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Micropolar Nanofluid Over a MHD Heat Transfer Porous Shrinking Sheet

Research Article

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Abstract: This analysis investigated with the boundary layer flow and heat transfer aspects of a micropoloar nanofluid over a porous shrinking sheet with thermal radiation. The boundary layer equations governed by the partial differential equations are transformed in to a set of ordinary differential equations with the help of suitable local similarity transformations. The coupled nonlinear ordinary differential equations are solved by the implicit finite difference method along with the Thamous algorithm. Dual solutions of dimensionless velocity, angular velocity, temperature and concentration profiles are analyzed by the effect of various controlling flow parameters viz., Lewis number Le, thermophoresis Nt, Brownian motion parameter Nb, Radiation parameter R, Prandtl number Pr, material parameter K, mass suction parameter S, magnetic parameter M.

Keywords: MHD, micropolar fluid flow, thermal radiation, nanofluid, shrinking sheet.

1. Introduction

In last few decades, the research interest in micropolar fluid theory has significantly increased due to its enormous applications in many industrial processes. A micropolar fluid is the fluid with internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. It is a hydro dynamical framework suitable for angular systems which consist of particles with macroscopic size. Unlike the other fluids, micropolar fluids may be described as non-Newtonian fluids consisting of dumb-bell molecules or short rigid cylindrical element, polymer fluids, fluid suspension, etc. In addition with the classical velocity field, a microrotation vector and a gyration parameter are introduced in the micropolar fluid model in order to investigate the kinematics of microrotation. The theory of micropolar fluids, first proposed by Eringen [1, 2] can be used to study the behaviors of exotic lubricants, polymeric suspensions, muddy and biological fluids, animal blood, colloidal solutions, liquid crystals with rigid molecules, etc. BS Malga and Kishan.N [3] studied the unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous plate with suction by taking into account the viscous dissipation is considered when the plate accelerates in its own plane. Guram and Smith [4] investigated the stagnation point flow of a micropolar fluid in an infinite plate with two different boundary conditions - vanishing spin and vanishing spin gradient. The free convective boundary layer flow of a thermomicropolar fluid over a non-isothermal vertical flat plate was discussed by Jena and Mathur [5]. Gorla [6] studied the micropolar boundary layer flow near a stagnation point on a moving wall.

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In a continuation study of flow over a stretching sheet, considerable interest has been placed on fluid flow over a shrinking sheet. The study of viscous flow over a shrinking sheet with suction effect at the boundary was first investigated by Miklavcic and Wang [7].

The "nanofluid" term was first introduced by Choi [8] to describe the mixture of nanoparticles and base fluid such as water and oil. The addition of nanoparticle into the base fluid is able to change the transport properties; flow and heat transfer capability of the liquids and indirectly increase the low thermal conductivity of the base fluid which is identified as the main obstacle in heat transfer performance. This mixture has attracted the interest of numerous researchers because of its many significant applications such as in the medical applications, transportations, microelectronics, and chemical engineering, aerospace and manufacturing. Hunegnaw D and Kishan.N [9] studied the the magnetohydrodynamic boundary layer flow and heat transfer of a nanofluid past a nonlinearly permeable stretching/shrinking sheet with thermal radiation and suction effect in the presence of chemical reaction.

Srinivas Maripala and Kishan.N [10], studied the Unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reaction. Recently, Krishnendu Bhattacharyya et al. [11] studied the Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet.

The main goal of the present study in to investigate the MHD flow and heat transfer in micropolar nanofluid over a porous shrinking sheet with thermal radiation are studied. It is very interesting to investigate the simultaneous effects of the thermal radiation and the microrotation on the steady flow. The nonlinear self-similar ordinary differential equations obtained here are solved numerically by finite difference technique with the help of Cranck-Nicklson method using Thomus algoritham. The complete effects of several parameters are discussed in detail.

2. Analysis of the Flow Problem

Consider, unsteady two-dimensional laminar boundary layer flow of incompressible electrically conducting viscous microploar nanofluid and heat transfer a porous shrinking sheet with thermal radiation. The flow is subjected to a transverse magnetic field of strength B_0, which is assumed to be applied in the positive $y-$direction, normal to the surface. It is assumed that the velocity of the shrinking sheet $U_W = -C_X$ with $c > 0$ being shrinking constant. Using boundary layer approximation, the equations of motion for the micropolar nanofluid and heat transfer may be written in usual notation as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(v + \frac{k}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\Upsilon}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} - \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{dT}{dy}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{dT}{dy}\right)^2 \tag{5}$$

Subject to the boundary conditions:

$$u = U_w,\ -cx = v_w,\ N = -m\frac{\partial u}{\partial y},\ T = T_w,\ C = C_w \ \text{ at } y = 0 \tag{6}$$

And $u \to 0$, $N \to 0$, $T \to T_\infty$ and $C \to C_\infty$ as y tends to ∞, Where u and v are velocity components in x and y direction respectively, $v(= \mu/\rho)$ is the kinemetic fluid viscosity, ρ is the fluid density, μ is the dynamic viscosity, N is the microrotation or angular velocity whose direction is normal to the xy-plane, j is microinertia per unit mass, γ is spin gradient viscosity, k is the vortex viscosity (gryo-viscosity), T is the temperature, C is the concentration of the fluid, k is the thermal conductivity of the fluid, C_p is the specific heat, q_r is the radiative heat flux, T_w and C_w-the temperature and concentration of the sheet, T_∞ and C_∞-the ambient temperature and concentration, D_B-the Brownian diffusion coefficient, D_T the thermophoresis coefficient, B_0-the magnetic induction, $(\rho C)_p$-the heat capacitance of the nanoparticles, $(\rho C)_f$-the heat capacitance of the base fluid, and $\tau = \frac{(\rho C)_p}{(\rho C)_f}$ is the ratio between the effective heat capacity of the nanoparticles material and heat capacity of the fluid. Here, v_w is the wall mass transfer velocity with $v_w < 0$ for mass suction and $v_w > 0$ for mass injection. We note that m is constant such that $0 \leq m \leq 1$. The case $m = 0$ indicates $N = 0$ at the surface. We assumed that spin gradient viscosity y is given by :

$$\gamma = \left(\mu + \frac{k}{2}\right) j = \left(\mu\left(1 + \frac{k}{2}\right)\right) j, \tag{7}$$

where $K = \frac{k}{\mu}$ is the material parameter. This assumption is in voked o allow the field of equations to predict the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity. Using Rosseland's approximation for radiation, we obtain $q_r = -\left(\frac{4\sigma}{3k_1}\right)\frac{\partial T^4}{\partial y}$ where σ the Stefan-Boltzman constant is, k_1 is the absorption coefficient. We presume that the temperature variation within the flow is such that T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher order terms we get, $T^4 = 4T_\infty^3 T - 3T_\infty^4$. Now Equation (4) reduces to:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k_1\rho C_p}\frac{\partial^2 T}{\partial y^2} - \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{dT}{dy}\right)^2\right] \tag{8}$$

The following transformations are introduced :

$$\psi = (cv)^{\frac{1}{2}} X f(\eta),\ N = cX\left(\frac{c}{v}\right)^{\frac{1}{2}},\ h(\eta) = T_\infty + (T_w - T_\infty)\theta(\eta),\ C = C_\infty + (C_w - C_\infty)\phi(\eta),\ \eta = \left(\frac{c}{v}\right)^{\frac{1}{2}} y \tag{9}$$

Where ψ is the stream function defined in the usual notation as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ and η is the similarity variable. Now, Equation (1) is identically satisfied and the Equations (2), (3) and (8) reduce to the following nonlinear self-similar ordinary differential equations:

$$(1+K)f''' + ff'' - f'^2 + Kh' - Mf' = 0 \tag{10}$$

$$\left(1 + \frac{K}{2}\right)h'' + fh' - f'h - K(2h + f') = 0 \tag{11}$$

$$(3R+4)\theta'' + 3RPrf\theta' + Pr(Nb\theta' + Nt\theta'^2) = 0 \tag{12}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{13}$$

Where primes denote differential with respect to η. $M - Pr = \frac{\mu C_p}{k^*}$ is the Prandtl number, $Nb = \tau D_B(C_w - C_\infty)/v$ is Brownian motion parameter, $Nt = \tau D_T(T_w - T_\infty)/vT_\infty$ is thrmophoresis parameter, $Le = v/D_B$ is Lewis number and $R = \frac{k^* K_1}{4\sigma T_\infty^3}$ is the thermal radiation parameter. The transformed boundary conditions are $f(\eta) = S$, $f'(\eta) = -1$, $h(\eta) = -mf''(\eta)$, $\theta(\eta) = 1$, $\phi(\eta) = 1$ at $\eta = 0$. And $f'(\eta)$ tends to 0, $h'(\eta)$ tends to 0, $\theta'(\eta)$ tends to 0, $\phi'(\eta)$ tends to 0 as η tends to ∞. And where $S = -\frac{v_w}{(cv)^{\frac{1}{2}}}$ is wall mass transfer parameter, $S > 0$ corresponds to mass suction and $S < 0$ corresponds to mass injection.

3. Results

The computations have been carried out for various flow parameters such as mass suction parameter S, magnetic parameter M, Lewis number Le, thermophoresis parameter Nt, Brownian motion parameter Nb, radiation parameter R, Prandtl number Pr, material parameter K. The values of skin frication coefficient $f''(0)$, stress coefficient $h'(0)$, Nusselt number coefficient $-\theta'(0)$, Sherwood number coefficient $-\phi'(0)$ for different values of thermophoresis parameter Nt, Brownian motion parameter Nb, Lewies number Le, magnetic parameter M are calculated.

Figure 1 shows that for both first and second solution for velocity, angular velocity, temperature, and concentration profiles respectively, for different values of mass suction parameter S. From Figure 1 it is noticed that the angular velocity increases with the increasing of mass suction parameter S for both dual solutions. The angular velocity decreases with the increase of m is observed from Figure 2. In Figure 3, the effects of magnetic parameter M enhance the dual angular velocity profiles for first and second solutions.

The variations in angular velocity profiles distribution for several values of thermophoresis parameter Nt respectively. The effect of thermophoresis parameter Nt reduces as increases with the increase of thermophoresis parameter Nt for both solutions. It is observed that an increase of thermophoiresis parameter Nt leads to increased the angular velocity profiles for both first and second solutions. The effect of Brownian motion parameter Nb on dual angular velocity profiles are shown in Figure 5.

It is also noticed from Figure 5 the influence of Brownian motion parameter Nb is to enhance the angular velocity for second solution, where as reduce for first solution. The reverse phenomenon is observed for both the solutions for accuracy from the boundary. The variations in angular velocity profiles for different values of material parameter K are demonstrated in Figure 6, the dual angular velocity profiles $h(\eta)$ in Figure 6, show that the angular velocity values increases as material parameter K decreases for both first and second solution.

3.1. Graphs

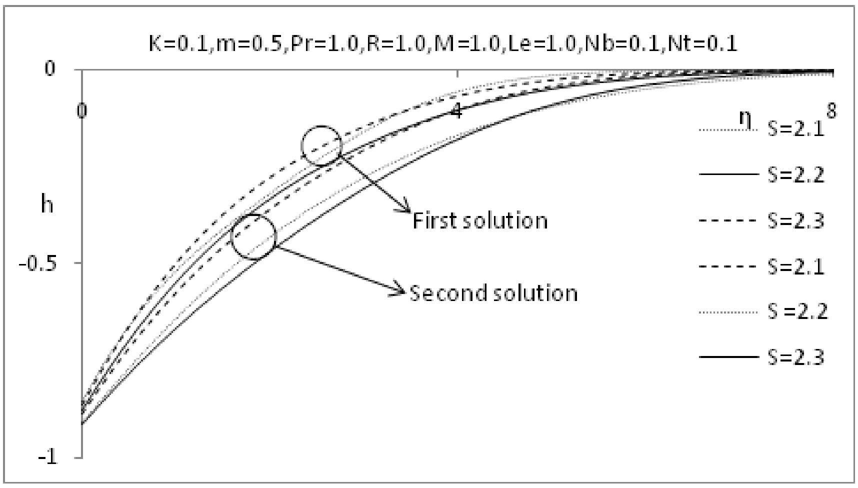

Figure 1: Dual angular velocity profiles for several values of S

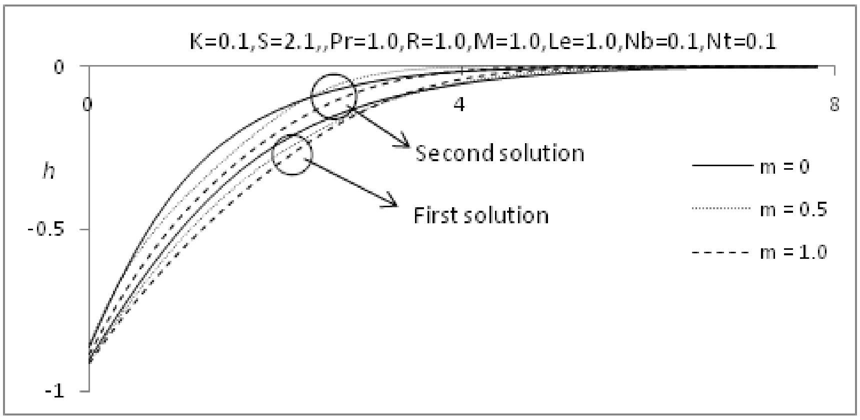

Figure 2: Dual angular velocity profiles for several values of m

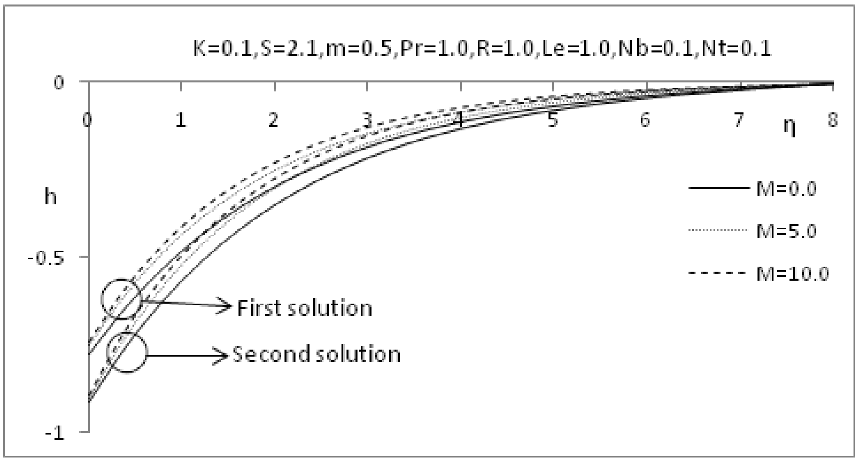

Figure 3: Dual angular velocity profiles for several values of M

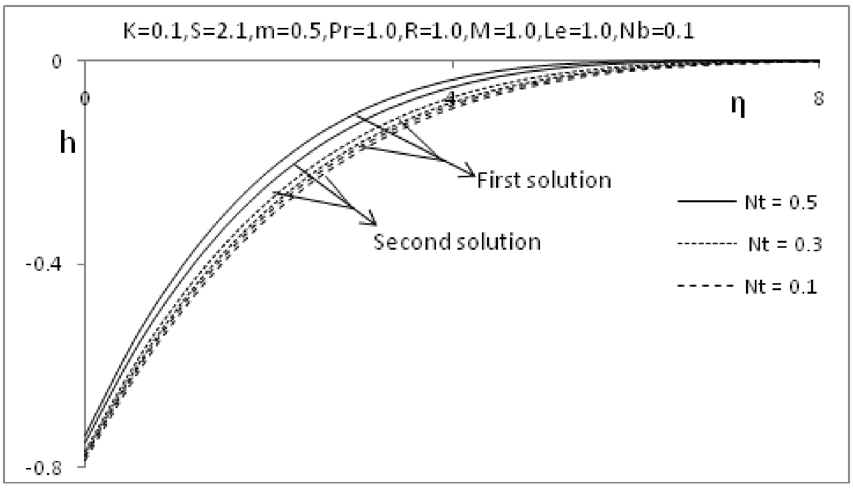

Figure 4: Dual angular velocity profiles for several values of Nt

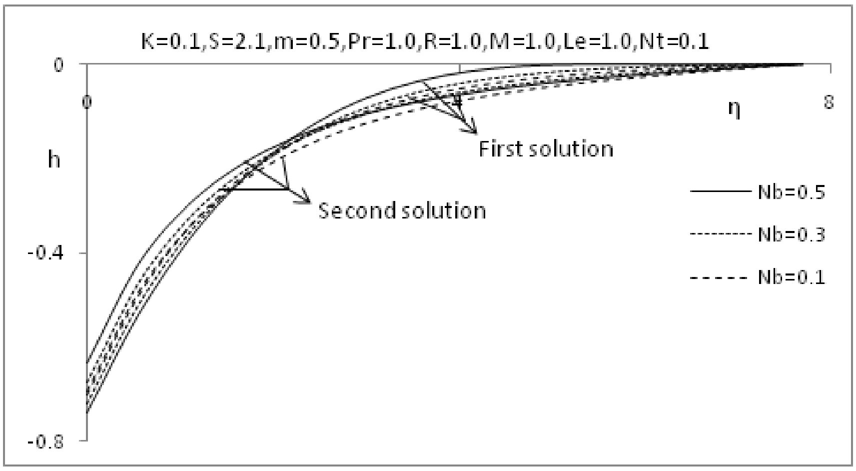

Figure 5: Dual angular velocity profiles for several values of Nb

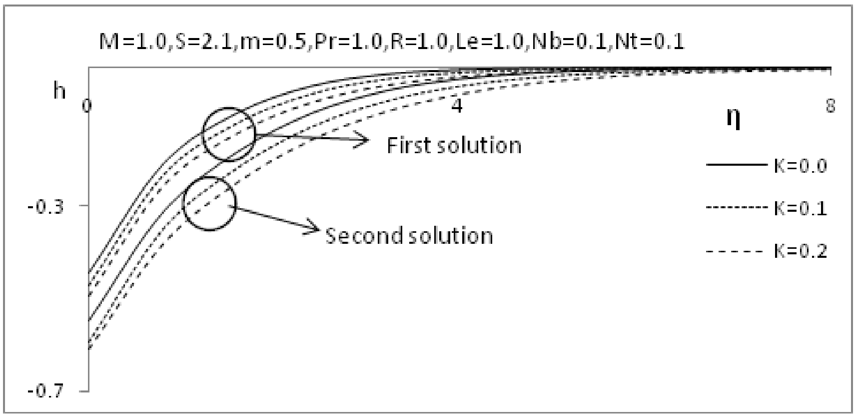

Figure 6: Dual angular velocity profiles for several values of K

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