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Twig and Cycle Related Near Mean Cordial Graphs

Research Article

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Abstract: Let $G = (V, E)$ be a simple graph. A Near Mean Cordial Labeling of G is a function in $f : V(G) \to \{1, 2, 3, \ldots, p-1, p+1\}$ such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1, & \text{if } (f(u) + f(v)) \equiv 0 \ (mod\ 2); \\ 0, & \text{else.} \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a Near Mean Cordial Graph if it admits a near mean cordial labeling. In this paper, it is to be proved that $Twig\ T(n)$, $\langle C_n : C_{n-1}\rangle$ and W_n (When $n \equiv 0, 2, 3\ (mod\ 4)$) are Near Mean Cordial graphs. Also, W_n (When $n \equiv 1\ (mod\ 4)$) are not Near Mean Cordial graphs.

MSC: 05C78.

Keywords: Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.

1. Introduction

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology, Harary [4] and G.J. Gallian [1] are referred. A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge uv a label depending on the vertex labels of u and v. A graph G is said to be labeled if the n vertices are distinguished from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa [3] in 1967 and subsequently by Golomb [2]. In this paper, It is to be proved that $Twig\ T(n)$, $\langle C_n : C_{n-1}\rangle$ and W_n (When $n \equiv 0, 2, 3\ (mod\ 4)$) are Near Mean Cordial graphs. Also, W_n (When $n \equiv 1\ (mod\ 4)$) are not Near Mean Cordial graph.

2. Preliminaries

Definition 2.1. *Let $G = (V, E)$ be a simple graph. Let $f : V(G) \to \{0, 1\}$ and for each edge uv , assign the label $|f(u) - f(v)|$. f is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the number of edges labeled 1 differ by atmost 1. A graph is called Cordial if it has a cordial labeling.*

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Definition 2.2. *Let $G = (V, E)$ be a simple graph. G is said to be a mean cordial graph if $f : V(G) \to \{0, 1, 2\}$ such that for each edge uv the induced map f^* defined by $f^*(uv) = \left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where $\lfloor x \rfloor$ denote the least integer which is $\leq x$ and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with zero label, $e_f(1)$ is the number of edges with one label.*

Definition 2.3. *Let $G = (V, E)$ be a simple graph. A Near Mean Cordial Labeling of G is a function in $f : V(G) \to \{1, 2, 3, \ldots, p-1, p+1\}$ such that the induced map f^* defined by*

$$f^*(uv) = \begin{cases} 1, & \text{if } (f(u) + f(v)) \equiv 0 \ (mod\ 2); \\ 0, & \text{else.} \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a Near Mean Cordial Graph if it admits a near mean cordial labeling.

Definition 2.4. *The graph obtained from a path by attaching exactly two pendant edges to each internal vertex of the path is called a Twig and it is denoted by $T(n)$.*

Definition 2.5. *A closed trail whose origin and internal vertices are distinct is called a cycle C_n. At one point of C_n is attached with another cycle C_{n-1} of length $n-1$. It is denoted by $\langle C_n : C_{n-1} \rangle$.*

Definition 2.6. *A Wheel on n ($n > 4$) vertices is a graph obtained from a cycle C_n by adding a new vertex and edges joining it to all the vertices of the cycle; the new edges are called the spokes of the wheel. Also, $W_n = C_n + K_1$, ($n > 4$).*

3. Main Results

Theorem 3.1. *Twig $T(n)$ is a Near Mean Cordial Graph.*

Proof. Let $V(T(n)) = \{u_i : 1 \leq i \leq n, v_i : 1 \leq i \leq n-2, w_i : 1 \leq i \leq n-2\}$. Let $E(T(n)) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i u_{i+1} : 1 \leq i \leq n-2\} \cup \{w_i u_{i+1} : 1 \leq i \leq n-2\}$.

When n is odd: Define $f : V(T(n)) \to \{1, 2, 3, \ldots, 3n-5, 3n-3\}$ by

$$\begin{aligned} f(u_i) &= 2i, & 1 \leq i \leq n \\ f(w_i) &= 2i - 1, & 1 \leq i \leq n-2 \\ f(v_i) &= 2n + 2i - 5, & 1 \leq i \leq \frac{n-1}{2} \\ f(v_{n-(i+1)}) &= 3n - 1 - 2i, & 1 \leq i \leq \frac{n-3}{2} \end{aligned}$$

When n is even: Define $f : V(T(n)) \to \{1, 2, 3, \ldots, 3n-5, 3n-3\}$ by

$$\begin{aligned} f(u_i) &= 2i - 1, & 1 \leq i \leq n \\ f(w_i) &= 2i, & 1 \leq i \leq n-2 \\ f(v_i) &= 2n - 4 + 2i, & 1 \leq i \leq \frac{n}{2} - 1 \\ f(v_{n-(i+1)}) &= 3n - 1 - 2i, & 1 \leq i \leq \frac{n}{2} - 1 \end{aligned}$$

The induced edge labelings are

$$f^*(u_iu_{i+1}) = \begin{cases} 1, & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \ (mod\ 2); \\ 0, & \text{else.} \end{cases} \quad 1 \leq i \leq n-1$$

$$f^*(w_iw_{i+1}) = \begin{cases} 1, & \text{if } f(w_i) + f(w_{i+1}) \equiv 0 \ (mod\ 2); \\ 0, & \text{else.} \end{cases} \quad 1 \leq i \leq n-1$$

$$f^*(v_iv_{i+1}) = \begin{cases} 1, & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \ (mod\ 2); \\ 0, & \text{else.} \end{cases} \quad 1 \leq i \leq n-1$$

When n is odd: Let $n = 2k+1$, $(k \in N)$. Here, $e_f(0) = e_f(1) = n + k - 2$.

When n is even: Let $n = 2k$, $(k \in N)$. Here, $e_f(1) = n + k - 2$ and $e_f(0) = n + k - 3$.

So, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. Hence, $T(n)$ is a Near Mean Cordial Graph. □

For example, the Near Mean Cordial Labeling of $T(8)$ and $T(9)$ are shown in Figures 1 and 2.

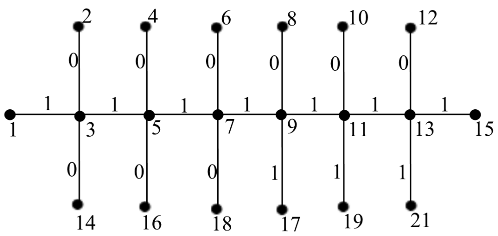

Figure 1:

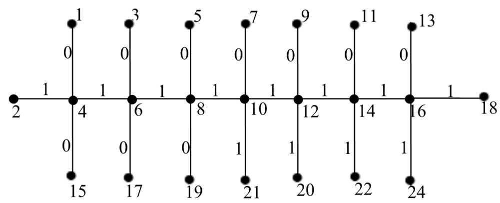

Figure 2:

Theorem 3.2. *$\langle C_n : C_{n-1}\rangle$ is a Near Mean Cordial Graph.*

Proof. Let $V(\langle C_n : C_{n-1}\rangle) = \{u_1, u_2, \ldots, u_n = v_1, v_2, \ldots, v_{n-1}\}$. Let $E(\langle C_n : C_{n-1}\rangle) = \{(u_iu_{i+1}) : 1 \leq i \leq n-1\} \cup \{(v_iv_{i+1}) : 1 \leq i \leq n-1\} \cup \{u_nu_1\} \cup \{v_nv_1\}$.

When n is odd: Define $f : V(\langle C_n : C_{n-1}\rangle) \to \{1, 2, 3, \ldots, 2n-3, 2n-1\}$ by

$$\begin{aligned} f(u_{2i-1}) &= i, & 1 \leq i \leq \frac{n+1}{2} \\ f(u_{2i}) &= \frac{n+1}{2} + i, & 1 \leq i \leq \frac{n-1}{2} \\ f(v_{2i}) &= \frac{3n-1}{2} + (i-1), & 1 \leq i \leq \frac{n-3}{2} \\ f(v_{2i+1}) &= n + i, & 1 \leq i \leq \frac{n-3}{2} \\ f(v_n) &= 2n-1 \quad & and \ \ f(u_n) = f(v_1) \end{aligned}$$

When n is even: Define $f : V(\langle C_n : C_{n-1}\rangle) \to \{1, 2, 3, \ldots, 2n-3, 2n-1\}$ by

$$\begin{aligned} f(u_{2i-1}) &= i, & 1 \leq i \leq \frac{n}{2} \\ f(u_{2i}) &= \frac{n}{2} + i, & 1 \leq i \leq \frac{n}{2} \\ f(v_{2i}) &= \frac{3n}{2} + (i-1), & 1 \leq i \leq \frac{n-4}{2} \\ f(v_{2i+1}) &= n + i, & 1 \leq i \leq \frac{n-2}{2} \\ f(v_{n-2}) &= 2n-1 \quad & and \ \ f(u_n) = f(v_1) \end{aligned}$$

In both the cases, The induced edge labelings are

$$f^*(u_iu_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases} \quad 1 \le i \le n-1$$

$$f^*(u_nu_1) = \begin{cases} 1 & \text{if } f(u_n) + f(u_1) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases}$$

$$f^*(u_nv_1) = \begin{cases} 1 & \text{if } f(u_n) + f(v_1) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases} \quad 1 \le i \le n-1$$

$$f^*(u_nv_n) = \begin{cases} 1 & \text{if } f(u_n) + f(v_n) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} 1 & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases} \quad 1 \le i \le n-1$$

Here, $e_f(1) = n$ and $e_f(0) = n-1$ (when n is odd). Here, $e_f(0) = n$ and $e_f(1) = n-1$ (when n is even). So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$. Hence, $\langle C_n : C_{n-1} \rangle$ is a Near Mean Cordial Graph. □

For example, the Near Mean Cordial Labeling of $\langle C_9 : C_8 \rangle$ and $\langle C_{10} : C_9 \rangle$ are shown in Figures 3 and 4.

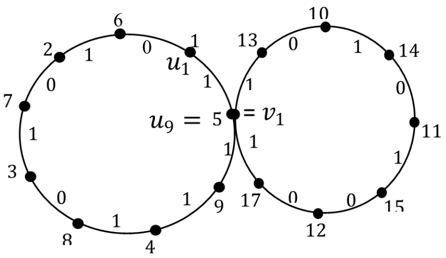

Figure 3:

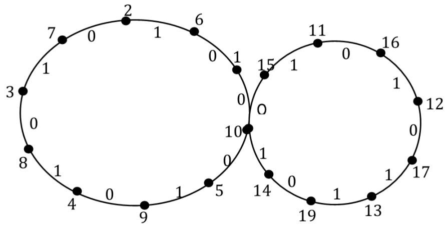

Figure 4:

Theorem 3.3. *W_n is a Near Mean Cordial Graph when $n \equiv 0, 2, 3 \ (mod\ 4)$.*

Proof. Let $V(W_n) = \{u, u_i : 1 \le i \le n\}$. Let $E(W_n) = \{(uu_i) : 1 \le i \le n\} \cup \{(u_iu_{i+1}) : 1 \le i \le n-1\} \cup \{u_nu_1\}$.
Case (1): when $n \equiv 0 \ (mod\ 4)$. Define $f : V(W_n) \to \{1, 2, 3, \ldots, n, n+2\}$ by let $f(u) = n+2$

$$f(u_{2i-1}) = i, \qquad 1 \le i \le \frac{n}{2}$$
$$f(u_{2i}) = \frac{n+2}{2} + (i-1), \quad 1 \le i \le \frac{n}{2}$$

Case (2): when $n \equiv 2 \ (mod\ 4)$. Define $f : V(W_n) \to \{1, 2, 3, \ldots, n, n+2\}$ by let $f(u) = n-1$, $f(u_1) = 1$, $f(u_2) = 2$ and $f(u_3) = 4$

$$f(u_{4i}) = 4i+2, \qquad 1 \le i \le \frac{n-2}{4}$$
$$f(u_{4i+1}) = 4(i+1), \quad 1 \le i \le \frac{n-2}{4}$$
$$f(u_{4i+2}) = 4i-1, \qquad 1 \le i \le \frac{n-2}{4}$$
$$f(u_{4i+3}) = 4i+1, \qquad 1 \le i \le \frac{n-6}{4}$$

Case (3): when $n \equiv 3 \ (mod\ 4)$. Define $f : V(W_n) \to \{1, 2, 3, \ldots, n, n+2\}$ by let $f(u) = n+2$, $f(u_n) = \frac{n+1}{2}$

$$f(u_{2i-1}) = i, \qquad 1 \le i \le \frac{n-1}{2}$$
$$f(u_{2i}) = \frac{n+3}{2} + (i-1), \quad 1 \le i \le \frac{n-1}{2}$$

The induced edge labelings are

$$f^*(u_iu_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(u_nu_1) = \begin{cases} 1 & \text{if } f(u_n) + f(u_1) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases}$$

$$f^*(uu_i) = \begin{cases} 1 & \text{if } f(u) + f(u_i) \equiv 0 \ (mod\ 2) \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

Here, $e_f(0) = e_f(1) = n$. Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$. Hence, W_n is a Near Mean Cordial Graph. □

For example, the Near Mean Cordial Labeling of W_8, W_{10} and W_{11} are shown in Figures 5, 6 and 7.

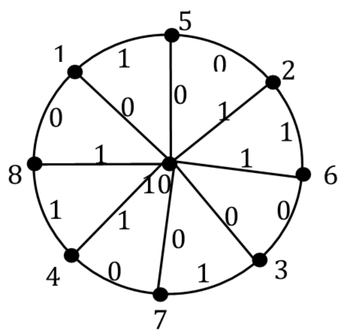

Figure 5:

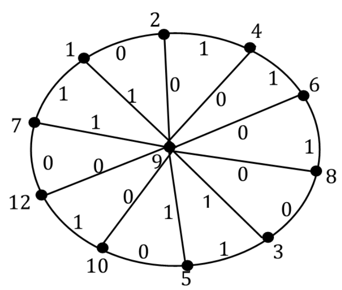

Figure 6:

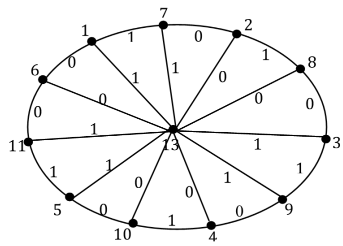

Figure 7:

Theorem 3.4. *W_n (when $n \equiv 1 \ (mod\ 4)$) is not a Near Mean Cordial Graph.*

Proof. Let $V(W_n) = \{u, u_i : 1 \le i \le n\}$. Let $E(W_n) = \{uu_i : 1 \le i \le n\} \cup \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_nu_1\}$ where u is called central vertex and $u_1, \ldots, u_n$ are called rim vertices. Consider W_9. Now, the vertex labels are 1, 2, 3, 4, 5, 6, 7, 8, 9, 11. If a pair consisting of the same parity, it gives edge labeling 1 otherwise the edge labeling is 0.

Case (1): Suppose a central vertex is labeled by an odd number. Number of pairs consisting of a central vertex and a rim vertex is 9. It contributes 4 zeros and 5 ones as edge labeling. The rim vertices are assigned only 4 even labels. It can be partitioned into 5 subcases.

(1). 4 even labeled vertices are consecutive on rim

(2). Only 3 even labeled vertices are consecutive on rim

(3). All the 4 even labeled vertices are not consecutive on rim

(4). Only 2 even labeled vertices are consecutive on rim

(5). Two pair of consecutive even label vertices separated by atleast an odd label vertex on rim.

Subcase (1): Suppose 4 even labeled vertices are consecutive on rim. In this case, we have 7 ones and 2 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 12 ones and 6 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Subcase (2): Suppose only 3 even labeled vertices are consecutive on rim. In this case, we have 5 ones and 4 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 10 ones and 8 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Subcase (3): Suppose all the 4 even labeled vertices are not consecutive on rim. In this case, we have 1 ones and 8 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 6 ones and 12 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Subcase (4): Suppose only 2 even labeled vertices are consecutive on rim. In this case, we have 3 ones and 6 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 8 ones and 10 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Subcase (5): Suppose two pair of consecutive even label vertices separated by an odd label vertices on rim. In this case, we have 5 ones and 4 zeros as edge labels for rim vertices . On the whole (including the central vertex), we have 10 ones and 8 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Case (2): Suppose a central vertex is labeled by an even number. Number of pairs consisting of a central vertex and a rim vertex is 9. It contributes 6 zeros and 3 ones as edge labeling. The rim vertices are assigned 3 even labels. It can be partitioned into 3 subcases.

(1). 3 even labeled vertices are consecutive on rim

(2). Only 2 even labeled vertices are consecutive on rim

(3). All the 3 even labeled vertices are not consecutive on rim

Subcase (1): Suppose 3 even labeled vertices are consecutive on rim. In this case, we have 7 ones and 2 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 10 ones and 8 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Subcase (2): Suppose only 2 even labeled vertices are consecutive on rim. In this case, we have 5 ones and 4 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 8 ones and 10 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

Subcase (3): Suppose all the 3 even labeled vertices are not consecutive on rim. In this case, we have 3 ones and 6 zeros as edge labels for rim vertices. On the whole (including the central vertex), we have 6 ones and 12 zeros as edge labels. Clearly, in this case $|e_f(0) - e_f(1)| > 1$.

In a similar manner, it can be verified that W_5 and W_n $(n \equiv 1\ (mod\ 4))$ for all $n > 9$ and we have $|e_f(0) - e_f(1)| > 1$. Hence W_n (when $n \equiv 1\ (mod\ 4)$)is not Near Mean Cordial Graph. □

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