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A New Iterative Method for Solving Nonlinear Equations Using Simpson Method

Research Article

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Abstract: In this paper, we introduced the new iterative method for solving nonlinear equations. This method based on the combination of mid-point with Simpson quadrature formulas and using Newton's method. The convergence analysis of our method are discussed. It is established that the new method has convergence order three. Numerical tests shows that the new method is comparable with the well-known existing single step iterative methods and provides better results.

Keywords: Nonlinear equations, iterative methods, Simpson method, Newton's method, convergence.

1. Introduction

Numerical analysis is the area of mathematics and computer sciences that creates, analyzes and numerically the problems of continuous mathematics. Such problems originate generally from real-world applications of algebra, geometry and calculus and they involves variable which very continuously: these problems occur throughout the natural sciences, social sciences, engineering, medicine and business. A new iterative method for finding the approximate solutions of the nonlinear equation $(x) = 0$. This numerical method has been constructed using different techniques such as Taylor series, homotopy, quadrature formula and decomposition method for more details see [1–3, 6–14]. This paper based on a combination of mid-point with Simpson quadrature formulas. There are several different methods in the literature for computation of the root of the nonlinear equation. The most famous of these methods is the classical Newton's method (NM) [4, 5].

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n)} \tag{1}$$

Therefore the Newton's method was modified by Steffensen's method [4]

$$x_{n+1} = x_n - \frac{[f(x_n)]^2}{f\left(x_n + f(x_n)\right) - f(x_n)}$$

Newton's method and Steffensen's method are of second order converges [4, 5].

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2. Iterative Methods

Consider the nonlinear equations of the type

$$f(x) = 0 \tag{2}$$

For simplicity , we assume that α is a simple root of (2) and γ is an initial guess sufficiently close to α. Now using the simpson quadrature formulas [4, 5] and fundamental theorem of calculus, one can show that the function $f(x)$ can be approximated by the series [4].

$$f(x) = f(\gamma) + \left(\frac{x-\gamma}{6}\right)\left[f'(x) + 4f'\left(\frac{x+\gamma}{2}\right) + f'(\gamma)\right] \tag{3}$$

Where $f'(x)$ is the differential of. From equation (2) and (3), we have

$$x = \gamma - \frac{6f(\gamma)}{\left[f'(x) + 4f'\left(\frac{x+\gamma}{2}\right) + f'(\gamma)\right]} \tag{4}$$

Using (4), one can suggest the following iterative method for solving the nonlinear equation (2).

Theorem 2.1. *For a given initial choice x_0, find the approximate solution x_{n+1} by the iterative schemes.*

$$x_{n+1} = x_n - \frac{6f(x_n)}{\left[f'(x_n) + 4f'\left(\frac{x_{n+1}+x_n}{2}\right) + f'(x_n)\right]}, \quad n = 0, 1, 2, 3, \ldots. \tag{5}$$

Theorem 2.2. *For a given initial choice x_0, find the approximate solution x_{n+1} by the iterative schemes. From equation* (1) *and* (5)

$$x_{n+1} = x_n - \frac{6f(x_n)}{\left[f'\left(x_n - \frac{f(x_n)}{f(x_n)}\right) + 4f'\left(x_n - \frac{1}{2}\frac{f(x_n)}{f(x_n)}\right) + f'(x_n)\right]}, \quad n = 0, 1, 2, 3, \ldots.$$

Theorem 2.2 is called the New Iterative method (NIM) and has third order convergence.

3. Convergence Analysis

Let us now discuss the convergence analysis of the above Theorem 2.2.

Theorem 3.1. *Let $\alpha \in I$ be a simple zero of sufficiently differential function $f : I \subseteq R \to R$ for an open interval I, if x_0 is sufficiently close to α then the New Iterative Method (NIM) defined by Theorem 2.2 third order convergence.*

Proof. Let α be a simple zero of f. Than by expanding $f(x_n)$ and $f'(x_n)$ about α we have

$$f(x_n) = e_n c_1 + e_n^2 c_2 + e_n^3 c_3 + \ldots \tag{6}$$

$$f'(x_n) = c_1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + \ldots \tag{7}$$

Where $c_k = \frac{1}{k!} f^{(k)}(\alpha)$, $k = 1, 2, 3, \ldots$ and $e_n = x_n - \alpha$. From equation (6) and equation (7), we have

$$\frac{f(x_n)}{f'(x_n)} = e_n - \frac{c_2}{c_1}e_n^2 - \left(\frac{2c_2^2}{c_1^2} + \frac{2c_3}{c_1}\right)e_n^3 - \ldots \tag{8}$$

From equation (8), we have

$$f'\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) = c_1 + \frac{2c_2^2}{c_1}e_n^2 + \frac{3c_2^2 c_3}{c_1^2}e_n^4 + \ldots \tag{9}$$

From equation (6) and equation (7), we have

$$\frac{1}{2}\frac{f(x_n)}{f'(x_n)} = \frac{e_n}{2} - \frac{c_2}{2c_1}e_n^2 - \left(\frac{c_2^2}{c_1^2} + \frac{c_3}{c_1}\right)e_n^3 - \dots \tag{10}$$

From equation (10), we have

$$4f'\left(x_n - \frac{1}{2}\frac{f(x_n)}{f'(x_n)}\right) = 4c_1 + 4c_2e_n + \ 3c_3e_n^2 + \dots \tag{11}$$

From equation (7), equation (9) and equation (11), we have

$$f'\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) + 4f'\left(x_n - \frac{1}{2}\frac{f(x_n)}{f'(x_n)}\right) + f'(x_n) = 6c_1 + 6c_2e_n + 6\left(c_3 + \frac{c_2^2}{c_1}\right)e_n^2 + \dots \tag{12}$$

From equation (6) and equation (12), we have

$$\frac{6f(x_n)}{\left[f'\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) + 4f'\left(x_n - \frac{1}{2}\frac{f(x_n)}{f'(x_n)}\right) + f'(x_n)\right]} = e_n - \frac{2c_2^2}{c_1^2}e_n^3 - \left(\frac{2c_2c_3}{c_1^2} + \frac{c_2^3}{c_1^3}\right)e_n^4 - \dots \tag{13}$$

$$x_{n+1} = \alpha + \frac{2c_2^2}{c_1^2}e_n^3 + \left(\frac{2c_2c_3}{c_1^2} + \frac{c_2^3}{c_1^3}\right)e_n^4 + \dots$$
$$e_{n+1} = \frac{2c_2^2}{c_1^2}e_n^3 + \left(\frac{2c_2c_3}{c_1^2} + \frac{c_2^3}{c_1^3}\right)e_n^4 + \dots$$

This shows that Theorem 2.2 is third order convergence. □

4. Numerical Results

We present some example to illustrate the root of the newly developed new iterative method, see Table 1. We compare the Newton method (NM) and Steffensen's method (SM). All computations are performed using MATLAB. The following examples are used for numerical testing.

$$f_1(x) = \cos(x) - x$$
$$f_2(x) = \sin(x) - 1 + x$$
$$f_3(x) = e^x - 3x$$
$$f_4(x) = x\tan(x) + 1$$
$$f_5(x) = 2\sin(x) - x$$
$$f_6(x) = x + \sin(x) - x^3$$
$$f_7(x) = \cos(x) - xe^x$$
$$f_8(x) = x^2 - 9$$
$$f_9(x) = e^x - 1.5 - \tan^{-1}(x)$$
$$f_{10}(x) = x^3 - 6x + 4$$

As for the convergence criteria, it was required that the distance of two consecutive approximations δ and also displayed is the number of iterations to approximate the zero (IT), the approximate zero x_n and the value $f(x_n)$.

method	IT	x_n	$f(x_n)$	δ
f_1, $x_0 = -2$				
NM	7	0.739085633215161		6.36046770807752e-013
SM	6	0.739085633215161	-8.36806107518129e-007	2.20046203480706e-013
NIM	5	0.739085633215161		3.12775260136178e-008
f_2, $x_0 = 1$				
NM	4	0.510973429388569		2.01926610987613e-008
SM	5	0.510973429388569	-1.11022302462516e-016	3.21703210737212e-008
NIM	3	0.510973429388569		1.99881600160268e-009
f_3, $x_0 = 0$				
NM	5	0.619061286735945		3.38063199656347e-009
SM	4	0.619061286735945	2.22044604925031e-016	2.15426398141787e-009
NIM	4	0.619061286735945		3.99680288865056e-015
f_4, $x_0 = 2.5$				
NM	3	2.79838604578389		9.02370224986626e-006
SM	6	2.79838604578389	7.54951656745106e-015	5.83542991705599e-009
NIM	2	2.79838604578389		0.000947485289019667
f_5, $x_0 = 2.9$				
NM	5	1.89549426703398		9.56376000615933e-009
SM	5	1.89549426703398	1.55431223447522e-015	6.99440505513849e-014
NIM	4	1.89549426703398		2.99760216648792e-014
f_6, $x_0 = 1$				
NM	6	1.31716296100603		7.95099541761601e-011
SM	7	1.31716296100603	9.76996261670138e-015	2.227600326421e-010
NIM	4	1.31716296100603		3.44498007898153e-009
f_7, $x_0 = 1$				
NM	6	0.51775736382458		9.99200722162641e-016
SM	6	0.51775736382458	9.99200722162641e-016	5.81991899117895e-010
NIM	4	0.51775736382458		8.70414851306123e-014
f_8, $x_0 = 2.5$				
NM	4	3		2.79904401878639e-008
SM	4	3	0	6.60830057341855e-010
NIM	3	3		2.54458987214434e-009
f_9, $x_0 = 1$				
NM	5	0.767653266201279		1.49391610193561e-012
SM	5	0.767653266201279	0	9.99200722162641e-016
NIM	3	0.767653266201279		2.32757084939195e-007
f_{10}, $x_0 = 1$				
NM	5	0.732050807568677		1.58995039356569e-012
SM	5	0.732050807568677	8.88178419700125e-016	6.99440505513849e-015
NIM	3	0.732050807568677		3.25426974034926e-007

Table 1. **Numerical Examples and Comparison**

5. Conclusion

With the comparative study of newly developed techniques (NIM) is faster than Newton's and Steffensen's method. Our method can be considered as significant improvement of Newton's and Steffensen's method and can be considered as alternative method of solving nonlinear equations.

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