International Journal of Mathematics And its Applications
Volume 5, Issue 4–B (2017), 121–127.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On Generalized c*-open Sets and Generalized c*-open Maps in Topological Spaces

Research Article

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Abstract: The aim of this paper is to introduce the notion of generalized c*-open sets and generalized c*-open maps in topological spaces and study their basic properties.

Keywords: gc*-open sets and gc*-open maps.

1. Introduction

In 1963, Norman Levine introduced the concept of generalized closed sets in topological spaces. Also in 1970, he introduced semi-open sets. Bhattacharya and Lahiri introduced and study semi-generalized closed (briefly sg-closed) sets in 1987. Palaniappan and Rao introduced regular generalized closed (briefly, rg-closed) sets in 1993. In the year 1996, Andrijevic introduced and studied b-open sets. Gnanambal introduced generalized pre-regular closed (briefly gpr-closed) sets in 1997. In this paper we introduce generalized c*-open sets, generalized c*-open maps in topological spaces and study their basic properties.
Section 2 deals with the preliminary concepts. In section 3, generalized c*-open sets are introduced and study its basic properties. The generalized c*-open maps in topological spaces are introduced in section 4.

2. Preliminaries

Throughout this paper X denotes a topological space on which no separation axioms are assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A, pcl(A) denotes the pre-closure of A and bcl(A) denotes the b-closure of A. Further X\A denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition 2.1. *A subset A of a topological space X is called*

1. *a semi-open set [6] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.*
2. *a pre-open set [12] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.*

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3. *a regular-open set [14] if $A = int(cl(A))$ and a regular-closed set if $EA = cl(int(A))$.*

4. *a γ-open set [8] (b-open set [1]) if $A \subseteq cl(int(A)) \cup int(cl(A))$ and a γ-closed set (b-closed set) if $int(cl(A)) \cap cl(int(A)) \subseteq A$.*

Definition 2.2 ([10])**.** *A subset A of a topological space X is said to be a c*-open set if $int(cl(A)) \subseteq A \subseteq cl(int(A))$.*

Definition 2.3. *A subset A of a topological space X is called*

1. *a generalized closed set (briefly, g-closed) [7] if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is open in X.*
2. *a regular-generalized closed set (briefly, rg-closed) [13] if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is regular-open in X.*
3. *a generalized pre-regular closed set (briefly, gpr-closed) [4] if $pcl(A) \subseteq H$ whenever $A \subseteq H$ and H is regular-open in X.*
4. *a regular generalized b-closed set (briefly, rgb-closed) [11] if $bcl(A) \subseteq H$ whenever $A \subseteq H$ and H is regular-open in X.*
5. *a regular weakly generalized closed set (briefly, rwg-closed) [16] if $cl(int(A)) \subseteq H$ whenever $A \subseteq H$ and H is regular-open in X.*
6. *a semi-generalized b-closed set (briefly, sgb-closed) [5] if $bcl(A) \subseteq H$ whenever $A \subseteq H$ and H is semi-open in X.*
7. *a weakly closed set (briefly, w-closed) [15] (equivalently, $\hat{g}$-closed [17]) if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is semi-open in X.*
8. *a semi-generalized closed set (briefly, sg-closed) [2] if $scl(A) \subseteq H$ whenever $A \subseteq H$ and H is semi-open in X.*

The complements of the above mentioned closed sets are their respectively open sets.

Definition 2.4 ([10])**.** *A subset A of a topological space X is said to be a generalized c*-closed set (briefly, gc*-closed set) if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is c*-open.*

Definition 2.5. *A function $f : X \to Y$ is said to be*

1. *a g-open map [9] if $f(U)$ is g-open in Y for every open set U of X.*
2. *a semi-generalized open (briefly, sg-open) [3] map if f(U) is sg-open in Y for every open set U of X.*
3. *a $\hat{g}$-open map [17] if f(U) is $\hat{g}$-open in Y for every open set U of X.*

3. Generalized c*-open Sets

The complement of a gc*-closed set need not be gc*-closed. This leads to the definition of gc*-open set. In this section we introduce gc*-open sets and study its basic properties. Now, begin with the definition of gc*-open set.

Definition 3.1. *A subset A of a space X is said to be a generalized c*-open (briefly, gc*-open) set if its complement is gc*-closed.*

Example 3.2. *Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\phi, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, d, e\}, X\}$. Then the gc*-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{e\}$, $\{a, d\}$, $\{a, e\}$, $\{b, c\}$, $\{d, e\}$, $\{a, d, e\}$, $\{a, b, d, e\}$, $\{a, c, d, e\}$, X.*

Proposition 3.3. *Let X be a topological space. Then every open set is gc*-open.*

Proof. Let A be an open set. Then $X\backslash A$ is a closed set. By Proposition 4.3 [10], $X\backslash A$is gc*-closed. Therefore, A is gc*-open. ☐

The converse of the Proposition 3.3 need not be true as seen from the following example.

Example 3.4. *Let* $X = \{a, b, c, d\}$ *with topology* $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$. *Then the gc*-open sets are* ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{c, d\}$, $\{a, b, c\}$, X. *Here the subset* $\{c\}$ *is gc*-open but not open.*

Proposition 3.5. *Let X be a topological space. Then every w-open (equivalently,* $\hat{g}$*-open) set is gc*-open.*

Proof. Let A be a w-open set. Then $X\backslash A$is w-closed. By Proposition 4.5 [10], $X\backslash A$ is gc*-closed. Therefore, A is gc*-open. ☐

The converse of the Proposition 3.5 need not be true as seen from the following example.

Example 3.6. *In Example 3.4, the subset* $\{c, d\}$ *is gc*-open but not w-open.*

Proposition 3.7. *Let X be a topological space. Then every clopen set is gc*-open.*

Proof. Let A be a clopen set in X. Then $X\backslash A$ is a clopen set in X. By Proposition 3.9 [10], $X\backslash A$ is c*-open. Also, cl($X\backslash A$)=$X\backslash A$. Thus $X\backslash A$ is a c*-open set containing $X\backslash A$ and $cl(X\backslash A) \subseteq X\backslash A$. Therefore, $X\backslash A$ is gc*-closed. Hence A is gc*-open. ☐

The converse of the Proposition 3.7 need not be true as seen from the following example.

Example 3.8. *In Example 3.4, the subset* $\{a, b, c\}$ *is gc*-open but not clopen.*

Proposition 3.9. *Let X be a topological space. Then every gc*-open set is rg-open.*

Proof. Let A be a gc*-open set. Then $X\backslash A$ is gc*-closed. By Proposition 4.7 [10], $X\backslash A$ is rg-closed. Therefore, A is rg-open. ☐

The converse of the Proposition 3.9 need not be true as seen from the following example.

Example 3.10. *Let* $X = \{a, b, c, d, e\}$ *with topology* $\tau = \{\phi, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, d, e\}, X\}$. *Then the subset* $\{c, d, e\}$ *is rg-open but not gc*-open.*

Proposition 3.11. *Let X be a topological space. Then every gc*-open set is gpr-open.*

Proof. Let A be a gc*-open set. Then $X\backslash A$ is gc*-closed. By Proposition 4.9 [10], $X\backslash A$is gpr-closed. Therefore, A is gpr-open. ☐

The converse of the Proposition 3.11 need not be true as seen from the following example.

Example 3.12. *In Example 3.10, the subset* $\{c, d, e\}$ *is gpr-open but not gc*-open.*

Proposition 3.13. *Let X be a topological space. Then every gc*-open set is rgb-open.*

Proof. Let A be a gc*-open set. Then $X\backslash A$ is gc*-closed. By Proposition 4.11 [10], $X\backslash A$ is rgb-closed. Therefore, A is rgb-open. ☐

The converse of the Proposition 3.13 need not be true as seen from the following example.

Example 3.14. *In Example 3.4, the subset is* $\{b, d\}$ *rgb-open but not gc*-open.*

Proposition 3.15. *Let X be a topological space. Then every gc*-open set is rwg-open.*

Proof. Let A be a gc*-open set. Then $X\backslash A$ is gc*-closed. By Proposition 4.13 [10], $X\backslash A$ is rwg-closed. Therefore, A is rwg-open. □

The converse of the Proposition 3.15 need not be true as seen from the following example.

Example 3.16. *In Example 3.4, the subset* $\{b,\ c,\ d\}$ *is rwg-open but not gc*-open.*

The g-open and gc*-open sets are independent. For example, let X = {a, b, c, d, e} with topology τ = {ϕ, {a}, {d}, {e}, {a, d}, {a, e}, {d, e}, {a, d, e}, X}. Then the subset {c, d, e} is g-open but not gc*-open and the subset{b, c} is gc*-open but not g-open.

The semi-generalized b-open and gc*-open sets are independent. For example, let X = {a, b, c, d, e} with topology τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, c, d}, {a, b, c, e}, X}. Then the subset {b, c, d, e} is sgb-open but not gc*-open and the subset {c, d, e} is gc*-open but not sgb-open.

Proposition 3.17. *Let X be a topological space and A,B be subsets of X. If A and B are gc*-open, then* $A\cap B$ *is gc*-open.*

Proof. Let A and B be gc*-open sets. Then $X\backslash A$ and $X\backslash B$ are gc*-closed sets. By Proposition 4.15 [10], $(X\backslash A)\cup(X\backslash B)$ is gc*-closed. Therefore, $X\backslash[(X\backslash A)\cup(X\backslash B)]$ is gc*-open. Hence $A\cap B$ is gc*-open. □

The union of two gc*-open subsets of a space X need not be gc*-open. For example, let $X=\{a,b,c,d,e\}$ with topology $\tau=\{\phi,\{a\},\{d\},\{e\},\{a,d\},\{a,e\},\{d,e\},\{a,d,e\},X\}$. Then, the subsets $\{c\}$ and $\{d\}$ are gc*-open but their union $\{c,d\}$ is not gc*-open.

Theorem 3.18. *Let A be a subset of a topological space X. Then A is gc*-open if and only if* $H\subseteq int(A)$ *whenever* $H\subseteq A$ *and H is c*-open.*

Proof. Assume that A is gc*-open. Then $X\backslash A$ is gc*-closed. Let H be a c*-open set and $H\subseteq A$. Then $X\backslash H$ is the c*-open set and $X\backslash A\subseteq X\backslash H$. Since $X\backslash A$ is gc*-closed, we have $cl(X\backslash A)\subseteq X\backslash H$. This implies, $H\subseteq X\backslash[cl(X\backslash A)]$. That is, $H\subseteq int(A)$. Conversely, let H be a c*-open set containing $X\backslash A$. This implies, $X\backslash H$ is c*-open and $X\backslash H\subseteq A$. Therefore, by hypothesis, $X\backslash H\subseteq int(A)$. This implies, $cl(X\backslash A)\subseteq H$. Thus $X\backslash A$ is gc*-closed. Hence A is gc*-open. □

Theorem 3.19. *Let X be a topological space. Then for any element* $p\in X$*, the set* $\{p\}$ *is either gc*-open or c*-open.*

Proof. Suppose $\{p\}$ is not a c*-open set. Then $X\backslash\{p\}$ is not a c*-open set. By Proposition 4.18 [10], $X\backslash\{p\}$ is gc*-closed. Therefore,$\{p\}$ is gc*-open. □

Theorem 3.20. *Let X be a topological space. If A is a gc*-open subset of X such that* $int(A)\subseteq B\subseteq A$*, then B is gc*-open in X.*

Proof. Let A be a gc*-open set and $int(A)\subseteq B\subseteq A$. Then $X\backslash A$ is a gc*-closed set and $X\backslash A\subseteq X\backslash B\subseteq cl(X\backslash A)$. Then by Proposition 4.20 [10], $X\backslash B$ is gc*-closed. Hence B is gc*-open. □

Theorem 3.21. *Let X be the topological space and A,B be subsets of X. If A is open and B is gc*-open, then* $A\cap B$ *is gc*-open.*

Proof. Assume that A is open and B is gc*-open. Then $X\backslash A$ is closed and $X\backslash B$ is gc*-closed. Therefore, by Proposition 4.24 [10], $(X\backslash A)\cup(X\backslash B)$ is gc*-closed. That is, $X\backslash(A\cap B)$ is gc*-closed. Hence $A\cap B$ is gc*-open. □

4. Generalized c*-open Maps

In this section, we introduce generalized c*-open maps in topological spaces. Also, we derive some of its basic properties.

Definition 4.1. *Let X and Y be two topological spaces. A function $f : X \to Y$ is said to be generalized c*-open map (briefly, gc*-open map) if f(U) is gc*-open in Y for every open set U in X.*

Example 4.2. *Let $X = \{a,\ b,\ c\}$ with topology $\tau = \{\phi,\ \{a\},\ X\}$ and Y={1, 2, 3} with topology $\sigma = \{\phi,\ \{1\},\ \{1,\ 2\},\ Y\}$. Define $f : X \to Y$ by $f(a) = 2,\ f(b) = 3,\ f(c) = 1$. Then f is gc*-open.*

Proposition 4.3. *Let X, Y be two topological spaces. A bijective function $f : X \to Y$ is a gc*-open if and only if the image of each closed subset of X is gc*-closed in Y.*

Proof. Assume that $f : X \to Y$ is a gc*-open map. Let V be a closed set in X. Then $X\backslash V$ is open in X. Therefore, by our assumption, $f(X\backslash V)$ is gc*-open in Y. This implies, $Y\backslash f(V)$ is gc*-open in Y. Hence, $f(V)$ is gc*-closed in Y. Conversely, assume that the image of each closed subset of X is gc*-closed in Y. Let U be an open set in X. Then $X\backslash U$ is closed in Y. Therefore, by our assumption, $f(X\backslash U)$ is gc*-closed in Y. This implies, $Y\backslash f(U)$ is gc*-closed in Y. This implies, $f(U)$ is gc*-open in Y. Therefore, f is a gc*-open map. □

Proposition 4.4. *Let X, Y be two topological spaces. Then every open map is gc*-open.*

Proof. Let $f : X \to Y$ be an open map and U be an open set in X. Then $f(U)$ is open in Y. By Proposition 3.3, $f(U)$ is a gc*-open set. Therefore, f is a gc*-open map. □

The converse of the Proposition 4.4 need not be true as seen from the following example.

Example 4.5. *Let $X = \{a,\ b,\ c\}$ and $Y = \{1,\ 2,\ 3\}$. Then, clearly $\tau = \{\phi,\ \{b\},\ \{c\},\ \{b,\ c\},\ X\}$ is a topology on X and $\sigma = \{\phi,\ \{1\},\ \{1,\ 2\},\ \{1,\ 3\},\ Y\}$ is a topology on Y. Define $f : X \to Y$ by $f(a) = 2,\ f(b) = 3,\ f(c) = 1$. Clearly, f is a gc*-open map. But f is not an open map, since the image of an open set $\{b\}$ under f is $\{3\}$, which is not open in Y.*

Proposition 4.6. *Let X, Y be two topological spaces. Then every $\hat{g}$-open map is gc*-open.*

Proof. Let $f : X \to Y$ be a $\hat{g}$-open map. Let U be an open set in X. Then $f(U)$ is $\hat{g}$-open in Y. Therefore, by Proposition 3.5, $f(U)$ is a gc*-open set. Therefore, f is a gc*-open map. □

The converse of the Proposition 4.6 need not be true as seen from the following example.

Example 4.7. *Let $X = \{a,\ b,\ c\}$ and $Y = \{1,\ 2,\ 3\}$. Then, clearly $\tau = \{\phi,\ \{b\},\ \{c\},\ \{b,\ c\},\ X\}$ is a topology on X and $\sigma = \{\phi,\ \{1\},\ \{1,\ 2\},\ \{1,\ 3\},\ Y\}$ is a topology on Y. Define $f : X \to Y$ by $f(a) = 2,\ f(b) = 3,\ f(c) = 1$. Then f is a gc*-open map. Consider the open set $\{b\}$ in X. Then $f(\{b\}) = \{3\}$, which is not a $\hat{g}$-open set in Y. Therefore, f is not a $\hat{g}$-open map.*

The g-open and gc*-open maps are independent. For example, let $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4, 5\}$. Then, clearly $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$ is a topology on X and $\sigma = \{\phi, \{1\}, \{4\}, \{5\}, \{1, 4\}, \{1, 5\}, \{4, 5\}, \{1, 4, 5\}, Y\}$ is a topology on Y. Define $f : X \to Y$ by $f(a) = 1,\ f(b) = 2,\ f(c) = f(d) = 3$. Then f is a g-open map. Consider the open set $\{a, c\}$ in X. Then $f(\{a, c\}) = \{1, 3\}$, which is not a gc*-open set in Y. Hence f is not a gc*-open map. Define $g : X \to Y$ by $g(a) = g(b) = 2,\ g(c) = 3,\ g(d) = 5$. Then g is a gc*-open map. Consider the open set $\{a, c\}$ in X. Then $g(\{a, c\}) = \{2, 3\}$, which is not a g-open set in Y. Therefore, f is not a g-open map.

The sg-open and gc*-open maps are independent. For example, let $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4, 5\}$. Then, clearly $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$ is a topology on X and $\sigma = \{\phi, \{1\}, \{4\}, \{5\}, \{1, 4\}, \{1, 5\}, \{4, 5\}, \{1, 4, 5\}, Y\}$ is a topology on Y. Define $f : X \to Y$ by $f(a) = 1$, $f(b) = 4$, $f(c) = f(d) = 3$. Then f is a sg-open map. Consider the open set $\{a, c\}$ in X. Then $f(\{a, c\}) = \{1, 3\}$, which is not a gc*-open set in Y. Hence f is not a gc*-open map. Define $g : X \to Y$ by $g(a) = g(b) = 2$, $g(c) = 3$, $g(d) = 5$. Then g is a gc*-open map. Consider the open set $\{b\}$ in X. Then $g(\{b\}) = \{2\}$, which is not a sg-open set in Y. Therefore, g is not a sg-open map.

Proposition 4.8. *Let X, Y and Z be topological spaces. If $f : X \to Y$ is an open map and $g : Y \to Z$ is a gc*-open map, then $g \circ f : X \to Z$ is gc*-open map.*

Proof. Let U be an open set in X. Since f is an open map, $f(U)$ is open in Y. Then $g(f(U)) = (g \circ f)(U)$ is a gc*-open set in Z. Therefore, $g \circ f$ is a gc*-open map. □

Proposition 4.9. *Let X, Y and Z be topological spaces. If $f : X \to Y$ and $g : Y \to Z$ are open maps, then $g \circ f : X \to Z$ is a gc*-open map.*

Proof. Let U be an open set in X. Since f is an open map, $f(U)$ is open in Y. Also, since g is an open map, $g(f(U))$ is open in Z. That is, $(g \circ f)(U)$ is a open set in Z. By proposition 3.3, $(g \circ f)(U)$ is a gc*-open set in Z. Therefore, $g \circ f$ is a gc*-open map. □

Proposition 4.10. *Let X, Y and Z be topological spaces. If $f : X \to Y$ is an open map and $g : Y \to Z$ is a $\hat{g}$-open map, then $g \circ f : X \to Z$ is a gc*-open map.*

Proof. Let U be an open set in X. Since f is an open map, $f(U)$ is open in Y. Then $g(f(U))$ is a $\hat{g}$-open set in Z. That is, $(g \circ f)(U)$ is a $\hat{g}$-open set in Z. Therefore, by Proposition 3.5, $(g \circ f)(U)$ is a gc*-open set in Z. Hence $g \circ f$ is a gc*-open map. □

Proposition 4.11. *Let X, Y be two topological spaces. A surjective function $f : X \to Y$ is a gc*-open map if and only if for each subset B of Y and for each closed set U containing $f^{-1}(B)$, there is a gc*-closed set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.*

Proof. Suppose $f : X \to Y$ is a surjective gc*-open map and B is a subset of Y. Let U be a closed set in X such that $f^{-1}(B) \subset U$. Then $V = Y \backslash f(X \backslash U)$ is a gc*-closed subset of Y containing B and $f^{-1}(V) \subset U$. Conversely, suppose F is an open subset of X. Then $X \backslash F$ is closed. Also, $f^{-1}(Y \backslash f(F)) = X \backslash f^{-1}(f(F)) \subset X \backslash F$. Therefore, by hypothesis, there exists a gc*-closed set V of Y such that $Y \backslash f(F) \subset V$ and $f^{-1}(V) \subset X \backslash F$. This implies, $F \subset X \backslash f^{-1}(V)$. Therefore, $f(F) \subset f(X \backslash f^{-1}(V)) = Y \backslash V$. Also, $f(F) \supset Y \backslash V$. Therefore, $f(F) = Y \backslash V$, which is gc*-open in Y. Therefore, f is a gc*-open map. □

5. Conclusion

In this paper we have introduced gc*-open sets and gc*-open maps in topological spaces and studied some of its basic properties. Also, we have studied the relationship between gc*-open sets with some generalized sets in topological spaces.

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