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Interior Domination on Subdivision and Splitting Graphs of Graphs

Research Article

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Abstract: A subset D of the vertex set $V(G)$ of a graph G is said to be a dominating set if every vertex in V-D is adjacent to some vertex in D. A dominating set D is said to be an interior dominating set if every vertex $v \in D$ is an interior vertex of G. The minimum cardinalities among the interior dominating sets of G is called the interior domination number $\gamma_{Id}(G)$ of G. In this paper we discuss interior domination on the subdivision graph $S(G)$ of various graphs.

MSC: 05C12, 05C69.

Keywords: Interior dominating set, Subdivision graph, Splitting graph.

1. Introduction

Let G be a finite, simple, undirected connected graph with vertex set $V(G)$ and edge set $E(G)$. We refer the basic definitions and theorems used in Chartrand [3]. A set $D \subseteq V(G)$ is a dominating set of G, if every vertex in V-D is adjacent to some vertex in D. The dominating set D is a minimal dominating set if no proper subset D' of D is a dominating set. The cardinality of the minimum dominating set is known as the domination number and is denoted by $\gamma(G)$. Let x and z be two distinct vertices in G. A vertex y distinct from x and z is said to lie between x and z if $d(x,z) = d(x,y) + d(y,z)$. A vertex v is an interior vertex of G if for every vertex u distinct from v, there is a vertex w such that v lies between u and w. If the vertices of a dominating set D of G are interior vertices of G then D is said to be an interior dominating set of G. The cardinality of the minimum interior dominating set is known as the interior domination number and is denoted by $\gamma_{Id}(G)$. If anyone vertex of D is not an interior vertex of G then D is not an interior dominating set of G. If there is no interior dominating set in G then we can say that the interior domination number of G is zero. For example, the interior domination number of a complete graph is zero. S. Arumugam and Paulraj Joseph [2] introduced the concept domination in subdivision graph. A subdivision of an edge e = u v of a graph G is the replacement of an edge e by a path (u, v, w). The graph obtained from a graph G by subdividing every edge e of G exactly once is called the subdivision graph of G and denoted by $S(G)$.

2. Interior Dominating Sets in Sub Division Graphs of Graphs

We introduce the new concept interior domination on subdivision graph of various graphs [1].

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Definition 2.1. *A subdivision of an edge $e = uv$ of a graph G is the replacement of the edge e by a path (u, v, w). The graph obtained from G by subdividing every edge e of G exactly once, is called the subdivision graph of G and is denoted by $S(G)$. If D is a minimum dominating set of $S(G)$ and if all the vertices of D are interior vertices of $S(G)$ then D is said to be the minimum interior dominating set of $S(G)$ and the cardinality of the minimum interior dominating set is called as the interior domination number and is denoted by $\gamma_{Id}(S(G))$.*

Example 2.2.

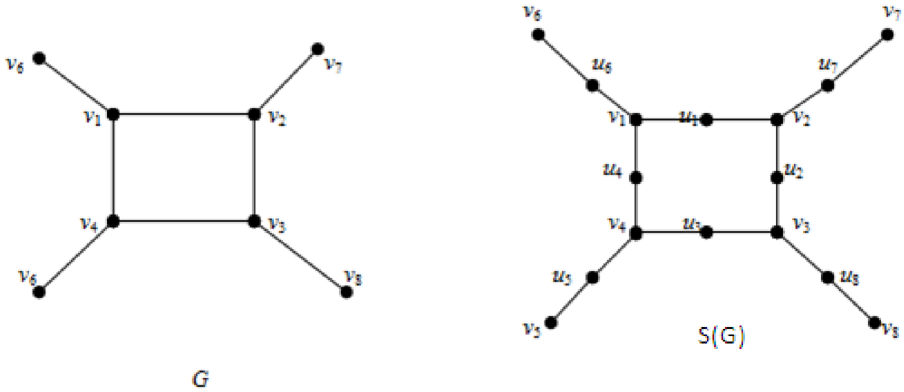

Figure 1. **A graph G and S (G)**

In the above graph, the set $\{v_1, v_2, v_3, v_4\}$ is a minimum interior dominating set of G and so $\gamma_{Id}(G) = 4$. Similarly, the set $\{v_2, v_4, u_6, u_7, u_8, u_5\}$ is a minimum interior dominating set of $S(G)$ and so $\gamma_{Id}(S(G)) = 6$.

Theorem 2.3. *For any positive integer $n \geq 2$, $\gamma_{Id}(S(K_{1,n})) = n$.*

Proof. Let u be the central vertex of $K_{1,n}$. Let $v_1, v_2, \ldots, v_n$ be the vertices adjacent with u. Each edge is subdivided into two edges uu_i and u_iv_i, $i = 1, 2, \ldots, n$ as shown in Figure 2. Then $S(K_{1,n})$ will have $2n + 1$ vertices and $2n$ edges. Every newly added vertex u_i dominates u and v_i $(1 \leq i \leq n)$. Hence the set $D = \{u_1, u_2, \ldots, u_n\}$ is a dominating set of $S(K_{1,n})$. If we delete a vertex u_i from D then $D - \{u_i\}$ will not dominate $S(K_{1,n})$ and so D is a minimum dominating set of $S(K_{1,n})$. We claim that every vertex u_i of D is an interior vertex. For every v_i there exists u such that $d(v_i, u) = d(v_i, u_i) + d(u_i, u)$, for $1 \leq i \leq n$. Also for every u_j there exists v_i such that $d(u_j, v_i) = d(u_j, u_i) + d(u_i, v_i)$, and for the vertex u there exists v_i such that $d(u, v_i) = d(u, u_i) + d(u_i, v_i)$

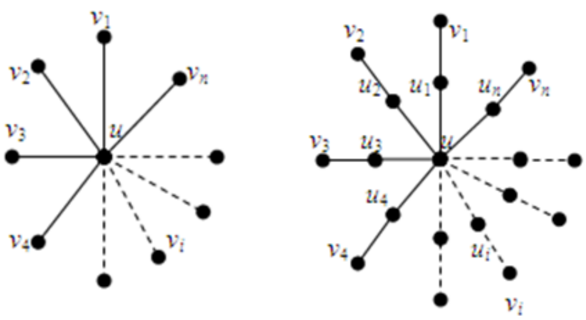

Figure 2. **A graph $K_{1,n}$ and $S(K_{1,n})$**

Hence every vertex of D is an interior vertex of $S(K_{1,n})$. Therefore D is a minimum interior dominating set of $S(K_{1,n})$. Therefore $\gamma_{Id}(S(K_{1,n})) = n$. □

Theorem 2.4. *For $B_{m,n}$ a bi-star of $m+1$ and $n+1$ vertices, $\gamma_{Id}(S(B_{m,n})) = m+n+1$.*

Proof. Let u and v be the central vertices of $B_{m,n}$ Let $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ be the vertices adjacent with u and v respectively as in Figure 3. Subdivide the graph $B_{m,n}$ as shown in Figure 3. Let $u'_1, u'_2, \ldots, u'_m, v'_1, v'_2, \ldots, v'_n$ and w be the subdivided vertices of $B_{m,n}$ Then $S(B_{m,n})$ will have $2(m+n)+3$ vertices. Here the newly added vertices $u'_1, u'_2, \ldots, u'_m$ and $v'_1, v'_2, \ldots, v'_n$ dominate u'_i and v'_i, respectively and w dominates itself. Hence the set $D = \{u'_1, u'_2, \ldots, u'_m, v'_1, v'_2, \ldots, v'_n, w\}$ is a dominating set of $B_{m,n}$ If we delete a vertex u'_1 or v'_i or w from D then D will not dominate $S(B_{m,n})$ and so D is a minimum dominating set of $S(B_{m,n})$.

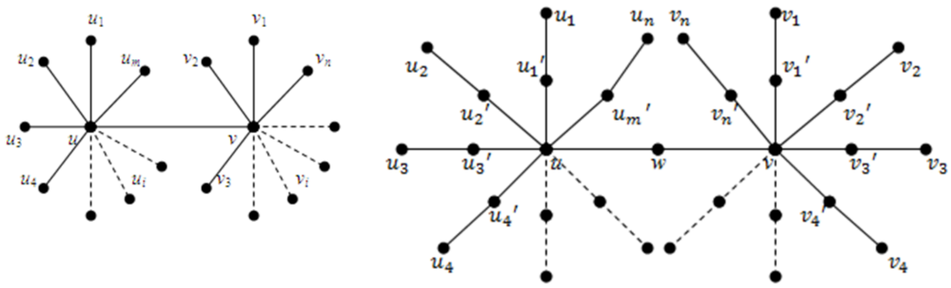

Figure 3. **A graph $B_{m,n}$ and $S(B_{mn})$**

We claim that every vertex of D is an interior vertex. First we prove that u'_i $(1 \le i \le m)$ is an interior vertex. For every u_i $(1 \le i \le m)$ there exists u such that $d(u_i, u) = d(u_i, u'_i) + d(u'_i, u)$. For every v_i $(1 \le i \le n)$ there exists u_i such that $d(v_i, u_i) = d(v_i, u'_i) + d(u'_i, u_i)$. For every v'_i $(1 \le i \le n)$ there exists u_i, such that $d(v'_i, u_i) = d(v'_i, u'_i) + d(u'_i, u_i)$. For every $x = \{u, v, w\}$ there exists u_i $(1 \le i \le m)$, such that $d(x, u_i) = d(x, u'_i) + d(u'_i, u_i)$. Also for every u'_j $(1 \le j \le m)$ with $i \ne j$, there exists u_i such that $d(u'_j, u_i) = d(u'_j, u'_i) + d(u'_i, u_i)$. Thus the vertices u'_i $(1 \le i \le m)$ are interior vertices. Similarly, we can prove that v'_i $(1 \le i \le n)$ and w are interior vertices. Hence every vertex of D is an interior vertex of $S(B_{m,n})$. Therefore D is a minimum interior dominating set of $S(B_{m,n})$ and so $\gamma_{Id}(S(B_{m,n})) = m+n+1$. □

Definition 2.5. *The corona product $G \circ H$ of two graphs G and H is obtained by taking one copy of G and $|V(G)|$ Copies of H and by joining each vertex of the i-th copy of H to the i-th vertex of G, where $1 \le i \le |V(G)|$.*

Theorem 2.6. *If $S(P_n \circ K_1)$ is a subdivision graph of the corona graph $P_n \circ K_1$, for $n \ge 4$ then*

$$\gamma_{Id}(S(P_n \circ K_1)) = \begin{cases} \frac{3n-1}{2}, & \textit{if n is odd;} \\ \frac{3n}{2}, & \textit{if n is even.} \end{cases}$$

Proof. Let $\{v_1, v_2, \ldots, v_n\}$ and $\{e_1, e_2, \ldots, e_{n-1}\}$ be the vertex set and edge set of P_n respectively. Let $\{u_1, u_2, \ldots, u_n\}$ be the set of pendent vertices adjacent to $v_1, v_2, \ldots, v_n$ respectively. Subdivide the corona graph $P_n \circ K_1$. Let $v'_1, v'_2, \ldots, v'_{n-1}$ and $u'_1, u'_2, \ldots, u'_n$ be the subdivided vertices of $e_1, e_2, \ldots, e_{n-1}$ and $e'_1, e'_2, \ldots, e'_n$ respectively in $S(P_n \circ K_1)$ as shown in Figure 4. Then the subdivided graph $S(P_n \circ K_1)$ will have $(4n-1)$ vertices. In $S(P_n \circ K_1)$ the newly added vertex u'_i $(1 \le i \le n)$ dominates u_i and v_i $(1 \le i \le n)$ and then the remaining vertices $v'_1, v'_2, \ldots, v'_{n-1}$ are dominated by $v_2, v_4, \ldots, v_{n-1}$ taken alternatively in v_i's Hence $D = \{u'_1, u'_2, \ldots, u'_n, v_2, v_4, \ldots, v_{n-1}\}$ is a dominating set. If we remove any vertex from the set D then it will not be a dominating set of $S(P_n \circ K_1)$ and so D is a minimum dominating set of $S(P_n \circ K_1)$. We claim that every vertex of D is an interior vertex of $S(P_n \circ K_1)$. For every v_i there exists u_i such that $d(v_i, u_i) = d(v_i, u'_i) + d(u'_i, u_i)$, $(1 \le i \le n)$. For every u_i there exists v_i such that $d(u_i, v_i) = d(u_i, u'_i) + d(u'_i, v_i)$, $(1 \le i \le n)$ For every u'_j $(j \ne i)$ there exists u_i such that $d(u'_j, u_i) = d(u'_j, u'_i) + d(u'_i, u_i)$, $(1 \le i,\ j \le n)$. Thus u'_i $(1 \le i \le n)$ are interior vertices. Similarly we can prove $v_2, v_4, \ldots, v_{n-1}$ are interior vertices. Thus every vertex of D is an interior vertex of $S(P_n \circ K_1)$. Therefore D is a minimum interior dominating set of $S(P_n \circ K_1)$.

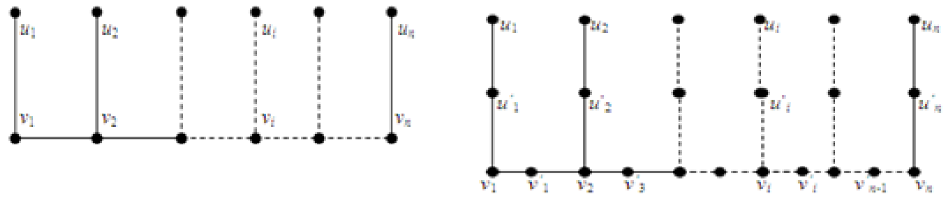

Figure 4. **The corona graph $P_n \circ K_1$ and $S(P_n \circ K_1)$**

Case (1): Suppose that $n = 2k$, for $k \geq 2$. Now $S(P_n \circ K_1)$ is a subdivision graph with $2k$ elements. Since $D = \{u'_1, u'_2, \ldots, u'_{2k}, v_2, v_4, \ldots, v_{2k}\}$ is an interior dominating set. Hence $|D| = 2k + \frac{2k}{2} = 3k = \frac{3n}{2}$ and so $\gamma_{Id}(S(P_n \circ K_1)) = \frac{3n}{2}$.
Case (2): Suppose that $n = 2k+1$, for $k \geq 2$. Now $S(P_n \circ K_1)$ is the subdivision graph of $P_n \circ K_1$ with $2k+1$ elements. Since $D = \{u'_1, u'_2, \ldots, u'_{2k+1}, v_2, v_4, \ldots, v_{2k}\}$ is a interior dominating set. Hence $|D| = 2k + 1 + \frac{2k}{2} = 3k + 1 = \frac{3n-1}{2}$ and so $\gamma_{Id}(S(P_n \circ K_1)) = \frac{3n-1}{2}$. □

Note that $\gamma_{Id}(S(P_3 \circ K_1)) = 4$ and $\gamma_{Id}(S(P_2 \circ K_1)) = 3$ by the above theorem.

Theorem 2.7. *For $n \geq 4$,*

$$\gamma_{Id}(S(C_n \circ K_1)) = \begin{cases} \frac{3n+1}{2}, & \text{if } n \text{ is odd;} \\ \frac{3n}{2}, & \text{if } n \text{ is even.} \end{cases}$$

Proof. Let $\{v_1, v_2, \ldots, v_n\}$ and $\{e_1, e_2, \ldots, e_{n-1}\}$ be the vertex set and edge set of C_n respectively. Let $\{u_1, u_2, \ldots, u_n\}$ the set of pendent vertices added to $v_1, v_2, \ldots, v_n$ respectively and let $e'_1, e'_2, \ldots, e'_n$ be the newly added edges for the corona graph $C_n \circ K_1$ as shown in Figure 5 and let $e'_1, e'_2, \ldots, e'_n$ be the newly added edges.

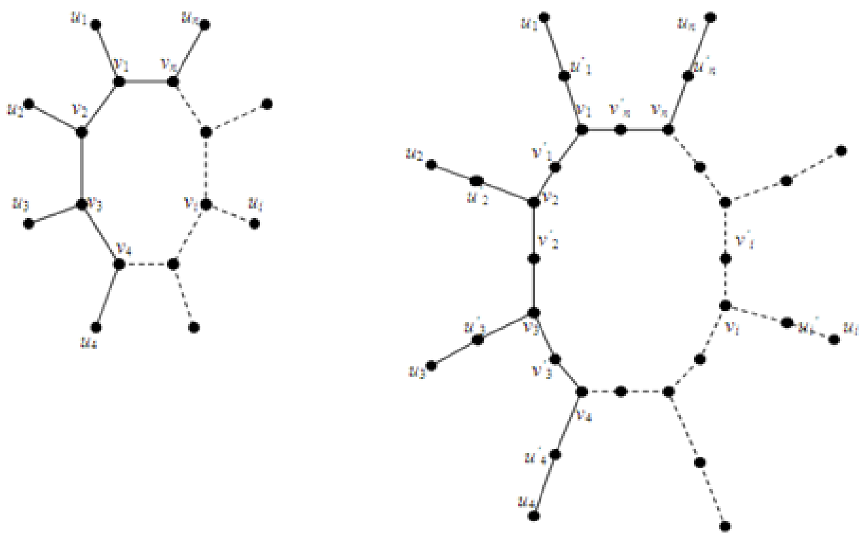

Figure 5. **The Corona graph $C_n \circ K_1$ and $S(C_n \circ K_1)$**

Subdivide the corona graph $C_n \circ K_1$. Let $v'_1, v'_2, \ldots, v'_n$ and $u'_1, u'_2, \ldots, u'_n$ be the subdivided vertices of $e_1, e_2, \ldots, e_n$ and $e'_1, e'_2, \ldots, e'_n$ respectively in $S(C_n \circ K_1)$. Then the subdivided graph $S(C_n \circ K_1)$ will have $4n = (n + n + n + n)$ vertices.

Take $D = \{u'_1, u'_2, \ldots, u'_n, v_2, v_4, \ldots, v_n\}$ and it dominates all the vertices of $S(C_n \circ K_1)$. If we remove a vertex from the set D then it will not be a dominating set of $S(C_n \circ K_1)$. Thus D is a minimum dominating set of $S(C_n \circ K_1)$. We claim that every vertex of D is an interior vertex of $S(C_n \circ K_1)$. First we prove that u'_i $(1 \le i \le n)$ is an interior vertex. For every v_i there exists u_i such that $d(v_i, u_i) = d(v_i, u'_i) + d(u'_i, u_i)$ and for $1 \le i \le n$. Also for every u_i there exists v_i such that $d(u_i, v_i) = d(u_i, u'_i) + d(u'_i, v_i)$, $(1 \le i \le n)$ and for every v'_i there exists u_i such that $d(v'_i, u_i) = d(v'_i, u'_i) + d(u'_i, u_i)$. For every vertex u_j, there exists a vertex $u_i$such that $d(u_j, u_i) = d(u_j, u'_i) + d(u'_i, u_i)$, $(1 \le i,\ j \le n)$ and $j \ne i$. Similarly we can prove $v_2, v_4, \ldots, v_n$ are interior vertices. Hence every vertex of D is an interior vertex of $S(C_n \circ K_1)$. Therefore D is the minimum interior dominating set of $S(C_n \circ K_1)$.

Case (1): Suppose that n is odd. We have prove that $D = \{u'_1, u'_2, \ldots, u'_n, v_2, v_4, \ldots, v_{n-1}, v_n\}$ is a minimum interior dominating set. Therefore $\gamma_{Id}(S(C_n \circ K_1)) = n + (\frac{n-1}{2}) + 1 = \frac{2n+n-1+2}{2} = \frac{3n+1}{2}$.

Case (2): Suppose that n is even. Here $D = \{u'_1, u'_2, \ldots, u'_n, v_2, v_4, \ldots, v_n\}$ is a minimum interior dominating set and so $\gamma_{Id}(S(C_n \circ K_1)) = n + \frac{n}{2} = \frac{3n}{2}$. □

Theorem 2.8. *If $S(K_{1,n} \circ K_1)$ is the subdivision graph of the corona $(K_{1,n} \circ K_1)$ then $\gamma_{Id}(S(K_{1,n} \circ K_1)) = n + 2$, for any positive integer $n \ge 2$.*

Proof. Let u be the central vertex of $K_{1,n}$. Let $v_1, v_2, \ldots, v_n$ be the vertices adjacent with u in $K_{1,n}$. Let $\{w, u_1, u_2, \ldots, u_n\}$ be the set of pendent vertices adjacent to $u, v_1, v_2, \ldots, v_n$ respectively. Subdivide the corona graph $(K_{1,n} \circ K_1)$ as shown in Figure 6. Let u', v'_1, v'_2, v'_n and $u'_1, u'_2, \ldots, u'_n$ be the subdivided vertices. Then the subdivided graph $S(K_{1,n} \circ K_1)$ will have $(4n + 3)$ vertices. Take $D = \{u'_1, u'_2, \ldots, u'_n\}$ then D dominates all the vertices of $S(K_{1,n} \circ K_1)$. If we remove a vertex from the set D it will not be a dominating set of $S(K_{1,n} \circ K_1)$.

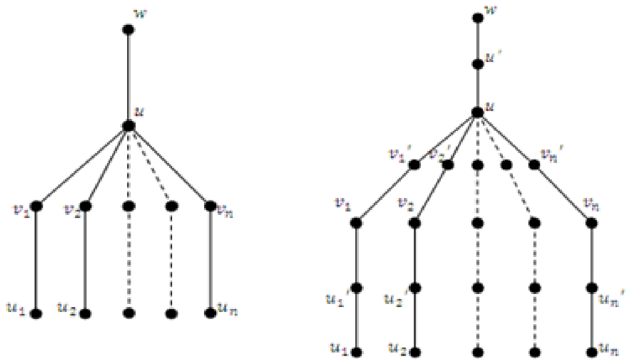

Figure 6. **The corona graph $K_{1,n} \circ K_1$ and $S(K_{1,n} \circ K_1)$**

Therefore D is the minimum dominating set of $S(K_{1,n} \circ K_1)$. We claim that every vertex of D is an interior vertex of $S(K_{1,n} \circ K_1)$. For every u_i there exists w such that $d(u_i, w) = d(u_i, u'_i) + d(u'_i, w)$ $(1 \le i \le n)$. Also for every v_i there exists u_i such that $d(v_i, u_i) = d(v_i, u'_i) + d(u'_i, u_i)$, for every v'_i where i $(1 \le i \le n)$ there exists u_i such that $d(v'_i, u_i) = d(v'_i, u'_i) + d(u'_i, u_i)$. For every i $(1 \le i \le n)$. For u there exists v_i such that $d(u, v'_i) = d(u, u'_i) + d(u'_i, v_i)$, for every i $(1 \le i \le n)$. For $x = \{u', w\}$ there exists u_i such that $d(x, u_i) = d(x, u'_i) + d(u'_i, u_i)$, for every i $(1 \le i \le n)$. Hence u'_i are interior vertices. Similarly u and u' are the interior vertices. Hence every vertex of D is an interior vertex of $S(K_{1,n} \circ K_1)$ and so D is a minimum interior dominating set of $S(K_{1,n} \circ K_1)$. Hence $\gamma_{Id}(S(K_{1,n} \circ K_1)) = n + 2$. □

Theorem 2.9. *Let G be any connected graph with n vertices. Let $(G \circ K_1)$ be a corona graph of G. Then $\gamma_{Id}(G \circ K_1) = n$.*

Proof. Let G be any connected graph with n vertices. Consider the corona graph of G as $G' = G \circ K_1$. The set of all vertices of G are the interior vertices of G' since each vertex of G has pendent edge. In G', all the vertices other than the vertices of G are dominated by the vertices of G. Hence they form a minimum interior dominating set of G'. Hence $\gamma_{Id}(G \circ K_1) = n$. □

Theorem 2.10. *Let G be any connected graph with n vertices and m edges. Let $S(G \circ K_1)$ be the subdivision graph of the corona graph $(G \circ K_1)$ of G. Then $\gamma_{Id}(S(G \circ K_1)) \leq n + m$.*

Proof. Let G be any connected graph with $v_1, v_2, \ldots, v_n$ as vertices .Constract the corona graph $G \circ K_1$ of G as in the above theorem. Construct the subdivision graph of $G \circ K_1$ as shown in Figure 7. The end vertices $u_1, u_2, \ldots, u_n$ and the vertices of G, $v_1, v_2, \ldots, v_n$ are dominated by the subdivided vertices $u'_1, u'_2, \ldots, u'_n$ of the edges $u_1v_1, u_2v_2, \ldots, u_nv_n$. Let them be $u'_1, u'_2, \ldots, u'_n$. Let $w_1, w_2, \ldots, w_m$ be the subdivided vertices of the m edges of G.

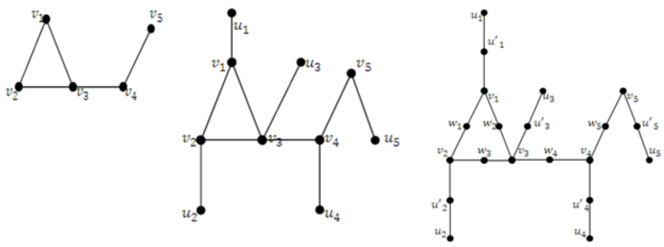

Figure 7. **A graph G with five vertices, $G \circ K_1$ and $S(G \circ K_1)$**

Then the set of vertices $D = \{u'_1, u'_2, \ldots, u'_n, w_1, w_2, \ldots, w_m\}$ is a dominating set of $S(G \circ K_1)$. Here the vertices $u'_1, u'_2, \ldots, u'_n$ are the subdivided vertices of the pendent edges and so they are interior vertices. Using the pendent edges, we can show that $w_1, w_2, \ldots, w_m$ are also interior vertices. Since $|D| = n + m$, we have $\gamma_{Id}(S(G \circ K_1)) \leq n + m$ by taking the minimum dominating set. □

3. Interior Dominating Sets in Splitting Graphs of Graphs

Definition 3.1. *Let G be a given graph .Then the splitting graph of G is obtained by adding a new vertex v' of G to each vertex v such that v' is adjacent to every vertex adjacent to v in G. If n is the order of G then 2n is the order of $S_p(G)$. We say that the vertices $v_1, v_2, \ldots, v_n$ are duplicated by $v'_1, v'_2, \ldots, v'_n$.*

Example 3.2.

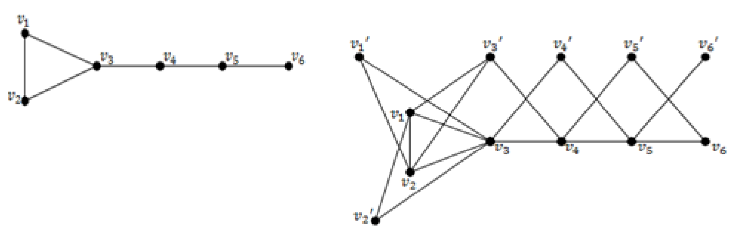

Figure 8. **A graph G and its splitting graph $S_p(G)$**

Remark 3.3. *We discuss about the interior domination on splitting graph of complete graph K_n.*

We know that $\gamma(K_n) = 1$. Since there exist no interior vertex in K_n, $\gamma_{Id}(K_n) = 0$. Consider the splitting graph of K_n. It has $2n$ number of vertices in $S_P(K_n)$. Here v_i, v_i' dominates $S_P(K_n)$ and so $\gamma(S_P(K_n)) = 2$. But there are no interior vertices since for every v_j, there exists no vertex $v = v_k$ or any duplicated vertex v_p such that $d(v_j, v) = d(v_j, v_i) + d(v_i, v)$. Hence $\gamma_{Id}(S_P(K_n)) = 0$. Next we are going to add an edge with K_n.

Theorem 3.4. *Let G be a graph having a complete graph K_n and an edge $v_i\ u$ where $v_i \in V(K_n)$. Then $\gamma_{Id}(S_P(G)) = 0$.*

Proof. Consider a graph G having K_n and a pendent edge v_iu where $v_i \in V(K_n)$. Then $\gamma(G) = 1$ and $\gamma_{Id}(G) = 1$ since $\{v_i\}$ is a minimum interior dominating set in G. Figure 9 shows the splitting graph of G. In G the edge v_iu is the newly added edge and the vertices u', v_i' $(1 \le i \le n)$ are the duplicated vertices of the vertices of G. The vertex v_i dominates all other vertices of G except its duplicate vertex v_i'. Hence the set $\{v_i, v_i'\}$ dominates G and is minimum and so $\gamma_{Id}(G) = 2$. Though v_i is an interior vertex in G, the vertex v_i' is not an interior vertex since there exists no vertex v in G such that $d(v_i, v) = d(v_i, v_i') + d(v_i', v)$. Hence $\gamma_{Id}(S_P(G)) = 0$.

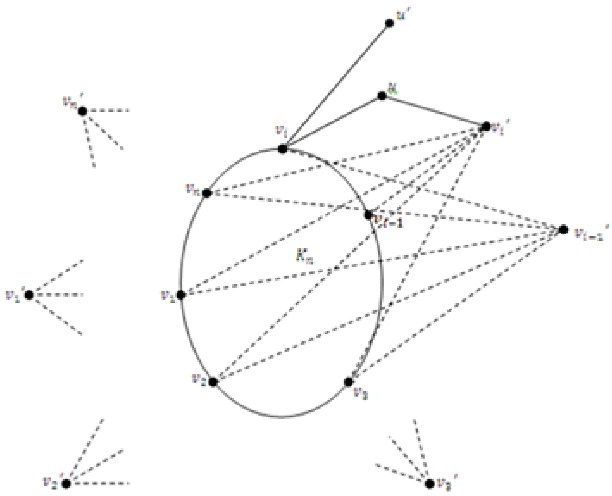

Figure 9. **Splitting graph of G having K_n and an edge joined with a vertex of K_n.**

□

Theorem 3.5. *If $S_P(P_n)$ is a splitting graph of P_n, then*

$$\gamma_{Id}(S_P(P_n)) = \begin{cases} \frac{n}{2}, & \text{if } n = 4i,\ i = 1, 2, \ldots; \\ \frac{n+1}{2}, & \text{if } n = 4i+1,\ i = 1, 2, \ldots; \\ \frac{n+2}{2}, & \text{if } n = 4i+2,\ i = 1, 2, 3, \ldots; \\ \frac{n+1}{2}, & \text{if } n = 4i+3,\ i = 1, 2, \ldots \end{cases}$$

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n which are duplicated by the vertices $v_1', v_2', \ldots, v_n'$ respectively in $S_P(P_n)$.

Case (1): Suppose that $n = 4i$, for $i = 1, 2, \ldots$. Here v_2, dominates v_1, v_3, v_1', v_3' and the neighbour vertex v_3 dominates v_2, v_4, v_2', v_4'. Next v_6, and v_7 dominates v_5, v_7, v_5', v_7'. Consider group of every four vertices as v_1, v_2, v_3, v_4; $v_5, v_6, v_7, v_8; \ldots v_{4i-2}, v_{4i-1}, v_{4i}$ the vertices v_3, v_4; $v_7, v_8; \ldots v_{4i-2}, v_{4i-1}$, form i number of groups dominate all vertices of $S_p(G)$. Thus $2i$ number of vertices dominate the whole graph $S_p(G)$. That is, $\frac{n}{2}$ number of vertices dominate $S_p(G)$ is minimum. All the vertices in the dominating set are interior vertices. To verify v_2 is an interior vertex, take v_1' and then there exists a vertex v_3' such that $d(v_1', v_3') = d(v_1', v_2) + d(v_2, v_3')$.

Case (2): Suppose that $n = 4i + 1$, $i = 1, 2, \ldots$ By Case (1), the set $D = \{v_2, v_3, v_6, v_7, \ldots, v_{4i-1}\}$ is a minimum interior dominating set of $S_p(G)$ with $\frac{n}{2}$ vertices. In this case we consider one more vertex v_{4i+1}. As the vertex v_{4i-1} dominates the vertices v'_{4i} and v_{4i} another vertex needs to dominates v_{4i+1} and v'_{4i+1}. Hence the vertex v_{4i} can be included in the minimum dominating set and also v_{4i} is an interior vertex. Hence $\gamma_{Id}(S_P(P_n)) = \{(n+1)/2\} + 1$.

Case (3): Assume that $n = 4i + 2$, $i = 1, 2, \ldots.$ In this case, we have to add two more vertices v_{4i+1} and v_{4i+2} in the graph given in Case (1) Apart from the graph given in Case (1), v_{4i+1} and v_{4i+2} vertices are needed to form a minimum dominating set and also v_{4i+1} and v_{4i+2} are interior vertices. Hence $\gamma_{Id}(S_P(P_n)) = \frac{(n+1)}{2}$.

Case (4): Assume that $n = 4i + 3$, $i = 1, 2, \ldots.$ In this case, we have to add three more vertices v_{4i+1}, v_{4i+2} and v_{4i+3} in the graph given in Case (1). Apart from the graph given in Case (1) v_{4i+1} and v_{4i+2} vertices are needed to form a minimum dominating set and also v_{4i+1} and v_{4i+2} are interior vertices. Hence $\gamma_{Id}(S_P(P_n)) = \frac{(n+2)}{2}$. □

Theorem 3.6. *If $S_P(C_n \circ K_1)$ is a splitting graph of corona graph $C_n \circ K_1$, then $\gamma_{Id}(S_P(C_n \circ K_1)) = n$.*

Proof. Let $X = \{v_1, v_2, \ldots, v_n\}$ be the set of vertices of C_n and $Y = \{u_1, u_2, \ldots, u_n\}$ be the set of pendant vertices attached to $v_1, v_2, \ldots, v_n$ respectively. Let $u'_1, u'_2, \ldots, u'_n, v'_1, v'_2, \ldots, v'_n$ be the

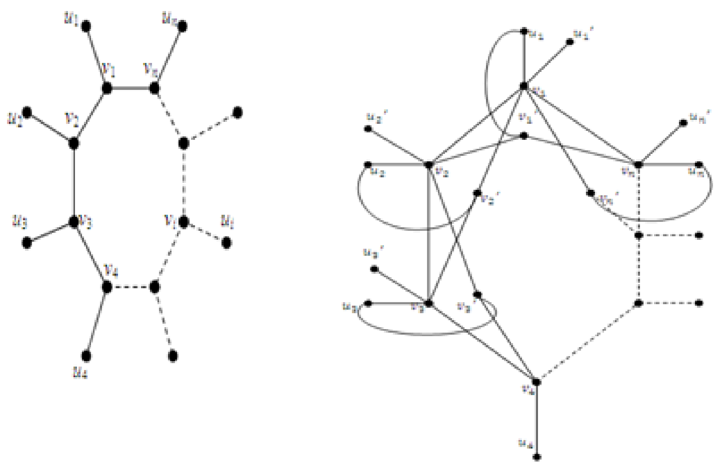

Figure 10. **The corona graph $C_n \circ K_1$ and $S_p(C_n \circ K_1)$**

duplicate vertices of $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ respectively in $S_P(C_n \circ K_1)$. Then the resulting graph $S_P(C_n \circ K_1)$ will have 4n vertices. In $S_p(C_n \circ K_1)$ the vertex v_i dominates $v_{i-1}, v_{i+1} u_i, u'_i, v'_{i-1}, v'_{i+1}$. The vertex set $\{v_1, v_2, \ldots, v_n\}$ is required to dominate all the vertices $\{u_1, u_2, \ldots, u_n\}$ added for corona. Hence for dominating the other $2n$ vertices, the vertex set of C_n is required. The vertices $v_1, v_2, \ldots, v_n$ are interior vertices since for example for every vertex v'_i there exists u_{i+1} such that $d(v'_i, u_{i+1}) = d(v'_i, v_{i+1}) + d(v_{i+1}, u_{i+1})$. then v_{i+1} is a interior vertex. Hence $\{v_1, v_2, \ldots, v_n\}$ is an interior dominating set of $S_p(C_n \circ K_1)$. Hence $\gamma_{Id}(S_P(C_n \circ K_1)) = n$. □

4. Conclusion

The study of domination on graphs is a wide area and it is seen that all types of connected graphs, including complete graphs have dominating sets. But some types of graphs have no interior dominating sets and we can say that interior domination number is zero for those graphs. For example, the domination number of a complete graph K_n is one and the interior domination number of K_n is zero. Also, for the splitting graph of K_n, the interior domination number is zero. In this paper we have studied the interior dominating sets of special types of graphs like subdivided and splitting graphs of graphs. This theory can be extended to special types of graphs like permutation graphs and hyper graphs and algorithms also can be designed for finding the interior dominating sets of graphs.

References

[1] A.Anto Kinsely and C.Caroline Selvaraj, *A study on interior domination in graphs*, IOSR Journal of Mathematics (IOSR-JM), 12(2-VI)(2016), 55-59.

[2] S.Arumugam and Paulraj Joseph, *Domination in Subdivision graphs*, International Journal of Management and System, 11(1)(1996), 111-116.

[3] J.E.Cockayne and S.T.Hedetniemi, *Towards a theory of domination in graphs*, Networks, 7(1997), 247-261.

[4] M.Dettlaff, M.Lemanska, J.Topp and P.Zylinski, *Coronas and domination subdivision number of a graph*, Bulletin of the Malaysian Mathematical Sci. Society, Springer's, DOI10.1007/40840-016-0417-0 (2016).

[5] Gary Chartand and Ping Zhang, *Introduction to Graph theory*, McGraw Hill Education (India) Private Limited, New Delhi, (2006).

[6] F.Harary and F.Buckley, *Distance in graphs*, Addison-Wesley Publishing Company, (1990).

[7] W.Teresa Haynes, T.Stephen Hedetniemi and Peter J.Slater, *Fundamentals of Domination in graphs*, Marcel Dekker, Inc., (1998).

[8] S.Velammal and S.Arumugam, *Domination and Subdivision in graphs*, Indian Journal of Applied Research, 1(3)(2011), 180-183.