International Journal of Mathematics And its Applications
Volume 5, Issue 4–A (2017), 81–87.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On the Geometry of Pseudo-slant Submanifolds of LP-Cosymplectic Manifold

Research Article

M.S. Siddesha[1], C.S. Bagewadi[2]*, D. Nirmala[2] and N. Srikantha[2]

1 Department of mathematics, New Horizon College of Engineering, Bangalore, India.

2 Department of Mathematics, Kuvempu University, Shankaraghatta, Shimoga, Karnataka, India.

Abstract: In this paper, we study pseudo-slant submanifolds of LP-cosymplectic manifold and obtained some interesting characterization results.

MSC: 53C42, 53C25, 53C40.

Keywords: Slant submanifold, pseudo-slant submanifold and LP-cosymplectic manifold.
© JS Publication.

1. Introduction

The slant immersion was initially studied by Chen ([6], [7]) as a generalization of both holomorphic and totally real submanifolds in almost Hermitian manifolds. Lotta [15] extended the concept of slant immersion into almost contact metric manifolds. As generalization, Papaghiuc [17](resp. Cabrerizo et al. [5]) introduced a new class of submanifolds called semi-slant submanifolds which is the generalization of CR-submanifolds and slant submanifolds in almost Hermitian manifold(resp. almost contact metric manifolds). Carriazo introduced and studied bi-slant submanifolds in almost Hermitian manifolds and simultaneously to anti-slant submanifolds in almost Hermitian manifolds [4]. Later V.A. Khan et al. renamed it as pseudo-slant submanifolds [11]. Since then many geometers like ([8], [11], [12], [13], [19], [20]) have studied slant and their generalized submanifolds of various manifolds. Motivated by these studies we study pseudo-slant submanifolds of LP-cosymplectic manifolds.

The paper is organized as follows; In section 2, we give a brief account of LP-cosymplectic manifold and submanifold. In section 3, we consider M is a pseudo-slant submanifold of LP-cosymplectic manifold $\tilde{M}$ and proved some characterization results

2. Preliminaries

Let $\tilde{M}$ be an a $(2n+1)$-dimensional paracontact manifold, equipped with a Lorentzian paracontact metric structure (ϕ, ξ, η, g) that is ϕ a (1,1) tensor field, ξ is a vector field, η a contact form and g Lorentzian metric of type (0.2) with signature

* *E-mail: prof_bagewadi@yahoo.co.in*

$(-,+,+,...,+)$ on $\tilde{M}$, satisfying

$$\phi^2(X) = X + \eta(X)\xi, \tag{1}$$
$$\phi\xi = 0,\ \eta\cdot\phi = 0,\ \eta(\xi) = -1, \tag{2}$$
$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \quad \eta(X) = g(X,\xi), \tag{3}$$

$\forall\ X, Y \in \Gamma(T\tilde{M})$ [16]. A Lorentzian paracontact metric structure on $\tilde{M}$ is said to be Lorentzian cosymplectic as simply LP-cosymplectic manifold, if

$$(\tilde{\nabla}_X\phi)Y = 0, \tag{4}$$

for any vector fields X, Y on $\tilde{M}$, where $\tilde{\nabla}$ denotes the Levi-Civita connection with respect to g. From (1) and (4), it follows that

$$(\tilde{\nabla}_X\xi) = 0, \tag{5}$$

for all $X, Y \in \Gamma(T\tilde{M})$. Now, let M be a submanifold of a Lorentzian almost paracontact manifold $\tilde{M}$ with the induced metric g. Then the Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y), \tag{6}$$
$$\tilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{7}$$

respectively, where ∇ and $\nabla^{\perp}$ are the induced connections on the tangent bundle TM and normal bundle $T^{\perp}M$ of M respectively. Here σ and A_V are the second fundamental form and shape operator corresponding to the normal vector field V and are related by

$$g(A_V X, Y) = g(\sigma(X,Y), V) \tag{8}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$. A submanifold M of a Lorentzian almost paracontact manifold $\tilde{M}$ is said to be totally umbilical if

$$\sigma(X,Y) = g(X,Y)H, \tag{9}$$

where H is the mean curvature.

3. Pseudo-slant Submanifold of a LP-cosymplectiic Manifold

In this section, we consider M is a pseudo-slant submanifold of a LP-cosyplectic manifold and obtain some characterization results.

For $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, we can set

$$\phi X = TX + FX \textit{ and } \phi V = tV + fV, \tag{10}$$

where TX (respectively tV) and FX (respectively fV) are the tangential and normal components of ϕX (respectivelyϕV). A submanifold M is said to be invariant (respectively anti-invariant) if F (respectively T) is identically zero i.e., $\phi X \in \Gamma(TM)$ (respectively $\phi X \in \Gamma(T^{\perp}M)$) for all $X \in \Gamma(TM)$. Thus by using (1) and (10) one can get

$$T^2 = I - tF + \eta\otimes\xi \quad FT + fF = 0, \tag{11}$$
$$f^2 = I - Ft \quad Tt + tf = 0. \tag{12}$$

We define the covariant derivatives of the tensor field T, F, t and f as

$$(\nabla_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{13}$$

$$(\nabla_X F)Y = \nabla^{\perp}_X FY - F\nabla_X Y, \tag{14}$$

$$(\nabla_X t)V = \nabla_X tV - t\nabla^{\perp}_X V, \tag{15}$$

$$(\nabla_X f)V = \nabla^{\perp}_X fV - f\nabla^{\perp}_X V, \tag{16}$$

respectively. Moreover, for any $X, Y \in \Gamma(TM)$, we have $g(TX, Y) = g(X, TY)$ and for $U, V \in \Gamma(T^{\perp}M)$, we have $g(U, fV) = g(fU, V)$. These imply that T and f are also symmetric tensor fields. Further for any $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M))$, we have

$$g(FX, V) = g(X, tV), \tag{17}$$

which is the relation between F and t. The Gauss and weingarten formulas together with (4) and (10) yield

$$(\nabla_X T)Y = A_{FY}X + t\sigma(X, Y), \tag{18}$$

$$(\nabla_X F)Y = f\sigma(X, Y) - \sigma(X, TY), \tag{19}$$

$$(\nabla_X t)V = A_{fV}X - TA_V X \tag{20}$$

and

$$(\nabla_X f)V = -\sigma(tV, X) - FA_V X, \tag{21}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$. Since $\xi \in TM$ by virtue of (5), (6), (8) and (10), we obtain

$$\nabla_X \xi = 0, \;\; \sigma(X, \xi) = 0, \;\; A_V \xi = 0, \tag{22}$$

for all $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M))$.

Definition 3.1 ([7]). *Let M be any submanifold, then M is said to be slant submanifold if for any $x \in M$ there exists a constant angle $\theta \in [0, \pi/2]$ between T_xM and ϕX for all $X \neq 0$ called slant angle of M. Hence one can see that slant submanifolds are generalization of invariant and anti-invariant submanifolds i.e., if $\theta = 0$ then M becomes invariant and if $\theta = \pi/2$ then M becomes anti-invariant submanifold.*

Theorem 3.2 ([21]). *Let M be a submanifold of an LP-cosymplectic manifold $\tilde{M}$ such that ξ is tangent to M. Then M is slant submanifold if and only if there exists a constant $\lambda \in [0, 1]$ such that*

$$T^2 = \lambda(I + \eta \bigotimes \xi). \tag{23}$$

Furthermore, in this case if θ is the slant angle of M, then $\lambda = cos^2\theta$.

From [5], we have

$$g(TX, TY) = \cos^2\theta g(X, Y) + \eta(X)\eta(Y) \tag{24}$$

$$g(FX, FY) = \sin^2\theta g(X, Y) + \eta(X)\eta(Y) \tag{25}$$

Let M be a slant submanifold of a lorentzian paracontact manifold $\tilde{M}$ with slant angle θ. Then for any $X \in \Gamma(TM)$, from (11) and (23), we have

$$tFX = \sin^2\theta(X + \eta(X)\xi) \tag{26}$$

and from (25), we have

$$F^2X = \sin^2\theta(X + \eta(X)\xi). \tag{27}$$

From (26) and (27), we obtain $F^2 = tF$.

Definition 3.3 ([11]). *A submanifold M of a LP-cosymplectic manifold $\tilde{M}$ is said to be pseudo-slant submanifold if there exists two orthogonal distributions D^θ and $D^\perp$ on M such that*

(1). $TM = D^\theta \oplus D^\perp \oplus <\xi>$,

(2). The distribution D^θ is slant distribution with slant angle $\theta \in [0, \pi/2]$,

(3). The distribution $D^\perp$ is anti-invariant i.e., $\phi X \in T^\perp M, \forall X \in TM$.

Theorem 3.4. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. Then the tensor F is parallel if and only if the tensor t is parallel*

Proof. By virtue of (8), (18) and (19) we have

$$\begin{aligned} g((\nabla_X F)Y, V) &= g(f\sigma(X,Y), V) - g(\sigma(X,TY), V) \\ &= g(\sigma(X,Y), fV) - g(A_V X, TY) \\ &= g(A_{fV}X, Y) - g(A_V X, TY) \\ &= g(A_{fV}X, Y) - g(TA_V X, Y) \\ &= g(A_{fV}X - TA_V X, Y) \\ &= g((\nabla_X t)V, Y), \end{aligned}$$

for every $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M))$. This completes the proof □

Theorem 3.5. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. Then the tensor F is parallel if and only if $A_{fV}Y = A_V TY$, for any $Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M))$.*

Proof. For $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^\perp M)$, by using (8) and (18) we obtain

$$\begin{aligned} g((\nabla_X F)Y, V) &= g(f\sigma(X,Y), V) - g(\sigma(X,TY), V) \\ &= g(\sigma(X,Y), fV) - g(A_V TY, X) \\ &= g(A_{fV}X, Y) - g(A_V TY, X) \\ &= g(A_{fV}Y, X) - g(A_V TY, X). \end{aligned}$$

This completes the proof. □

Theorem 3.6. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. The covariant derivation of T is symmetric i.e., $g((\nabla_X T)Y, Z) = g((\nabla_X T)Z, Y)$, $\forall X, Y, Z \in \Gamma(TM)$.*

Proof. By using (8), (17) and (18), we have

$$\begin{aligned} g((\nabla_X T)Y, Z) &= g(A_{FY}X + t\sigma(X,Y), Z) \\ &= g(\sigma(X,Z), FY) + g(\sigma(X,Y), FZ) \\ &= g(t\sigma(X,Z), Y) + g(A_{FZ}X, Y) \\ &= g(A_{FZ}X + g(t\sigma(X,Y), Y) \\ &= g((\nabla_X T)Z, Y), \end{aligned}$$

for any $X, Y, Z \in \Gamma(TM)$. This proves our assertion. □

Theorem 3.7. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. Then the tensor T is parallel if and only if $A_{FY}X = -A_{FX}Y$, $X, Y \in \Gamma(TM)$.*

Proof. By virtue of (8), (17) and (18), we have

$$\begin{aligned} g((\nabla_X T)Y, Z) &= g(A_{FY}X + t\sigma(X,Y), Z) \\ &= g(\sigma(X,Z), FY) + g(\sigma(X,Y), FZ) \\ &= g(A_{FY}Z, X) + g(A_{FZ}Y, X), \end{aligned}$$

for any $X, Y, Z \in \Gamma(TM)$. This completes the proof. □

Theorem 3.8. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. The covariant derivation of f is symmetric i.e., $g((\nabla_X f)V, U) = g((\nabla_X f)U, V)$, for every $X \in \Gamma(TM)$ and $U, V \in \Gamma(T^{\perp}M))$.*

Proof. By virtue of (8), (17) and (21) we have

$$\begin{aligned} g((\nabla_X f)V, U) &= g(-\sigma(tV, X) - FA_V X, U) \\ &= g(-A_U X, tV) - g(A_V X, tU) \\ &= -g(FA_U X, V) - g((\sigma(X, tU), V) \\ &= g(-FA_U X - \sigma(X, tU), V) \\ &= g((\nabla_X f)U, V), \end{aligned}$$

for any $X \in \Gamma(TM)$ and $V, U \in \Gamma(T^{\perp}M))$. This proves our assertion. □

Theorem 3.9. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. Then the tensor f is parallel if and only if the shape operator A_V of M satisfies*

$$A_V tU = -A_U tV \tag{28}$$

for any $U, V \in \Gamma(T^{\perp}M)$.

Proof. By virtue of (8), (17) and (21), we have

$$\begin{aligned} g((\nabla_X f)V, U) &= -g(\sigma(tV, X), U) - g(FA_V X, U) \\ &= -g(A_U tV, X) - g(A_V X, tU) \\ &= -g(A_V tU + A_U tV, X), \end{aligned}$$

for any $X \in \Gamma(TM)$ and $U, V \in \Gamma(T^{\perp}M))$. This completes the proof. □

Theorem 3.10. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. If tensor f is parallel then M totally geodesic submanifold of $\tilde{M}$.*

Proof. Since f is parallel, from (21), we have

$$\sigma(tV, X) + FA_V X = 0 \tag{29}$$

for all $X \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$. Applying ϕ to (29) and by virtue of (1) and (22), we obtain

$$\begin{aligned} 0 &= \phi^2 A_V X + \phi\sigma(tV, X) \\ &= A_V X + \eta(A_V X)\xi + t\sigma(tV, X) + n\sigma(tV, X). \end{aligned}$$

Comparing tangential component, we obtain

$$A_V X + t\sigma(tV, X) = 0.$$

On the other hand for $Z \in \Gamma(TM)$, by using (8), (3.3), (17) and (28), we get

$$\begin{aligned} g(A_V X, Z) &= -g(t\sigma(tV, X), Z) = -g(\sigma(tV, X), FZ) \\ &= -g(A_{FZ} tV, X) = -g(A_V tFZ, X). \end{aligned}$$

Taking account of $tFZ = Z + \eta(Z)\xi - T^2 Z$, we obtain

$$\begin{aligned} g(A_V Z, X) &= -g(A_V Z - \eta(Z)A_V\xi + A_V T^2 Z, X) \\ &= -g(A_V Z, X) + g(A_V X, T^2 Z). \\ \Longrightarrow g(T^2 A_V X, Z) &= 0. \end{aligned}$$

Hence, by applying (24), we get, $0 = g(TA_V X, TZ) = \cos^2\theta g(A_V X, Z)$. Since M is a proper pseudo-slant submanifold, we conclude that $A_V = 0$ i.e., M is totally geodesic in $\tilde{M}$. □

Definition 3.11. *A pseudo-slant submanifold M of a LP-cosymplectic manifold $\tilde{M}$ is said to be D^θ-geodesic(respectively $D^\perp$-geodesic) if $\sigma(X, Y) = 0$ for $X, Y \in \Gamma(D^\theta)$(respectively $\sigma(Z, W) = 0$ for $Z, W \in \Gamma(D^\perp)$). If $\sigma(X, Z) = 0$, M is called mixed geodesic submanifold for any $X \in \Gamma(D^\theta)$ and $Z \in \Gamma(D^\perp)$*

Theorem 3.12. *Let M be a proper pseudo-slant submanifold of a LP-cosymplectic manifold $\tilde{M}$. If t is parallel, then either M is a mixed geodesic or a totally real submanifold.*

Proof. By virtue of (19) and from Theorem (3.4), we obtain $f\sigma(X, Z) = 0$, for any $X \in \Gamma(D^\theta)$ and $Z \in \Gamma(D^\perp)$. Also by using (19) and (22), we conclude that $f\sigma(Y, TX) - \sigma(Y, T^2X) = -cos^2\theta\sigma(X, Y) = 0$. This proves our assertion. □

References

[1] M.Atçeken, *Semi-slant submanifolds of an almost paracontact metric manifold*, Canadian Mathematical Bulletin, 53(2)(2010), 206-217.

[2] D.E.Blair, *Contact manifolds in Riemannian geometry: Lecture Notes in Mathematics*, 509, Springer, Berlin, (1976).

[3] D.E.Blair, *Riemannian geometry of contact and symplectic manifolds*, Birkhuser, Boston, (2002).

[4] A.Carriazo, *New devolopments in slant submanifolds theory*, Narosa publishing House, New Delhi, India, (2002).

[5] J.L.Cabrerizo, A.Carriazo, L.M.Fernandez and M.Fernandez,*Slant submanifolds in Sasakian manifold*, Glasgow Math. J., 42(2000), 125-138.

[6] B.Y.Chen, *Geometry of slant submanifolds*, Katholieke Universiteit Leuven, Leuven, (1990).

[7] B.Y.Chen, *Slant immersions*, Bulletin of the Australian Mathematical Society, 41(1990), 135-147.

[8] U.C.De and A.Sarkar, *On pseudo-slant submanifolds of trans-Sasakian manifolds*, Proceedings of the Estonian Academy of Sciences, 60(1)(2011), 1-11.

[9] Y.Gündüzalp, *Slant submersions from almost paracontact riemannian manifolds*, Kuwait Journal of Science, 42(1)(2015), 17-29.

[10] S.Kaneyuki and F.L.Williams, *Almost Paracontact and Parahodge Structures on Manifolds*, Nagoya Mathematical Journal, 98(1985), 173-187.

[11] V.A.Khan and M.A.Khan, *Pseudo-slant submanifolds of a Sasakian manifold*, Indian Journal of Pure and Applied Mathematics, 38(1)(2007), 31-42.

[12] M.A.Khan, *Totally umbilical hemi-slant submanifolds of cosymplectic manifolds*, Mathematica Aeterna, 3(8)(2013), 645-653.

[13] M.A.Khan, F.R.Al-Solamy and A.A.Ishan, *Totally umbilical pseudo-slant submanifolds of Riemannian product manifolds*, Acta Universitatis Apulensis, 41(2015), 79-87.

[14] B.Laha and A.Bhattacharyya, *Totally umbilical hemislant submanifolds of LP-Sasakian manifold*, Lobachevskii Journal of Mathematics, 36(2)(2015), 127-131.

[15] A.Lotta, *Slant submanifolds in contact geometry*, Bulletin of Mathematical Society Romania, 39(1996), 183-198.

[16] K.Matsumoto, *On Lorentzian paracontact manifolds*, Bull. Yamagata Univ. Nat. Sci., 12(1989), 151-156.

[17] N.Papaghiuc, *Semi-slant submanifolds of Kaehlerian manifold*, An. St. Univ. Iasi. tom., XL, S.I. 9(1994), 55-61.

[18] A.Perrone, *Some results on almost paracontact metric manifolds*, Mediterranean Journal of Mathematics, 13(2016), 3311-3326.

[19] M.S.Siddesha and C.S.Bagewadi, *On slant submanifolds of* (k,μ)*-contact manifold*, Differential Geometry-Dynamical Systems, 18(2016), 123-131.

[20] M.S.Siddesha and C.S.Bagewadi, *Semi-slant submanifolds of* (k,μ)*-contact manifold*, Commun. Fac. Sci. Univ. Ser. A_1 Math. Stat., 67(2)(2017).

[21] K.Singh, S.Uddin and C.Ozel, *A class of totally umbilical slant submanifolds of Lorentzian para-Sasakian manifolds*, International Journal of Physical Sciences, 7(10)(2012), 1526-1529.

[22] S.Zamkovoy, *Canonical connections on paracontact manifolds*, Annals of Global Analysis and Geometry, 36(2009), 37-60.