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# T-S Fuzzy Stochastic Discrete-Time Neural Networks with Time-Varying Delays

**Research Article**

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**Abstract:** In this chapter, we intend to investigate the stability problem for discrete-time T-S fuzzy stochastic systems with time-varying delay. For a given T-S fuzzy stochastic system, our attention is focused on obtaining the sufficient conditions assuring its asymptotic stability controller for the unstable systems. By using a linear matrix inequality (LMI) approach is developed to derive several sufficient criteria ensuring the delayed neural networks to be globally. A numerical example is presented to show the effectiveness of the proposed method.

**Keywords:** Discrete-time neural network, Linear matrix inequality.


## 1. Introduction

During the past few decades, neural networks have received great attention due to their extensive applications in a variety of areas such as signal processing, pattern recognition, associative memories, parallel computation, optimization and other scientific areas. In hardware implementation of neural networks, time delays are inevitably encountered and they are a source of oscillation and instability to a great degree, which has driven many scholars to investigate the stability problem of delayed neural networks, see ([1]-[5]) references therein. The existence of time delay in a system may lead to instability or bad performance. Thus, it is important to investigate the stability and control of time-delay systems and a variety of approaches and results have been developed for the stability analysis and control synthesis of time-varying delay systems [6]. Fuzzy systems in the form of the Takagi-Sugeno model have attracted great interests in the past decade. It has shown that the T-S model method can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. And some nonlinear dynamic systems can be approximated by the overall fuzzy linear T-S models for the purposes of stability analysis and controller design [7].

It is now well known that stochastic modeling has come to play an important role in many branches of engineering applications. An area of particular interest has been the control of stochastic systems, with consequent emphasis being placed on the stabilization of the stochastic model in terms of various definitions of stochastic stability . So far, there are very few papers dealing with the reliable stabilization for general stochastic systems, not to mention the consideration of the case where time-delay, parameter nonlinear disturbance simultaneously exist in the system model, due to the complexity of such a challenging problem ([8]-[10]). This motivates us to investigate the multi objective realization problem of reliability for

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stochastic time-varying delay systems with nonlinear disturbances, that is, to generalize the results of the stochastic case. Using LKF method and stochastic analysis techniques. To the best of our knowledge, few results have been reported in the literature concerning the problem of robust stability for T-S fuzzy stochastic discrete-time neural networks with time-varying delays, which remains important and challenging one.

### 1.1. Notations

The notations that are used throughout this paper are fairly standard. The superscript $T$ stands for matrix transposition; $R^n$ denotes the n-dimensional Euclidean space; the notation $P > 0(\geq 0)$ means that P is real symmetric and positive definite (semi definite); $I_n$ and $0_{m\times n}$ represent an $n \times n$ identity matrix and an $m \times n$ zero matrix, respectively; *diag*... stands for a block-diagonal matrix; $\|.\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix; and in symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. Problem Formulation and Preliminaries

Consider a nonlinear system that can be presented by the following T-S fuzzy discrete-time neural networks time-varying delay model.

Plant Form:

Rule i: IF $\theta_1(k)$ is $M_{i1}$ and $\theta_2(k)$ is $M_{i2}$ and . . . and $\theta_p(k)$ is $M_{ip}$,

THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + A_{di} f(x(k-d(k))) + B_{1i}u(k) + [E_i x(k) + E_{di}x(k-d(k)) + B_{2i}u(k)]w(k) \\ x(l) &= \psi(l), l = -d, -d+1, ...0; \quad i\epsilon S \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state vector; $u(k) \in R^p$ is the control input; $w(k)$ is a $1-D$, zero mean Gaussian white noise sequence on a probability space with $\{w(k)\}$=0; and $E\{w^2(k)\} = 1$; $d(k)$ is the time delay, which is a positive integer and satisfies $1 \leq d \leq d(k) \leq \bar{d}$, where d and $\bar{d}$ are constant positive scalars that represent the minimum and maximum delays, respectively. Clearly, $d = \bar{d}$ means that the time delay $d(k)$ is time invariant. $\{\psi(l), l = -\bar{d}, -\bar{d}+1, ..., 0\}$ is the initial condition sequence. $M_{ij}$ is the fuzzy set; $S = \{1, 2, ..., r\}$, with r being the number of IF-THEN rules; $\theta(k)$=[$\theta_1(k)$, $\theta_2(k)$, . . . ,$\theta_p(k)$] is the premise variables vector. $A_i, A_{di}, B_{1i}, E_i, E_{di}$ , and $B_{2i}$ are known constant matrices with appropriate dimensions. The fuzzy basis functions are given by

$$h_i(\theta(k)) = \frac{\prod_{j=1}^{p} M_{ij}(\theta_j(k))}{\sum_{i=1}^{r} \prod_{j=1}^{p} M_{ij}(\theta_j(k))}, \quad i \in S \tag{2}$$

with $M_{ij}$ $\theta_j(k))$ representing the grade of membership of $\theta_j(k) \in M_{ij}$. For simplicity, we will replace $h_i(\theta(k))$ by $h_i$ in some places. By definition, the fuzzy basis functions satisfy $h_i \geq 0$ $(i\epsilon S)$ and $\sum_{i=1}^{r} h_i$=1. It is assumed that the premise variables do not depend on the input variable $u(k)$ explicitly. Then, the defuzzified output of the T-S fuzzy system (1) can be represented as

$$x(k+1) = \sum_{i=1}^{r} h_i[A_i x(k) + A_{di} f(x(k-d(k))) + B_{1i}u(k)] + \sum_{i=1}^{r} h_i[E_i x(k) + E_{di}x(k-d(k)) + B_{2i}u(k)]w(k), \tag{3}$$

We give the open-loop system of (3) in a compact form

$$x(k+1) = \bar{A}(k)x(k) + \bar{A}_d(k) f(x(k-d(k))) + [\bar{E}(k)x(k) + \bar{E}_d(k)x(k-d(k))]w(k), \tag{4}$$

where $\bar{A}(k) = \sum_{i=1}^{r} h_i A_i$ , $\bar{A}_d(k) = \sum_{i=1}^{r} h_i A_{di}$, $\bar{E}(k) = \sum_{i=1}^{r} h_i E_i$, $\bar{E}_d(k) = \sum_{i=1}^{r} h_i E_{di}$. Now, consider the following fuzzy control law.

Controller Form:

Rule i:IF $\theta_1(k)$ is $M_{i1}$ and $\theta_2(k)$ is $M_{i2}$ and . . . and $\theta_p(k)$ is $M_{ip}$,

THEN

$$u(k) = K_i x(k), i \in S$$

where $K_i$ is the gain matrix of the state-feedback controller in each rule; the state-feedback controller in (3) is given by

$$u(k) = \sum_{i=1}^{r} h_i K_i x(k). \tag{5}$$

Under control law (5), the closed-loop system is obtained as

$$x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j A_{ij} x(k) + \sum_{i=1}^{r} h_i A_{di} f(x(k-d(k))) + \left[\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j E_{ij} x(k) + \sum_{i=1}^{r} h_i E_{di} x(k-d(k))\right] w(k) \tag{6}$$

where $A_{ij} = A_i + B_{1i} K_j$ and $E_{ij} = E_i + B_{2i} K_j$. The compact form of a closed-loop system can be given as

$$x(k+1) = \hat{A}(k)x(k) + \bar{A}_d(k) f(x(k-d(k))) + [\hat{E}(k)x(k) + \bar{E}_d(k)x(k-d(k))]w(k) \tag{7}$$

where $\hat{A}(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j A_{ij}$ and $\hat{E}(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j E_{ij}$. Before presenting the main results of this paper, we first introduce the following definition for the fuzzy stochastic system in (3), which will be essential for our derivation.

**Definition 2.1.** *The T-S fuzzy stochastic system in (3) is said to be stochastically stable if under $u(k) = 0$, there exists a scalar $c > 0$ such that*

$$\sum_{k=0}^{\infty} \|x(k)\|^2 \le c\|\psi(0)\|_a^2$$

*where*

$$\varsigma(l) = \psi(l+1) - \psi(l), \quad \textit{and} \quad \|\psi(0)\|_a^2 = \max_{l=-\bar{d},\ldots,-1} \{\|\psi(0)\|^2, \|\psi(l)\|^2, \|\varsigma(l)\|^2\}.$$

Our purpose in this study is to design a state-feedback fuzzy controller in the form of (5), such that the closed-loop system in (7) is stochastically stable. To achieve that goal, first we will analyze the stability conditions of the open-loop system (4), and then find an available control law to stabilize the closed-loop system in (7).

# 3. Main Section

**Theorem 3.1.** *Given positive integers $\tau$, $m$, and $\bar{d}$, the system in (4) is stochastically stable if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_i > 0$, $S_{1i} > 0$, $S_{2i} > 0$, $R_{1i} > 0, R_{2i} > 0, M_i$, $N_i, X_i, Y_i, Z_i$, $G (i \epsilon S)$ and a scalar $\epsilon > 0$, such that for any integers $k$ and $s$, the following inequalities hold:*

$$\Psi(k)=\begin{bmatrix} \Pi_{11} & \Pi_{12} & X_3^T+Y_1 & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\ * & \Pi_{22} & -X_3^T+Y_2 & -X_4^T & \Pi_{25} & \Pi_{26} & \Pi_{27} & \Pi_{28} \\ * & * & 2Y_3 & Y_4^T & Y_5^T-Z_3 & Y_6^T-Y_3+Z_3 & \Pi_{37} & \Pi_{38} \\ * & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} & \Pi_{47} & \Pi_{48} \\ * & * & * & * & \Pi_{55} & \Pi_{56} & \Pi_{57} & \Pi_{58} \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} \\ * & * & * & * & * & * & \Pi_{77} & \Pi_{78} \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix}<0, \tag{8}$$

$$\begin{bmatrix} \bar{M}(k) & \bar{X}(k) \\ * & \epsilon\bar{R}_1(k) \end{bmatrix}\geq 0, \tag{9}$$

$$\begin{bmatrix} \bar{N}(k) & \bar{Y}(k) \\ * & \bar{R}_2(k) \end{bmatrix}\geq 0, \tag{10}$$

$$\begin{bmatrix} \bar{N}(k) & \bar{Z}(k) \\ * & \bar{R}_2(k) \end{bmatrix}\geq 0, \tag{11}$$

$$\bar{S}_1(s)-\bar{R}_1(k)<0, \tag{12}$$

$$\bar{S}_2(s)-\bar{R}_2(k)<0, \tag{13}$$

*Where* $\Pi_{11}=\bar{A}^TP_1\bar{A}+\bar{A}^TP_1\bar{E}+E^{\bar{T}}P_1\bar{A}+\bar{E}^TP_1\bar{E}-P+Q_1-Q_{11}+Q_2+(\bar{d}-\tau m)\bar{R}+2X_1$, $\Pi_{12}=Q_1-Q_{11}+X_2^T-X_1$, $\Pi_{13}=X_3^T+Y_1$, $\Pi_{14}=\bar{A}^TP_1\bar{A}_d+\bar{E}P_1\bar{A}_d+X_4^T$, $\Pi_{15}=\bar{A}^TP_1\bar{E}_d+X_5^T-Z_1$, $\Pi_{16}=\bar{E}P_1\bar{E}_d+X_6^T-Y_1+Z_1$, $\Pi_{17}=0$, $\Pi_{18}=0$, $\Pi_{22}=Q_1-Q_{11}-2X_2$, $\Pi_{23}=-X_3^T+Y_2$, $\Pi_{24}=-X_4^T$, $\Pi_{25}=-X_5^T-Z_2$, $\Pi_{26}=-X_6^T-Y_2+Z_2$, $\Pi_{27}=0$, $\Pi_{28}=0$, $\Pi_{33}=2Y_3$, $\Pi_{34}=Y_4^T$, $\Pi_{35}=Y_5^T-Z_3$, $\Pi_{36}=Y_6^T-Y_3+Z_3$, $\Pi_{37}=0$, $\Pi_{38}=0$, $\Pi_{44}=A_d^T\bar{P}_1\bar{A}_d$, $\Pi_{45}=-Z_4$, $\Pi_{46}=A_d^T\bar{P}_1\bar{E}_d+Y_4+Z_4$, $\Pi_{47}=0$, $\Pi_{48}=0$, $\Pi_{55}=-Q_{22}-2Z_5$, $\Pi_{56}=Y_5+Z_5-Z_5-Z_6^T$, $\Pi_{57}=0$, $\Pi_{58}=0$, $\Pi_{66}=-R_1-2Y_6+2Z_6+E_d^T\bar{P}_1\bar{E}_d$, $\Pi_{67}=0$, $\Pi_{68}=0$, $\Pi_{77}=\tau R_1$, $\Pi_{78}=0$, $\Pi_{88}=(\bar{d}-\tau m)R_2$.

*Proof.* Define the Lyapunov-Krasovskii function as

$$\begin{aligned}
V(k)=&V_1(k)+V_2(k)+V_3(k)+V_4(K),\\
V_1(k)=&x^T(k)P(k)x(k),\\
V_2(k)=&\sum_{i=k-\tau}^{k-1}\Upsilon^T(i)Q_1(i)\Upsilon(i)+\sum_{i=k-\bar{d}}^{k-1}x^T(i)Q_2(i)x(i),\\
V_3(k)=&\sum_{i=k-\bar{d}+1}^{-\tau m+1}\sum_{i=k+j-1}^{k-1}x^T(i)R(i)x(i),\\
V_4(k)=&\sum_{i=k-\tau}^{-1}\sum_{j=k+i}^{k-1}\delta^T(j)S_1(j)\delta(j)+\sum_{i=k-\bar{d}}^{-\tau m-1}\sum_{j=k+i}^{k-1}\delta^T(j)S_2(j)\delta(j),
\end{aligned}$$

$\Upsilon(k)=[x^T(k)x^T(k-\tau)\dots x^T(k-\tau m+\tau)]^T$, $\delta(j)=x(j+1)-x(j)$. By calculating the difference of $V(k)$ along the trajectory of system (7), we have

$$\begin{aligned}\Delta V(k) =& \sum_{i=1}^{4} \Delta V_i(k),\\ =& \sum_{i=1}^{4} V_i(k+1) - V_i(k)\\ \Delta V_1(k) =& x^T(k+1)P(k+1)x(k+1) - x^T(k)P(k)x(k), \quad (14)\\ \Delta V_2(k) =& \gamma^T(k)Q_1\gamma(k) + x^T(k)Q_2x(k) - \gamma^T(k-\tau)Q_1\gamma(k-\tau) - x^T(k-d)Q_2x(k-d), \quad (15)\\ \Delta V_3(k) =& (d-\tau m)x^T(k)Rx(k) - \sum_{i=k-d(k)}^{k-\tau m} x^T(i)Rx(i),\\ =& (d-\tau m)x^T(k)Rx(k) - x^T(k-d(k))Rx(k-d(k)), \quad (16)\end{aligned}$$

Considering (12) and (13), we have $S_1(s) > R_1(k)$ and $S_2(s) > R_2(k)$, for all $s$, $k \epsilon S$; then

$$\begin{aligned}\Delta V_4(k) =& \tau\delta^T(k)R_1(k)\delta(k) + (d-\tau m)\delta^T(k)R_2(k)\delta(k) - \sum_{j=k-\tau}^{k-1} \delta^T(j)R_1(k)\delta(j)\\ &- \sum_{j=k-d(k)}^{k-\tau m-1} \delta^T(j)R_2(k)\delta(j) - \sum_{j=k-d}^{k-d(k)-1} \delta^T(j)R_2(k)\delta(j), \quad (17)\end{aligned}$$

Summing up (14)-(17), we have

$$\begin{aligned}\Delta V(k) \leq& - \sum_{j=k-\tau}^{k-1} \delta^T(k)R_1(k)\delta(j) + \sum_{j=k-d(k)}^{k-\tau m-1} \delta^T(j)R_2(k)\delta(j) + \sum_{j=k-d}^{k-d(k)-1} \delta^T(j)R_2(k)\delta(j),\\ =& \sum_{j=k-\tau}^{k-1} \xi^T(k,j) \begin{bmatrix} 0 & 0 \\ 0 & R_1(k) \end{bmatrix} \xi(k,j) + \sum_{j=k-d(k)}^{k-\tau m-1} \xi^T(k,j) \begin{bmatrix} 0 & 0 \\ 0 & R_2(k) \end{bmatrix} \xi(k,j)\\ &+ \sum_{j=k-d(k)}^{k-d(k)-1} \xi^T(k,j) \begin{bmatrix} 0 & 0 \\ 0 & R_2(k) \end{bmatrix} \xi(k,j) + \eta^T(k)\Omega(k)\eta(k).\end{aligned}$$

Next, we will introduce several slack matrices to further reduce the conservatism. According to the definition of $\delta(j)$, for any matrices $\hat{X}(k), \hat{Y}(k)$, and $\hat{Z}(k)$, we have

$$0 = 2\eta^T(k)\hat{X}(k)\Big[x(k) - x(k-\tau) - \sum_{j=k-\tau}^{k-1} \delta(j)\Big], \quad (18)$$

$$0 = 2\eta^T(k)\hat{Y}(k)\Big[x(k-\tau m) - x(k-d(k)) - \sum_{j=k-d(k)}^{k-\tau m-1} \delta(j)\Big], \quad (19)$$

$$0 = 2\eta^T(k)\hat{Z}(k)\Big[x(k-d(k)) - x(k-d) - \sum_{j=k-d}^{k-d(k)-1} \delta(j)\Big], \quad (20)$$

Considering former definitions, we have

$$\begin{aligned}\eta^T(k)\Big(\hat{\Psi}(k) + \hat{\Psi}^T(k)\Big)\eta(k) =& \sum_{j=k-\tau}^{k-1} \xi^T(k,j) \begin{bmatrix} 0 & \hat{X}(k) \\ \hat{X}^T(k) & 0 \end{bmatrix} \xi(k,j) + \sum_{j=k-d(k)}^{k-\tau m-1} \xi^T(k,j) \begin{bmatrix} 0 & \hat{Y}(k) \\ \hat{Y}^T(k) & 0 \end{bmatrix} \xi(k,j)\\ &+ \sum_{j=k-d(k)}^{k-d(k)-1} \xi^T(k,j) \begin{bmatrix} 0 & \hat{Z}(k) \\ \hat{Z}^T(k) & 0 \end{bmatrix} \xi(k,j).\end{aligned}$$

Furthermore, for any matrices $M(k)$ and $N(k)$, we have

$$\begin{aligned}0 =&\tau\eta^T(k)\hat{M}(k)\eta(k) - \sum_{j=k-\tau}^{k-1}\eta^T(k)\hat{M}(k)\eta(k)\\ 0 =&(d-\tau m)\eta^T(k)\hat{N}(k)\eta(k) - \sum_{j=k-d(k)}^{k-\tau m-1}\eta^T(k)\hat{N}(k)\eta(k) - \sum_{j=k-d}^{k-d(k)-1}\eta^T(k)\hat{N}(k)\eta(k)\end{aligned}$$

$$\tau\eta^T(k)\hat{M}(k)\eta(k) =\eta^T(k)\Omega(k)\eta(k) + \sum_{j=k-\tau}^{k-1}\xi^T(k,j)\begin{bmatrix}\hat{M}(k) & 0\\ 0 & 0\end{bmatrix}\xi(k,j) \tag{21}$$

$$(d-\tau m)\eta^T(k)\hat{N}(k)\eta(k) = \sum_{j=k-d(k)}^{k-\tau m-1}\xi^T(k,j)\begin{bmatrix}\hat{N}(k) & 0\\ 0 & 0\end{bmatrix}\xi(k,j) + \sum_{j=k-d(k)}^{k-d(k)-1}\xi^T(k,j)\begin{bmatrix}\hat{N}(k) & 0\\ 0 & 0\end{bmatrix}\xi(k,j) \tag{22}$$

summing up (14)-(22)

$$\begin{aligned}\Delta V(k) \le&\eta^T(k)\Omega(k)\eta(k) + \xi^T(k,j)\begin{bmatrix}\hat{M}(k) & \hat{X}(k)\\ * & R_1(k)\end{bmatrix}\xi(k,j)\\ &+\sum_{j=k-d(k)}^{k-\tau m-1}\xi^T(k,j)\begin{bmatrix}\hat{N}(k) & \hat{Y}(k)\\ * & R_2(k)\end{bmatrix}\xi(k,j) + \sum_{j=k-d}^{k-d(k)-1}\xi^T(k,j)\begin{bmatrix}\hat{N}(k) & \hat{Z}(k)\\ * & R_2(k)\end{bmatrix}\xi(k,j).\end{aligned} \tag{23}$$

for all $\eta(k)\neq 0$ we have $\Delta V(k) \le \eta^T(k)\Omega\eta(k) < -\rho\|x(k)\|^2$ that is

$$V(N+1) - V(0) < -\rho\|x(k)\|^2 \tag{24}$$

Therefore,for any integer $N > 1$, summing up both sides of (27) from $k = 0$ to $k = N$ will result in

$$V(N+1) - V(0) < -\rho\sum_{k=0}^{N}\|x(k)\|^2$$

which means that

$$\sum_{k=0}^{N}\|x(k)\|^2 <\frac{1}{\rho}V(0) - V(N+1) \le V(0). \tag{25}$$

Reconsidering the former defined L-K functional, we have

$$\begin{aligned}V_1(0) =&x^T(0)P(0)x(0)\\ \le&\sum_{i=1}^{r}h_i(\theta(0))\lambda_{max}(P_i)x^T(0)G^{-T}G^{-1}x(0)\\ \le&\max_{i\epsilon S}\left\{\frac{\lambda_{max}(P_i)}{\lambda_{min}(G^TG)}\|\Psi(0)\|^2\right.\end{aligned} \tag{26}$$

$$\begin{aligned}V_2(0) =& \sum_{i=-r}^{-1}\Upsilon^T(i)Q_1(i)\Upsilon(i) + \sum_{i=-d}^{-1}x^T(i)Q_2(i)x(i)\\ \le&\max_{i\epsilon S}\lambda_{max}(Q_{1i})\sum_{j=-\tau}^{-1}\Upsilon^T(j)T_4^{-T}T_4^{-1}\Upsilon(j)+\\ &\max_{i\epsilon S}\lambda_{max}(Q_{2i})\sum_{j=-d}^{-1}x^T(j)G^{-T}G^{-1}x(j)\\ \le&\Big(max_{i\epsilon S}\Big\{\frac{\tau m\lambda_{max}(Q_{1i})}{\varepsilon\lambda_{min}(G^TG)}\Big\}+\\ &\max_{i\epsilon S}\left\{\frac{d\lambda_{max}(Q_{2i})}{\varepsilon\lambda_{min}(G^TG)}\right\}\Big) \times \max_{-d\le l\le 1}\|\Psi(l)\|^2\end{aligned} \tag{27}$$

$$V_3(0) = \sum_{j=-d+1}^{-\tau m+1} \sum_{i=j-1}^{-1} x^T(i)R(i)x(i)$$
$$\leq \max_{i\epsilon S}\left\{\frac{(d+\tau m)(d-\tau m+1)\lambda_{max}(R_i)}{2\varepsilon\lambda_{min}(G^TG)}\right\} \times \max_{-d\leq l\leq 1} \|\Psi(l)\|^2 \tag{28}$$
$$V_4(0) = \sum_{i=-\tau}^{-1} \sum_{j=i}^{-1} \delta^T(j)S_1(j)\delta(j) + \sum_{i=-d}^{-\tau m-1} \sum_{j=i}^{-1} \delta^T(j)S_2(j)\delta(j)$$
$$\leq \sum_{i=-\tau}^{-1} \sum_{j=i}^{-1} \frac{\delta^T(j)\delta(j)}{\lambda_{min}(S_1(j))} + \sum_{i=-d}^{-\tau m-1} \sum_{j=i}^{-1} \frac{\delta^T(j)\delta(j)}{\lambda_{min}(S_2(j))}$$
$$\leq \max_{i\epsilon S}\left\{\frac{(\tau m+d+1)(d-\tau m)}{2\lambda_{min}(S_{2i})}\right\} \max\|\varsigma(l)\|^2 + \max\left\{\frac{\tau^2+1}{2\lambda_{min(S_{1i})}}\right\} \max\|\varsigma(l)\|^2. \tag{29}$$

summing up (26)-(29), we have

$$V(0) = V_1(0) + V_2(0) + V_3(0) + V_4(0) \leq k\|\psi(0)\|_a^2 \tag{30}$$

where

$$k = \max_{i\epsilon S}\left\{\frac{(\tau m+d+1)(d-\tau m)}{2\lambda_{min}(S_{2i})}\right\} + \max_{i\epsilon S}\left\{\frac{d\lambda_{max}(Q_{2i})}{\varepsilon\lambda_{min}(G^TG)}\right\} + max\left\{\frac{\lambda_{max}(P_i)}{\lambda_{min}(G^TG)}\right\} + max\left\{\frac{\tau m\lambda_{max}(Q_{1i})}{\varepsilon\lambda_{min}(G^TG)}\right\}$$
$$+ \max\left\{\frac{(d+\tau m)(d-\tau m+1)\lambda_{max}(R_i)}{2\varepsilon\lambda_{min}(G^TG)}\right\} + \max\left\{\frac{\tau^2+1}{2\lambda_{min(S_{1i})}}\right\}. \tag{31}$$

From (30) and (31), we have

$$\sum_{k=0}^{N} \|x(k)\|^2 < \frac{k}{\rho}|\psi(0)\|_a^2 = c\|\psi(0)\|_a^2, \tag{32}$$

where $c = \frac{k}{\rho}$.

$$\lim_{N\to\infty} \sum_{k=0}^{N} \|x(k)\|^2 \leq c\psi(0)\|_a^2. \tag{33}$$

□

**Theorem 3.2.** *Given positive integers $\tau$, $m$ ,and $d$, the system in (7) is stochastically stable if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $R_i > 0$, $S_{1i} > 0$, $S_{2i} > 0$, $R_{1i} > 0$, $R_{2i} > 0$, $M_i$, $N_i$, $X_i$, $Y_i$, $Z_i$, $G$, $H_i$, $(i\epsilon S)$, and a scalar$\varepsilon > 0$, such that for any $o, s, t, l, i, j\epsilon S$, the following inequalities hold:*

$$\Psi(k) = \begin{bmatrix} \Pi_{11} & \Pi_{12} & X_3^T + Y_1 & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\ * & \Pi_{22} & -X_3^T + Y_2 & -X_4^T & \Pi_{25} & \Pi_{26} & \Pi_{27} & \Pi_{28} \\ * & * & 2Y_3 & Y_4^T & Y_5^T - Z_3 & Y_6^T - Y_3 + Z_3 & \Pi_{37} & \Pi_{38} \\ * & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} & \Pi_{47} & \Pi_{48} \\ * & * & * & * & \Pi_{55} & \Pi_{56} & \Pi_{57} & \Pi_{58} \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} \\ * & * & * & * & * & * & \Pi_{77} & \Pi_{78} \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix} \tag{34}$$

*and the fuzzy controller is given as $u(k)$=$K(k)x(k) = H(k)G^{-1}x(k)$ with $K(k) = \sum_{i=1}^{r} h_iK_i$, $H(k) = \sum_{i=1}^{r} h_iH_i$.*

*Proof.* To stabilize the closed-loop system (7), we just need to replace $\bar{A}(k)$ and $\bar{A}_d(k)$ in (8)by $\hat{A}(k)$ and $\hat{A}_d(k)$. we have the following expressions:

$$\hat{A}(k) = \bar{A}(k) + \bar{B}_1(k)K(k) \tag{35}$$

$$\hat{E}(k) = \bar{E}(k) + \bar{B}_2(k)K(k) \tag{36}$$

Accordingly, inequality (8) will be replaced by

$$\hat{A}(k) = \bar{A}(k) + \bar{B}_1(k)K(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i + B_1 K)$$
$$\hat{E}(k) = \bar{E}(k) + \bar{B}_2(k)K(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (E_i + B_2 K)$$

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & X_3^T + Y_1 & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\ * & \Pi_{22} & -X_3^T + Y_2 & -X_4^T & \Pi_{25} & \Pi_{26} & \Pi_{27} & \Pi_{28} \\ * & * & 2Y_3 & Y_4^T & Y_5^T - Z_3 & Y_6^T - Y_3 + Z_3 & \Pi_{37} & \Pi_{38} \\ * & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} & \Pi_{47} & \Pi_{48} \\ * & * & * & * & \Pi_{55} & \Pi_{56} & \Pi_{57} & \Pi_{58} \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} \\ * & * & * & * & * & * & \Pi_{77} & \Pi_{78} \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix} < 0 \tag{37}$$

Then it follows from the analysis of Theorems 3.1 and 3.2 that the closed-loop system in (7) is stochastically stable. The proof is completed. □

# 4. Numerical Examples

Consider the following discrete-time neural networks T-S fuzzy stochastic system with time varying delay:

$$x(k+1) = A_i x(k) + A_{di} f(x(k-d(k))) + B_{1i}u(k) + [E_i x(k) + E_{di} x(k-d(k)) + B_{2i}u(k)]w(k) \tag{38}$$

with the following parameters:

$$A = \begin{bmatrix} -0.1 & 0 \\ 0.01 & -0.3 \end{bmatrix},\ A_d = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.2 \end{bmatrix},\ E = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ E_d = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} -0.1 & 0 \\ 0.01 & -0.3 \end{bmatrix},\ \text{m=3},\ d_1 = 2,\ d_2 = 5.$$

The purpose is the design of a state feedback controller such that the resulting closed-loop system is globally robustly stable with disturbance attenuation level for estimation of the deviation of the perturbed trajectory from the equilibrium point. The feasible solutions for LMI (38) can be found as follows

$$P = \begin{bmatrix} 0.7531 & -0.0004 \\ -0.0004 & 0.7457 \end{bmatrix}, Q_1 = \begin{bmatrix} -0.8030 & 0.0029 \\ 0.0029 & 0.7457 \end{bmatrix}, Q_2 = \begin{bmatrix} -0.0978 & 0.0013 \\ 0.0013 & -0.0786 \end{bmatrix},$$

$$R = \begin{bmatrix} 2.525 & -0.0344 \\ -0.0344 & 2.0448 \end{bmatrix}, R_1 = \begin{bmatrix} 0.0191 & 0.0005 \\ 0.0005 & -0.1972 \end{bmatrix}, R_2 = \begin{bmatrix} 0.7208 & 0.0004 \\ 0.0004 & 0.5881 \end{bmatrix}, X_1 = \begin{bmatrix} -0.0301 & 0.0013 \\ 0.0013 & -0.0246 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.8040 & -0.0004 \\ -0.0004 & -0.1389 \end{bmatrix}, Z_1 = \begin{bmatrix} -0.0073 & 0.0004 \\ 0.0004 & -0.0025 \end{bmatrix}, X_2 = \begin{bmatrix} 0.3743 & 0.0011 \\ 0.0011 & 0.3295 \end{bmatrix}, Y_2 = \begin{bmatrix} -0.6769 & -0.0023 \\ -0.0023 & -0.0877 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 0.0819 & 0.0002 \\ 0.0002 & 0.0077 \end{bmatrix}, X_3 = \begin{bmatrix} -0.5533 & 0.0012 \\ 0.0012 & -0.0878 \end{bmatrix}, Y_3 = \begin{bmatrix} -2.4886 & 0.0000 \\ 0.0000 & -0.6741 \end{bmatrix}, Z_3 = \begin{bmatrix} -0.3176 & 0.0004 \\ 0.0004 & -0.3615 \end{bmatrix}.$$

## 5. Conclusion

The delay-dependent stability analysis problem for discrete-time T-S fuzzy systems with state delay have been studied by using the fuzzy LKF approach. First, the delay-dependent stability condition is presented in terms of LMIs for closed-loop fuzzy systems. Then, the LMI-based delay-dependent stabilization conditions is given for the state feedback and observer-based control cases, the addressed neural networks have been established in terms of linear matrix inequalities, which can be checked numerically using the effective LMIs. The usefulness of our result is illustrated by a numerical example.

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