-open if* $A \subseteq Cl(\alpha Int(A))$ *and semi**α*-open if* $A \subseteq Cl^*(\alpha Int(A)))$.

(7). δ*-semi-open if* $A \subseteq Cl(\delta Int(A))$ *and semi**δ*-open* $A \subseteq Cl^*(\delta Int(A))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.2. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be*

(1). contra-continuous [4] if $f^{-1}(V)$ *is closed in* (X, τ) *for every open set* V *in* (Y, σ).

(2). contra-g-continuous [2] if $f^{-1}(V)$ *is g-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(3). contra-semi-continuous [5] if $f^{-1}(V)$ *is semi-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(4). contra-semi-continuous [10] if* $f^{-1}(V)$ *is semi*-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(5). contra-pre-continuous [7] if $f^{-1}(V)$ *is pre-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(6). contra-α*-continuous [6] if* $f^{-1}(V)$ *is* α*-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(7). contra-α^**-continuous [9] if* $f^{-1}(V)$ *is* α^**-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(8). contra-semi-pre-continuous [3] if $f^{-1}(V)$ *is semi-pre closed in* (X, τ) *for every open set* V *in* (Y, σ).

*(9). contra-semi*pre-continuous [8] if* $f^{-1}(V)$ *is semi*-pre closed in* (X, τ) *for every open set* V *in* (Y, σ).

(10). contra-semiα*-continuous [15] if* $f^{-1}(V)$ *is semi-*α*-closed in* (X, τ) *for every open set* V *in* (Y, σ).

*(11). contra-semi**α*-continuous [15] if* $f^{-1}(V)$ *is semi**α*-closed in* (X, τ) *for every open set* V *in* (Y, σ).

(12). contra-δ*-continuous [16] if* $f^{-1}(V)$ *is* δ*-closed in* (X, τ) *for every open set* V *in* (Y, σ).

Theorem 2.3 ([14])**.** *Every* δ*-open set is open.*

Theorem 2.4 ([11, 12])**.** *Every* δ*-open set is semi**δ*-open and every* δ*-closed set is semi**δ*-closed.*

Theorem 2.5 ([11])**.** *In any topological space,*

*(1). Every semi**δ*-open set is* δ*-semi-open.*

*(2). Every semi**δ*-open set is semi-open.*

*(3). Every semi**δ*-open set is semi*-open.*

*(4). Every semi**δ*-open set is semi*-preopen.*

*(5). Every semi**δ*-open set is semi-preopen.*

*(6). Every semi**δ*-open set is semi**α*-open*

*(7). Every semi**δ*-open set is semi*α*-open.*

Remark 2.6 ([12])**.** *Similar results for semi**δ*-closed sets are also true.*

Theorem 2.7 ([11])**.** *Arbitrary union of semi**δ*-open sets in* X *is also semi**δ*-open in* X.

Theorem 2.8 ([11]). *For a subset A of a topological space (X,τ) the following statements are equivalent:*

(1). A is semiδ-open.*

(2). $A \subseteq Cl^*(\delta Int(A))$.

(3). $Cl^*(\delta Int(A)) = Cl^*(A)$.

Theorem 2.9 ([12]). *For a subset A of a topological space (X,τ), the following statements are equivalent:*

(1). A is semiδ-closed.*

(2). $Int^*(\delta Cl(A)) \subseteq A$.

(3). $Int^*(\delta Cl(A)) = Int^*(A)$.

Theorem 2.10. *A subset A of a space X is*

(1). semiδ-open if and only if $s^*\delta Int(A) = A$ [11].*

(2). semiδ-closed if and only if $s^*\delta Cl(A) = A$ [12].*

3. Contra-Semi*δ-Continuous Functions

Definition 3.1. *A function $f : (X,\tau) \to (Y,\sigma)$ is called contra-semi*δ-continuous if $f^{-1}(V)$ is semi*δ-closed in (X,τ) for every open set V in (Y,σ).*

Example 3.2. *Let $X = Y = \{a,b,c,d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}, Y\}$. $S{*}\delta C(X,\tau) = \{\phi, \{c\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, X\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be defined by $f(a) = d$, $f(b) = c$, $f(c) = a$, $f(d) = b$. clearly, f is contra-semi*δ-continuous.*

Theorem 3.3. *Every contra-δ-continuous function is contra-semi*δ-continuous.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ be contra-δ-continuous. Let V be an open set in (Y,σ). Since f is contra-δ-continuous, $f^{-1}(V)$ is δ-closed in (X,τ). By Theorem 2.4, $f^{-1}(V)$ is semi*δ-closed in (X,τ). Hence f is contra-semi*δ-continuous. □

Remark 3.4. *It can be seen that the converse of the above theorem is not true.*

Definition 3.5. *A function $f : (X,\tau) \to (Y,\sigma)$ is called contra-δ-semi-continuous if $f^{-1}(V)$ is δ-semi-closed in (X,τ) for every open set V in (Y,σ).*

Theorem 3.6. *In any topological space,*

(1). Every contra-semiδ-continuous function is contra-δ-semi-continuous.*

(2). Every contra-semiδ-continuous function is contra-semi-continuous.*

(3). Every contra-semiδ-continuous function is contra-semi*-continuous.*

(4). Every contra-semiδ-continuous function is contra-semi*pre-continuous.*

(5). Every contra-semiδ-continuous function is contra-semi-pre-continuous.*

(6). Every contra-semiδ-continuous function is contra-semi*α-continuous.*

(7). Every contra-semiδ-continuous function is contra-semiα-continuous.*

Proof.

(1). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is δ-semi-closed in (X, τ). Hence f is contra-δ-semi-continuous.

(2). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is semi-closed in (X, τ). Hence f is contra-semi-continuous.

(3). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is semi*-closed in (X, τ). Hence f is contra-semi*-continuous.

(4). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is semi*pre -closed in (X, τ). Hence f is contra-semi*pre-continuous.

(5). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is semi-pre-closed in (X, τ). Hence f is contra-semi-pre-continuous.

(6). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is semi*α-closed in (X, τ). Hence f is contra-semi*α-continuous.

(7). Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let V be an open set in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X, τ). By Remark 2.6, $f^{-1}(V)$ is semiα-closed in (X, τ). Hence f is contra-semiα-continuous. □

Remark 3.7. *The converse of each of the statements in Theorem 3.6 is not true.*

Remark 3.8. *The concepts of contra-semi*δ-continuous and contra-continuous (resp. contra-g-continuous, contra-α-continuous, contra- precontinuous, contra-α*-continuous, contra-pre*-continuous) are independent.*

Remark 3.9. *The composition of two contra-semi*δ-continuous functions need not be contra-semi*δ-continuous and this can be shown by the following example.*

Example 3.10. *Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Y\}$ and $\eta = \{\phi, \{a\}, Z\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d$; $f(b) = c$; $f(c) = b$; $f(d) = a$ and define $g : (Y, \sigma) \to (Z, \eta)$ by $g(a) = c$; $g(b) = g(c) = g(d) = a$. Then f and g are contra-semi*δ-continuous but $g \circ f$ is not semi*δ-continuous. Since $\{a\}$ is open in (Z, η) but $(g \circ f)^{-1}(\{a\}) = f^{-1}(g^{-1}(\{a\})) = f^{-1}(\{b, c, d\}) = \{a, b, c\}$ which is not semi*δ-closed in (X, τ).*

Theorem 3.11. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent:*

(1). f is contra-semiδ-continuous.*

(2). For each $x \in X$ and each closed set F in Y containing f(x), there exists a semiδ-open set U in X containing x such that $f(U) \subseteq F$.*

*(3). The inverse image of each closed set in Y is semi*δ-open in X.*

(4). $Cl^*(\delta Int(f^{-1}(F))) = Cl^*(f^{-1}(F))$ *for every closed set F in Y.*

(5). $Int^*(\delta Cl(f^{-1}(V))) = Int^*(f^{-1}(V))$ *for every open set V in Y.*

Proof.
(1)⇒(2) Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-continuous. Let $x \in X$ and F be a closed set in Y containing $f(x)$. Then $V = Y \backslash F$ is an open set in Y not containing $f(x)$. Since f is contra-semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed set in X not containing x. That is, $f^{-1}(V) = X \backslash f^{-1}(F)$ is a semi*δ-closed set in X not containing x. Therefore $U = f^{-1}(F)$ is a semi*δ-open set in X containing x such that $f(U) \subseteq F$.
(2)⇒(3) Let F be a closed set in Y. Let $x \in f^{-1}(F)$, then $f(x) \in F$. By (2), there is a semi*δ-open set U_x in X containing x such that $f(x) \in f(U_x) \subseteq F$. That is, $x \in U_x \subseteq f^{-1}(F)$. Therefore $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$. By Theorem 2.7, $f^{-1}(F)$ is semi*δ-open in X.
(3)⇒(4) Let F be a closed set in Y. By (3), $f^{-1}(F)$ is a semi*δ-open set in X. By Theorem 2.8, $Cl^*(\delta Int(f^{-1}(F))) = Cl^*(f^{-1}(F))$.
(4)⇒(5) If V is any open set in Y, then $Y \backslash V$ is closed in Y. By (4), we have $Cl^*(\delta Int(f^{-1}(Y \backslash V))) = Cl^*(f^{-1}(Y \backslash V))$. Taking the complements, we get $Int^*(\delta Cl(f^{-1}(V))) = Int^*(f^{-1}(V))$.
(5)⇒(1) Let V be any open set in Y. Then by assumption, $Int^*(\delta Cl(f^{-1}(V))) = Int^*(f^{-1}(V))$. By Theorem 2.9, $f^{-1}(V)$ is semi*δ-closed. □

Theorem 3.12. *If* $f : (X, \tau) \to (Y, \sigma)$ *is semi*δ-continuous and* $g : (Y, \sigma) \to (Z, \eta)$ *is contra-continuous, then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is contra-semi*δ-continuous.*

Proof. Let V be an open set in (Z, η). Since g is contra-continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is semi*δ-continuous, and hence by Theorem 3.36 [13], $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi*δ-closed in (X, τ). Hence $g \circ f$ is contra-semi*δ-continuous. □

Theorem 3.13. *If* $f : (X, \tau) \to (Y, \sigma)$ *is contra-semi*δ-continuous and g:* $(Y, \sigma) \to (Z, \eta)$ *is continuous, then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is contra-semi*δ-continuous.*

Proof. Let V be an open set in (Z, η). Since g is continuous, $g^{-1}(V)$ is open in (Y, σ). Since f is contra-semi*δ-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi*δ-closed in (X, τ). Hence $g \circ f$ is contra- semi*δ-continuous. □

Theorem 3.14. *If* $f : (X, \tau) \to (Y, \sigma)$ *is contra-semi*δ-continuous and* $g : (Y, \sigma) \to (Z, \eta)$ *is contra-continuous, then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is semi*δ-continuous.*

Proof. Let V be an open set in (Z, η). Since g is contra-continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is contra-semi*δ-continuous, and hence by Theorem 3.11 $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi*δ-open in (X, τ). Therefore, $g \circ f$ is semi*δ-continuous. □

Theorem 3.15. *If* $f : (X, \tau) \to (Y, \sigma)$ *is semi*δ-irresolute and* $g : (Y, \sigma) \to (Z, \eta)$ *is contra semi*δ-continuous, then their composition* $g \circ f : (X, \tau) \to (Z, \eta)$ *is contra semi*δ-continuous.*

Proof. Let V be an open set in (Z, η). Since, g is contra semi*δ-continuous, then $g^{-1}(V)$ is semi*δ-closed in (Y, σ) and since f is semi*δ-irresolute, by invoking Theorem 4.5 [13], $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi*δ-closed in (X, τ). Therefore, $g \circ f$ is contra semi*δ-continuous. □

Theorem 3.16. *If $f : (X,\tau) \to (Y,\sigma)$ is contra-δ-continuous $g : (Y,\sigma) \to (Z,\eta)$ is continuous, then their composition $g \circ f : (X,\tau) \to (Z,\eta)$ is contra-semi*δ-continuous.*

Proof. Let V be an open set in (Z,η). Since, g is continuous, then $g^{-1}(V)$ is open in (Y,σ) and since f is contra-δ-continuous, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is δ-closed in (X,τ). Hence by Theorem 2.4, $(g\circ f)^{-1}(V)$ is semi*δ-closed in (X,τ). Therefore, $g \circ f$ is contra semi*δ-continuous. □

Theorem 3.17. *If $f : (X,\tau) \to (Y,\sigma)$ is contra semi*δ-continuous $g : (Y,\sigma) \to (Z,\eta)$ is δ-continuous, then their composition $g \circ f : (X,\tau) \to (Z,\eta)$ is contra-semi*δ-continuous.*

Proof. Let V be an open set in (Z,η). Since, g is δ-continuous, then $g^{-1}(V)$ is δ-open in (Y,σ) and by Theorem 2.3 $g^{-1}(V)$ is open in (Y,σ). Since f is contra-semi*δ-continuous, then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi*δ-closed in (X,τ). Therefore, $g \circ f$ is contra-semi*δ-continuous. □

Theorem 3.18. *Let X, Y be any topological spaces and Y be $T_{\frac{1}{2}}$ space. Then the composition $g \circ f : (X,\tau) \to (Z,\eta)$ of contra-semi*δ-continuous function $f : (X,\tau) \to (Y,\sigma)$ and the g-continuous function $g : (Y,\sigma) \to (Z,\eta)$ is contra-semi*δ-continuous.*

Proof. Let V be an closed set in (Z,η). Since g is g-continuous, then $g^{-1}(V)$ is g-closed in (Y,σ) and Y is $T_{\frac{1}{2}}$ space, $g^{-1}(V)$ is closed in (Y,σ). Since f is contra-semi*δ-continuous, by Theorem 3.11 $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is semi*δ-open in (X,τ). Therefore, again by Theorem 3.11 $g \circ f$ is contra-semi*δ-continuous. □

Definition 3.19. *A topological space (X,τ) is said to be $T_{S^*\delta}$-space, if every semi*δ-open set of X is open in X.*

Theorem 3.20. *Let $f : (X,\tau) \to (Y,\sigma)$ be a contra-semi*δ-continuous function and X be a $T_{S^*\delta}$-space. Then f is contra-continuous.*

Proof. Let V be any closed set in (Y,σ). Since f is contra semi*δ-continuous, $f^{-1}(V)$ is semi*δ-open in (X,τ). Then by assumption, $f^{-1}(V)$ is open in (X,τ). Therefore, f is contra-continuous. □

Theorem 3.21. *If a function $f : (X,\tau) \to (Y,\sigma)$ is semi*δ-continuous and if Y is locally indiscrete, then f is contra-semi*δ-continuous.*

Proof. Let V be an open set in (Y,σ). Since Y is locally discrete, V is closed in (Y,σ). Since, f is semi*δ-continuous, $f^{-1}(V)$ is semi*δ-closed in (X,τ). Therefore, f is contra-semi*δ-continuous. □

Theorem 3.22. *Let $f : (X,\tau) to (Y,\sigma)$ be a function and $g : X \to X \times Y$ the graph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then f is contra-semi*δ-continuous if g is contra- semi*δ-continuous.*

Proof. Let V be an open subset of (Y,σ). Then $X \times V$ is an open subset of $X \times Y$. Since g is a contra-semi*δ-continuous, then $g^{-1}(X \times V)$ is semi*δ-closed subset of X. Also, $g^{-1}(X \times V) = f^{-1}(V)$. Hence, f is contra-semi*δ-continuous. □

4. Contra Semi*δ-Irresolute Functions

Definition 4.1. *A function $f : (X,\tau) \to (Y,\sigma)$ is said to be contra-semi*δ-irresolute if $f^{-1}(V)$ is semi*δ-closed in (X,τ) for every semi*δ-open set V in (Y,σ).*

Theorem 4.2. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following are equivalent:*

(1). f is contra-semiδ-irresolute.*

(2). For each $x \in X$ and each semiδ-closed set F in Y with $f(x) \in F$, there exists a semi*δ-open set U in X such that $x \in U$ and $f(U) \subseteq F$.*

(3). The inverse image of each semiδ-closed set in Y is semi*δ-open in X.*

(4). $Cl^*(\delta Int(f^{-1}(F))) = Cl^*(f^{-1}(F))$ for every semiδ-closed set F in Y.*

(5). $Int^*(\delta Cl(f^{-1}(V))) = Int^*(f^{-1}(V))$ for every semiδ-open set V in Y.*

Proof.
(1)⇒(2) Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-irresolute. Let $x \in X$ and F be a semi*δ-closed set in Y containing $f(x)$. Then $V = Y \backslash F$ is semi*δ-open set in Y not containing $f(x)$. Since f is contra-semi*δ-irresolute, $f^{-1}(V)$ is semi*δ-closed set in X not containing x. That is, $f^{-1}(V) = f^{-1}(Y \backslash F) = X \backslash f^{-1}(F)$ is a semi*δ-closed set in X not containing x. Therefore $U = f^{-1}(F)$ is a semi*δ-open set in X containing x such that $f(U) \subseteq F$.
(2)⇒(3) Let F be a semi*δ-closed set in Y. Let $x \in f^{-1}(F)$, then $f(x) \in F$. By (2), there exists a semi*δ-open set U_x in X containing x such that $f(x) \in f(U_x) \subseteq F$. That is, $x \in U_x \subseteq f^{-1}(F)$. Therefore $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$. By Theorem 2.7, $f^{-1}(F)$ is semi*δ-open in X.
(3)⇒(4) Let F be a semi*δ-closed set in Y. By (3), $f^{-1}(F)$ is a semi*δ-open set in X. By Theorem 2.8, $Cl^*(\delta Int(f^{-1}(F))) = Cl^*(f^{-1}(F))$.
(4)⇒(5) If V is any semi*δ-open set in Y, then $Y \backslash V$ is semi*δ-closed in Y. By (4), we have $Cl^*(\delta Int(f^{-1}(Y \backslash V))) = Cl^*(f^{-1}(Y \backslash V))$. Taking the complements, we get $Int^*(\delta Cl(f^{-1}(V))) = Int^*(f^{-1}(V))$.
(5)⇒(1) Let V be any semi*δ-open set in Y. Then by (5), $Int^*(\delta Cl(f^{-1}(V))) = Int^*(f^{-1}(V))$. By Theorem 2.9, $f^{-1}(V)$ is semi*δ-closed . Therefore f is contra-semi*δ-irresolute. □

Theorem 4.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be semi*δ-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ be contra-semi*δ-irresolute. Then $g \circ f : (X, \tau) \to (Z, \eta)$ is contra-semi*δ-irresolute.*

Proof. Let V be a semi*δ-open set in (Z, η). Since g is contra-semi*δ-irresolute, $g^{-1}(V)$ is semi*δ-closed in (Y, σ). Since f is semi*δ-irresolute, by invoking Theorem 4.5 [13], $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi*δ-closed in (X, τ). Hence $g \circ f$ is contra-semi*δ-irresolute. □

Theorem 4.4. *Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ be semi*δ-irresolute. Then $g \circ f : (X, \tau) \to (Z, \eta)$ is contra-semi*δ-irresolute.*

Proof. Let V be a semi*δ-open set in (Z, η). Since g is semi*δ-irresolute, $g^{-1}(V)$ is semi*δ-open in (Y, σ). Since f is contra-semi*δ-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi*δ-closed in (X, τ). Hence $g \circ f$ is contra-semi*δ-irresolute. □

Theorem 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be contra-semi*δ-irresolute and $g : (Y, \sigma) \to (Z, \eta)$ be contra-semi*δ-irresolute. Then $g \circ f : (X, \tau) \to (Z, \eta)$ is semi*δ-irresolute.*

Proof. Let V be a semi*δ-open set in (Z, η). Since g is contra-semi*δ-irresolute, $g^{-1}(V)$ is semi*δ-closed in (Y, σ). Since f is contra-semi*δ-irresolute, by Theorem 4.2, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi*δ-open in (X, τ). Hence $g \circ f$ is semi*δ-irresolute. □

5. Open and Closed Functions Associated with Semi*δ-Open Sets

Definition 5.1. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be semi*δ-open if $f(V)$ is semi*δ-open in Y for every open set V in X.*

Definition 5.2. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be contra-semi*δ-open if f(V) is semi*δ-closed in Y for every open set V in X.*

Definition 5.3. *A function $f:X\to Y$ is said to be pre-semi*δ-open if f(V) is semi*δ-open in Y for every semi*δ-open set V in X.*

Definition 5.4. *A function $f:X\to Y$ is said to be contra-pre-semi*δ-open if $f(V)$ is semi*δ-closed in Y for every semi*δ-open set V in X.*

Definition 5.5. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be semi*δ-closed if f(F) is semi*δ-closed in Y for every closed set F in X.*

Definition 5.6. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be contra-semi*δ-closed if f(F) is semi*δ-open in Y for every closed set F in X.*

Definition 5.7. *A function $f:X\to Y$ is said to be pre-semi*δ-closed if f(F) is semi*δ-closed in Y for every semi*δ-closed set F in X.*

Definition 5.8. *A function $f:X\to Y$ is said to be contra-pre-semi*$\boldsymbol{\delta}$-closed if f(F) is semi*δ-open in Y for every semi*δ-closed set F in X.*

Remark 5.9. *The composition of two semi*δ-closed maps need not be semi*δ-closed in general as shown in the following example.*

Example 5.10. *Consider $X=Y=Z=\{a,\ b,\ c,\ d\ \}$, $\tau=\{\phi,\ \{a\},\ \{b\},\ \{a,\ b\},\ \{b,\ c\},\ \{a,\ b,\ c\},\ X\ \}$, $\sigma=\{\phi,\ \{a\},\ \{b\},\ \{c\},\ \{a,b\},\ \{a,c\},\ \{b,c\},\ \{a,b,c\},\ Y\}$ and $\eta=\{\phi,\ \{a\},\ \{b\},\ \{a,b\},\ \{a,b,c\},\ Z\}$. $S^*\delta C(Y,\sigma)=\{\phi,\ \{a\},\ \{b\},\ \{c\},\ \{d\},\ \{a,b\},\ \{a,c\},\ \{a,d\},\ \{b,c\},\ \{b,d\},\ \{c,d\}\{a,b,d\},\ \{a,c,d\},\ \{b,c,d\},\ Y\}$, $S^*\delta C(Z,\eta)=\{\phi,\ \{c\},\ \{a,c\},\ \{b,c\},\ \{c,d\},\ \{a,c,d\},\ \{b,c,d\},\ Z\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=b,\ f(b)=a,\ f(c)=d,\ f(d)=a$. Clearly, f is semi*δ-closed. Consider the map $g:(Y,\sigma)\to(Z,\eta)$ defined by $g(a)=g(b)=d,\ g(c)=a,\ g(d)=c$, clearly g is semi*δ-closed. But $g\circ f:(X,\tau)\to(Z,\eta)$ is not semi*δ-closed.Since $g\circ f(\{a,d\})=g(f\{a,d\})=g\{a,b\}=\{d\}$ which is not semi*δ-closed in (Z,η).*

Theorem 5.11. *Let $f:(X,\tau)\to(Y,\sigma)$ be a bijection. Then the following are equivalent:*

(1). f^{-1} is semiδ-continuous.*

(2). f is semiδ-open.*

(3). f is semiδ-closed.*

Proof.

(1)⇒(2) Let V be an open set in X. Since, f^{-1} is semi*δ-continuous map then $(f^{-1})^{-1}(V)=f(V)$ is semi*δ-open set in Y. Hence, f is semi*δ-open map.

(2)⇒(3) Let F be a closed set of X. Then $X\backslash F$ is an open set in X. By hypothesis $f(X\backslash F)$ is semi*δ-open in Y. Since, $f(X\backslash F)=Y\backslash f(F)$. Hence, $f(F)$ is semi*δ-closed in Y. Therefore, f is semi*δ-closed .

(3)⇒(1) Let F be a closed set in X. Then $f(F)$ is semi*δ-closed in Y. Since $(f^{-1})^{-1}(F) = f(F)$, which is semi*δ-closed set in Y. Therefore, f is semi*δ-continuous. □

Theorem 5.12. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be surjective.*

(1). If $(g \circ f)$ is semiδ-open and f is continuous, then g is semi*δ-open.*

(2). If $(g \circ f)$ is continuous and f is semiδ-open, then g is semi*δ-continuous.*

(3). If $(g \circ f)$ is semiδ-continuous and g is an open map, then f is semi*δ-continuous.*

(4). If $(g \circ f)$ is open and g is semiδ-continuous, then f is semi*δ-open.*

(5). If $(g \circ f)$ is semiδ-open, f is semi*δ-continuous, and (X, τ) is a $T_{S^*\delta}$-pace, then g is semi*δ-open.*

Proof.

(1). Let V be an open set in (Y, σ). Then, $f^{-1}(V)$ is an open set in (X, τ). Since $(g \circ f)$ is semi*δ-open, $(g \circ f)(f^{-1}(V)) = g(f(f^{-1}(V)) = g(V)$ is semi*δ-open in (Z, η). Therefore, g is semi*δ-open.

(2). Let V be any open set in (Z, η). Since $(g \circ f)$ is continuous $(g \circ f)^{-1}(V)$ is open in (X, τ). Since f is semi*δ-open $f((g \circ f)^{-1}(V))$ is semi*δ-open in (Y, σ). Since $f((g \circ f)^{-1}(V) = f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ which is semi*δ-open in (Y, σ). Therefore g is semi*δ-continuous.

(3). Let V be any open set in (Y, σ). Since g is open, $g(V)$ is open in (Z, η). Also since $(g \circ f)$ is semi*δ-continuous $(g \circ f)^{-1}(g(V)) = f^{-1}(g^{-1}(g(V))) = f^{-1}(V)$ is semi*δ-open in (X, τ). Therefore f is semi*δ-continuous.

(4). Let V be an open set in (X, τ). Since $(g \circ f)$ is open, $(g \circ f)(V)$ is an open set in (Z, η). Also since g is semi*δ-continuous, $g^{-1}(g \circ f)(V) = f(V)$ is semi*δ-open in (Y, σ). Hence, f is semi*δ-open.

(5). Let V be a open set in (Y, σ). Since f is semi*δ-continuous, $f^{-1}(V)$ is semi*δ-open in (X, τ). Since X is a $T_{S^*\delta}$-space, $f^{-1}(V)$ is open in (X, τ). Since $g \circ f$ is semi*δ-open and f is surjective, $(g \circ f)(f^{-1}(V)) = g(V)$ is semi*δ-open in (Z, η). Thus g is semi*δ-open map. □

Theorem 5.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(1). f is semiδ-open.*

(2). $f(Int(A)) \subseteq s^*\delta Int(f(A))$ for every subset A of X.

Proof.

(1)⇒(2) Let $f : (X, \tau) \to (Y, \sigma)$ be semi*δ-open. Let A be a subset of X. Then $Int(A)$ is an open set in X. Since f is a semi*δ-open map, $f(Int(A))$ is a semi*δ-open set in Y. We have $Int(A) \subseteq (A)$. Thus $f(Int(A)) \subseteq f(A)$. Then s*$\delta Int(f(Int(A))) \subseteq s^*\delta Int(f(A))$ which implies $f(Int(A)) \subseteq s^*\delta Int(f(A))$.

(2)⇒(1) Let A be any open set in X. Then $Int(A) = A$. Thus $f(Int(A)) = f(A)$. But $f(Int(A)) \subseteq s^*\delta Int(f(A))$. That is $f(A) \subseteq s^*\delta Int(f(A))$. Also s*$\delta Int(f(A)) \subseteq f(A)$. By Theorem 2.10 (1), $f(A)$ is semi*δ-open and hence f is semi*δ-open. □

Theorem 5.14. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(1). f is semiδ-closed.*

(2). s$\delta Cl(f(A)) \subseteq f(Cl(A))$ for every subset A of X.*

Proof.

(1)⇒(2) Let $f : (X, \tau) \to (Y, \sigma)$ be semi*δ-closed. Let A be a subset of X. Then $cl(A)$ is a closed set in X. Since f is a semi*δ-closed map, $f(cl(A))$ is a semi*δ-closed set in Y. We have $A \subseteq cl(A)$. Thus $f(A) \subseteq f(cl(A))$. Then s*$\delta cl(f(A)) \subseteq s^*\delta cl(f(cl(A))) = f(cl(A))$.

(2)⇒(1) Let A be any closed set in X. Then $A = cl(A)$. Thus $f(A) = f(cl(A))$. But s*$\delta cl(f(A)) \subseteq f(cl(A)) = f(A)$. Also $f(A) \subseteq s^*\delta cl(f(A))$. By Theorem 2.10 (2), $f(A)$ is semi*δ-closed and hence f is semi*δ-closed. □

Theorem 5.15. *A map $f : (X, \tau) \to (Y, \sigma)$ is semi*δ-open if and only if for each subset S of (Y, σ) and for each closed set F of (X, τ) containing $f^{-1}(S)$, there exists a semi*δ-closed set V of (Y, σ) such that $S \subseteq V$ and $f^{-1}(V) \subseteq F$.*

Proof. Suppose that f is semi*δ-open. Let $S \subseteq Y$ and F be a closed set of (X, τ) such that $f^{-1}(S) \subseteq F$. Now $X\backslash F$ is an open set in (X, τ). Since f is semi*δ-open map, $f(X\backslash F)$ is semi*δ-open set in (Y, σ). Then, $V = Y\backslash f(X\backslash F)$ is a semi*δ-closed set in (Y, σ). Note that $f^{-1}(S) \subset F$ implies $S \subset V$ and $f^{-1}(V) = X\backslash f^{-1}(X\backslash F) \subseteq X\backslash(X\backslash F) = F$. That is , $f^{-1}(V) \subseteq F$.

Conversely, let B be an open set of (X, τ). Then, $f^{-1}((f(B))^c) \subseteq B^c$ and B^c is a closed set in (X, τ). By hypothesis, there exists a semi*δ-closed set V of (Y, σ) such that $(f(B))^c \subseteq V$ and $f^{-1}(V) \subseteq B^c$ and so $B \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(B) \subseteq f((f^{-1}(V)))^c$ which implies $f(B) = V^c$. Since V^c is a semi*δ-open. $f(B)$ is semi*δ-open in (Y, σ) and therefore f is semi*δ-open. □

Theorem 5.16. *If $f : (X, \tau) \to (Y, \sigma)$ is closed map and $g : (Y, \sigma) \to (Z, \eta)$ is semi*δ-closed, then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ is semi*δ-closed map.*

Proof. Let V be any closed set in (X, τ). Since f is closed map, $f(V)$ is a closed set in (Y, σ). Since g is semi*δ-closed map, $g(f(V))$ is semi*δ-closed in (Z, η) which implies $(g \circ f)(V) = g(f(V))$ is semi*δ-closed in (Z, η) and hence $g \circ f$ is semi*δ-closed. □

Remark 5.17. *If f: $(X, \tau) \to (Y, \sigma)$ is semi*δ-closed map and $g : (Y, \sigma) \to (Z, \eta)$ is closed, then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ is not semi*δ-closed map as shown in the following example.*

Example 5.18. *Consider $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, Y\}$, $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Z\}$, $S^*\delta C(Y, \sigma) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\} \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d\}, Y\}$; $S^*\delta C(Z, \eta) = \{\phi, \{a\}, \{d\}, \{a, d\}, \{b, c\}, \{b, c, d\}, Z\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = a$. Clearly, f is semi*δ-closed. Consider the map $g : (Y, \sigma) \to (Z, \eta)$ defined by $g(a) = c$, $g(b) = d$, $g(c) = a$, $g(d) = d$, clearly g is closed. But $g \circ f : (X, \tau) \to (Z, \eta)$ is not semi*δ-closed, since $g \circ f(\{c, d\}) = g(f\{c, d\}) = g\{a, d\} = \{c, d\}$ which is not semi*δ-closed in (Z, η).*

Theorem 5.19. *If $f : (X, \tau) \to (Y, \sigma)$ is g-closed, $g : (Y, \sigma) \to (Z, \eta)$ is semi*δ-closed and (Y, σ) is $T_{\frac{1}{2}}$ space, then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ is semi*δ-closed.*

Proof. Let V be any closed set in (X, τ). Since f is g-closed, $f(V)$ is g-closed in (Y, σ) and since Y is $T_{\frac{1}{2}}$ space, $f(V)$ is closed in (Y, σ). Also g is semi*δ-closed, $g(f(V)) = (g \circ f)(V)$ is semi*δ-closed in (Z, η). Therefore, $g \circ f$ is semi*δ-closed. □

Theorem 5.20. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two mappings such that their composition $g \circ f : (X, \tau) \to (Z, \eta)$ be semi*δ-closed mapping. Then the following statements are true.*

*(1). If f is continuous and surjective, then g is semi*δ-closed.*

*(2). If g is semi*δ-irresolute and injective, then f is semi*δ-closed.*

*(3). If f is g-continuous, surjective and (X, τ) is a $T_{\frac{1}{2}}$ space, then g is semi*δ-closed.*

Proof.

(1). Let V be a closed set in (Y, σ). Since f is continuous, $f^{-1}(V)$ is closed in (X, τ). Also since $g \circ f$ is semi*δ-closed which implies $(g \circ f)(f^{-1}(V))$ is semi*δ-closed in $(Z, \circ)$. That is $g(V)$ is semi*δ-closed in $(Z, \circ)$, since f is surjective. Therefore, g is semi*δ-closed.

(2). Let V be a closed set in (X, τ). Since $g \circ f$ is semi*δ-closed, $(g \circ f)(V)$ is semi*δ-closed in $(Z, \circ)$. Also since g is semi*δ-irresolute, $g^{-1}(g \circ f(V))$ is semi*δ-closed in (Y, σ). That is $f(V)$ is semi*δ-closed in (Y, σ), since g is injective. Therefore, f is semi*δ-closed.

(3). Let V be a closed set in (Y, σ). Since, f is g-continuous , $f^{-1}(V)$ is g-closed in (X, τ) and X is a $T_{\frac{1}{2}}$ space, $f^{-1}(V)$ is closed in (X, τ). Since g ?f is semi*δ-closed, $(g \circ f)(f^{-1}(V))$ is semi*δ-closed in (Z, η). That is $g(V)$ is semi*δ-closed in (Z, η), since f is surjective. Therefore, g is semi*δ-closed. □

Theorem 5.21. *Let $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \eta)$ be semi*δ-open maps and (Y, σ) be $T_{S^*\delta}$-space. Then their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is semi*δ-open.*

Proof. Let V be an open set in (X, τ). By assumption $f(V)$ is semi*δ-open in (Y, σ). Since Y is a $T_{S^*\delta}$-space, $f(V)$ is open in (Y, σ) and again by assumption $g(f(V))$ is semi*δ-open in (Z, η). Thus $g \circ f(V)$ is semi*δ-open in (Z, η). Hence $g \circ f$ is semi*δ-open. □

Theorem 5.22. *Let $f : X \to Y$ and be $g : Y \to Z$ be functions.*

*(1). $g \circ f$ is pre-semi*δ-open if both f and g are pre-semi*δ-open.*

*(2). $g \circ f$ is semi*δ-open if f is semi*δ-open and g is pre-semi*δ-open.*

*(3). $g \circ f$ is pre-semi*δ-closed if both f and g are pre-semi*δ-closed.*

*(4). $g \circ f$ is semi*δ-closed if f is semi*δ-closed and g is pre-semi*δ-closed.*

Proof.

(1). Let V be any semi*δ-open set in (X, τ). Since f is pre-semi*δ-open, $f(V)$ is semi*δ-open set in (Y, σ). Also since g is pre-semi*δ-open, $g(f(V)) = (g \circ f)(V)$ is semi*δ-open set in (Z, η). Hence $g \circ f$ is pre-semi*δ-open.

(2). Let V be any open set in (X, τ). Since f is semi*δ-open, $f(V)$ is semi*δ-open set in (Y, σ). Also since g is pre-semi*δ-open, $g(f(V)) = (g \circ f)(V)$ is semi*δ-open set in (Z, η). Hence $g \circ f$ is semi*δ-open.

(3). Let F be any semi*δ-closed set in (X, τ). Since f is pre-semi*δ-closed, $f(F)$ is semi*δ-closed set in (Y, σ). Also since g is pre-semi*δ-closed, $g(f(F)) = (g \circ f)(F)$ is semi*δ-closed set in (Z, η). Hence $g \circ f$ is pre-semi*δ-closed.

(4). Let F be any closed set in (X, τ). Since f is semi*δ-closed, $f(F)$ is semi*δ-closed set in (Y, σ). Also since g is pre-semi*δ-closed, $g(f(F)) = (g \circ f)(F)$ is semi*δ-closed set in (Z, η). Hence $g \circ f$ is semi*δ-closed. □

Acknowledgment

The first author is thankful to University Grants Commission, New Delhi, for sponsoring this work under grants of Major Research Project-MRP-MATH-MAJOR-2013-30929. F.No. 43-433/2014(SR) Dt. 11.09.2015.

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