International Journal of Mathematics And its Applications
Volume 5, Issue 4–B (2017), 247–257.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Direct Product of General Doubt Intuitionistic Fuzzy Ideals of BCK/BCI-algebras with Respect to Triangular Binorm

Research Article

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Abstract: In this paper, we introduced the concept of $(\in, \in \vee q_k)$- doubt intuitionistic fuzzy subalgebra and $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideals in BCK-algebra with respect to triangular binorm by using the combined notion of not quasi coincidence $(\overline{q})$ of a fuzzy point to a fuzzy set and the notion of triangular binorm. We define direct product of $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy sets and direct product of $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebras/ideals of BCK/BCI-algebras and investigate some related properties.

MSC: 06F35, 03E72, 03G25.

Keywords: BCK-algebra, Doubt fuzzy ideal, $(\in, \in \vee q_k)$-doubt fuzzy subalgebra,$(\in, \in \vee q_k)$-doubt fuzzy ideal, $(\in, \in \vee q_k)$- doubt intuitionistic fuzzy subalgebra,$(\in, \in \vee q_k)$- doubt intuitionistic fuzzy ideal, Direct product.

1. Introduction

The name triangular norm, or simply t-norm originated from the study of generalized triangle inequalities for statistical metric spaces, hence the name triangular norm or simply t-norm. The name first appeared in a paper entitled statistical metrics [19] that was published on 27^{th} october in 1942. The real starting point of t-norms came in 1960, when Berthold Schweizer and Abe Sklar, (two students of Menger) published their paper, statistical metric spaces [25] After a very short time, Schweizer and Sklar [27] introduced several basic notions and properties. Namely, they introduced triangular conorms (briefly t-conorms) as a dual concept of t-norms. For a given t-norm T, its dual t-conorm S is defined by $S(a, b) = 1-T(1-a, 1-b)$. They pointed out that the boundary condition is the only difference between the t-norm and t-conorm axioms. In recent years, a systematic study concerning the properties and related matters of t-norms have been made by Klement et al. [15, 16].

The concept of fuzzy sets was first proposed by Zadeh [32] in 1965. Rosenfeld [24] was the first who consider the case of a groupoid in terms of fuzzy sets. Since then these ideas have been applied to other algebraic structures such as group, semigroup, ring, field, topology, vector spaces etc. Imai and Iseki [12] introduced BCK-algebra as a generalization of notion

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of the concept of set theoretic difference and propositional calculus and in the same year Iseki [14] introduced the notion of BCI-algebra which is a generalization of BCK-algebra. Xi Ougen [29] applied the concept of fuzzy set to BCK-algebra and discussed some properties. Since then B-algebras was introduced in [23] by Neggers and Kim and which is related to several classes of algebras such as BCI/BCK-algebras. Huang [11] fuzzified BCI-algebras in little different ways. Jun et al. [10, 31] renamed Huang's definition as doubt (anti) fuzzy ideals in BCK/BCI-algebras. Biswas [8] introduced the concept of anti fuzzy subgroup. The concept of doubt fuzzy BF-algebras was introduced by Saeid in [28] and the concept of doubt fuzzy ideal of BF-algebras was introduced by Barbhuiya [4].

The concept of fuzzy point introduced by Ming and Ming in [20] and also they introduced the idea of relation "belongs to" and "quasi coincident with" between fuzzy point and fuzzy set. Murali [21] proposed a definition of a fuzzy point belonging to fuzzy subset under natural equivalence on fuzzy subset. Bhakat and Das [6, 7] used the relation of "belongs to" and "quasi-coincident" between fuzzy point and fuzzy set to introduced the concept of $(\in, \in \vee q)$-fuzzy subgroup, $(\in, \in \vee q)$-fuzzy subring and $(\in \vee q)$-level subset. some properties of $(\in, \in \vee q)$-fuzzy ideals of d-algebra was discussed by Barbhuiya and Choudhury [3]. In [5] Barbhuiya introduced $(\in, \in \vee q)$-intuitionistic fuzzy ideals of BCK/BCI-algebras. In fact, the $(\in, \in \vee q)$-fuzzy subgroup is an important generalization of Rosenfeld's fuzzy subgroup. Further in [18] Larimi generalized $(\in, \in \vee q)$-fuzzy ideals to $(\in, \in \vee q_k)$-fuzzy ideals. Reza Ameri et al [2] introduced the notion of $(\overline{\in}, \overline{\in \wedge q_k})$-fuzzy subalgebras in BCK/BCI-algebras. In [9] Dutta et al. combined the notion of not quasi coincidence $\overline{q}$ of a fuzzy point to a fuzzy set and the notion doubt(anti) fuzzy ideals introduced the concept of generalized doubt fuzzy subalgebra and generalized doubt fuzzy ideal in BG-algebra. In this paper, we introduced the concept of $(\in, \in \vee q_k)$- doubt intuitionistic fuzzy subalgebra and $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideals in BCK-algebra with respect to triangular binorm by using the combined notion of not quasi coincidence $(\overline{q})$ of a fuzzy point to a fuzzy set and the notion of triangular binorm. We define direct product of $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy sets and direct product of $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebras/ideals of BCK/BCI-algebras and investigate some related properties.

1.1. Preliminaries

Definition 1.1 ([29–31]). *An algebra $(X, *, 0)$ of type (2, 0) is called a BCK-algebra if it satisfies the following axioms:*

(1). $((x * y) * (x * z)) * (z * y) = 0$;

(2). $(x * (x * y)) * y = 0$;

(3). $x * x = 0$;

(4). $0 * x = 0$;

(5). $x * y = 0$ *and* $y * x = 0 \Rightarrow x = y$ *for all* $x, y, z \in X$.

We can define a partial ordering " $\leq$ *" on* X *by* $x \leq y$ *iff* $x * y = 0$.

Definition 1.2 ([29–31]). *A BCK-algebra* X *is said to be commutative if it satisfies the identity* $x \wedge y = y \wedge x$ *where* $x \wedge y = y * (y * x)$ $\forall x, y \in X$. *In a commutative BCK-algebra, it is known that* $x \wedge y$ *is the greatest lower bound of* x *and* y. *In a BCK-algebra* X, *the following hold:*

(1). $x * 0 = x$;

(2). $(x * y) * z = (x * z) * y$;

(3). $x * y \leq x$;

(4). $(x * y) * z \leq (x * z) * (y * z)$;

(5). $x \leq y$ *implies* $x * z \leq y * z$ *and* $z * y \leq z * x$.

A non-empty subset S *of a BG-algebra* X *is called a subalgebra of* X *if* $x * y \in S$ *for all* $x, y \in S$. *A nonempty subset* I *of a BCK-algebra* X *is called an ideal of* X *if (i)* $0 \in I$ *and (ii)* $x * y \in I$ *and* $y \in I \Rightarrow x \in I$ *for all* $x, y \in X$.

Definition 1.3 ([6, 20]). *A fuzzy set* μ *of the form*

$$\mu(y) = \begin{cases} t & \text{if } y = x,\ t \in (0,1] \\ 0 & \text{if } y \neq x \end{cases}$$

is called a fuzzy point with support x *and value* t *and it is denoted by* x_t *[6, 20]. Let* μ *be a fuzzy set in* X *and* x_t *be a fuzzy point then*

(1). If $\mu(x) \geq t$ *then we say* x_t *belongs to* μ *and write* $x_t \in \mu$

(2). If $\mu(x) + t > 1$ *then we say* x_t *quasi-coincident with* μ *and write* $x_t q \mu$

(3). If $x_t \in \vee q \mu \Leftrightarrow x_t \in \mu$ *or* $x_t q \mu$

(4). If $x_t \in \wedge q \mu \Leftrightarrow x_t \in \mu$ *and* $x_t q \mu$

The symbol $x_t \overline{\alpha} \mu$ *means* $x_t \alpha \mu$ *does not hold and* $\overline{\in \wedge q}$ *means* $\overline{\in} \vee \overline{q}$. *For a fuzzy point* x_t. *and a fuzzy set* μ *in set X, Pu and Liu [20] gave meaning to the symbol* $x_t \alpha \mu$ *where* $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$.

Definition 1.4 ([2, 18]). *Let* μ *be a fuzzy set in* X *and* x_t *be a fuzzy point then*

(1). If $\mu(x) < t$ *then we say* x_t *does not belongs to* μ *and write* $x_t \overline{\in} \mu$.

(2). If $\mu(x) + t \leq 1$ *then we say* x_t *not quasi-coincident with* μ *and write* $x_t \overline{q} \mu$.

(3). If $x_t \overline{\in \vee q} \mu \Leftrightarrow x_t \overline{\in} \mu$ *and* $x_t \overline{q} \mu$.

(4). If $x_t \overline{\in \wedge q} \mu \Leftrightarrow x_t \overline{\in} \mu$ *or* $x_t \overline{q} \mu$.

Definition 1.5 ([2, 18]). *Let* μ *be a fuzzy set in* X *and* x_t *be a fuzzy point then*

(1). If $\mu(x) + t + k > 1$ *then we say* x_t *is k quasi-coincident with* μ *and write* $x_t q_k \mu$ *where* $k \in [01)$.

(2). If $x_t \in \vee q_k \mu \Leftrightarrow x_t \in \mu$ *or* $x_t q_k \mu$.

(3). If $x_t \in \wedge q_k \mu \Leftrightarrow x_t \in \mu$ *and* $x_t q_k \mu$.

Definition 1.6 ([2, 18]). *Let* μ *be a fuzzy set in* X *and* x_t *be a fuzzy point then*

(1). If $\mu(x) + t + k \leq 1$ *then we say* x_t *is not k quasi-coincident with* μ *and write* $x_t \overline{q_k} \mu$ *where* $k \in [01)$.

(2). If $x_t \overline{\in \vee q_k} \mu \Leftrightarrow x_t \overline{\in} \mu$ *and* $x_t \overline{q}_k \mu$.

(3). If $x_t \overline{\in \wedge q_k} \mu \Leftrightarrow x_t \overline{\in} \mu$ *or* $x_t \overline{q}_k \mu$.

Definition 1.7 ([30]). *A fuzzy set* μ *of a BG-algebra Xis said to be* (α, β)*-fuzzy ideal of* X *if*

(1). $x_t \alpha \mu \Rightarrow 0_t \beta \mu$ *for all* $x \in X$.

(2). $(x*y)_t, y_s \alpha \mu \Rightarrow x_{m(t,s)} \beta \mu$ *for all* $x, y \in X$ *Where* $\alpha \neq \in \wedge q, m\{t,s\} = min\{t,s\}$ *and* $t, s \in (0,1]$.

Definition 1.8 ([9]). *A fuzzy subset* μ *of a BG-algebra* X *is an* $(\in, \in \vee q_k)$*-doubt fuzzy subalgebra of* X *if*

$$\mu(x*y) \leq \max\left\{\mu(x), \mu(y), \frac{1-k}{2}\right\} \quad \text{for all} \quad x, y \in X.$$

Remark 1.9. *A fuzzy subset* μ *of a BG-algebra* X *is an* $(\in, \in \vee q)$*-doubt fuzzy subalgebra of* X *iff*

$$\mu(x*y) \leq M\{\mu(x), \mu(y), 0.5\}$$

Definition 1.10 ([9]). *A fuzzy subset* μ *of a BG-algebra* X *is an* $(\in, \in \vee q_k)$*-doubt fuzzy ideal of* X *if*

(1). $\mu(0) \leq max\{\mu(x), \frac{1-k}{2}\}$ *for all* $x \in X$.

(2). $\mu(x) \leq max\{\mu(x*y), \mu(y), \frac{1-k}{2}\}$ *for all* $x, y \in X$.

Remark 1.11. *A fuzzy subset* μ *of a BG-algebra* X *is an* $(\in, \in \vee q)$*-doubt fuzzy ideal of* X *iff*

$$\mu(0) \leq M\{\mu(x), 0.5\}$$
$$\mu(x) \leq M\{\mu(x*y), \mu(y), 0.5\}$$

Definition 1.12 ([1]). *An intuitionistic fuzzy set (IFS)* A *in a non-empty set* X *is an object of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ *where* $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$ *with the condition* $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. *The numbers* $\mu_A(x)$ *and* $\nu_A(x)$ *denote respectively the degree of membership and the degree of non membership of the element* x *in the set* A. *For the sake of simplicity, we shall use the symbol* $A = (\mu_A, \nu_A)$ *for the intuitionistic fuzzy set* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$.

Definition 1.13. *An intuitionistic fuzzy set* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ *of a BCK-algebra* X

$$x_{\alpha,\beta}(y) = \begin{cases} (\alpha, \beta) & \text{if } y = x, \\ (0,1) & \text{if } y \neq x \end{cases}$$

is said to be an intuitionistic fuzzy point with support x *and value* (α, β) *and is denoted by* $x_{(\alpha,\beta)}$. *A fuzzy point* $x_{(\alpha,\beta)}$ *is said to intuitionistic belongs to (resp., intuitionistic quasi-coincident) with intuitionistic fuzzy set* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ *written* $x_(\alpha, \beta) \in A$ *resp:* $x_{(\alpha,\beta)} qA$ *if* $\mu_A(x) \geq \alpha$ *and* $\nu_A(x) \leq \beta$ *(resp.*$\mu_A(x) + \alpha > 1$ *and* $\nu_A(x) + \beta < 1$*). By the symbol* $x_{(\alpha,\beta)} q_k A$ *we mean* $\mu_A(x) + \alpha + k > 1$ *and* $\nu_A(x) + \beta + k < 1$*, where* $k \in (0,1)$.

We use the symbol $x_t \in \mu_A$ implies $\mu_A(x) \geq t$ and $x_t \overline{\in} \nu_A$ implies $\nu_A(x) \leq t$ in the whole paper.

Definition 1.14 ([1, 5]). *If* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ *and* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in X\}$ *be any two IFS of a set* X *then:* $A \subseteq B$ *iff for all* $x \in X, \mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$*;* $A = B$ *iff for all* $x \in X, \mu_A(x) = \mu_B(x)$ *and* $\nu_A(x) = \nu_B(x)$*;* $A \cap B = \{\langle x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x)\rangle | x \in X\}$*, where* $(\mu_A \cap \mu_B)(x) = min\{\mu_A(x), \mu_B(x)\}$ *and* $(\nu_A \cup \nu_B)(x) = max\{\nu_A(x), \nu_B(x)\}$*;* $A \cup B = \{\langle x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x)\rangle | x \in X\}$*, where* $(\mu_A \cup \mu_B)(x) = max\{\mu_A(x), \mu_B(x)\}$ *and* $(\nu_A \cap \nu_B)(x) = min\{\nu_A(x), \nu_B(x)\}$.

An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ *of a BCK-algebra* X *is said to be an intuitionistic fuzzy ideal of* X *if*

(1). $\mu_A(0) \geq \mu_A(x)$

(2). $\nu_A(0) \leq \nu_A(x)$

(3). $\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

(4). $\nu_A(x) \leq max\{\nu_A(x * y), \nu_A(y)\}\ \ \forall\ \ x, y \in X.$

Definition 1.15. *A triangular norm(t-norm) is a function* $T : [0\ 1] \times [0\ 1] \to [0\ 1]$ *satisfying the following conditions:*

$(T1)$ $T(x, 1) = x, T(0, x) = 0$ *; (boundary conditions)*

$(T2)$ $T(x, y) = T(y, x)$ *; (commutativity)*

$(T3)$ $T(x, T(y, z)) = T(T(x, y), z)$ *; (associativity)*

$(T4)$ $T(x, y) \leq T(z, w)$ *;if* $x \leq z, y \leq w$ *for all* $x, y, z \in [0\ 1]$ *(monotonicity)*

Every t-norm T *satisfies* $T(x, y) \leq min(x, y)\quad \forall\, x, y \in [0,\ 1]$.

Example 1.16. *The four basic t-norms are:*

(1). The minimum is given by $T_M(x, y) = min(x, y)$.

(2). The product is given by $T_P(x, y) = xy$.

(3). The Lukasiewicz is given by $T_L(x, y) = max(x + y - 1, 0)$.

(4). The Weakest t-norm (drastic product) is given by

$$T_D(x, y) = \begin{cases} \min(x, y), & \textit{if } \max(x, y) = 1; \\ 0, & \textit{otherwise.} \end{cases}$$

Definition 1.17. *A s-norm* S *is a function* $S : [0\ 1] \times [0\ 1] \to [0\ 1]$ *satisfying the following conditions:*

$(S1)$ $S(x, 1) = 1, S(0, x) = x$ *; (boundary conditions)*

$(S2)$ $S(x, y) = S(y, x)$ *; (commutativity)*

$(S3)$ $S(x, S(y, z)) = S(S(x, y), z)$ *; (associativity)*

$(S4)$ $S(x, y) \leq S(z, w)$ *;if* $x \leq z, y \leq w$ *for all* $x, y, z \in [0\ 1]$ *(monotonicity)*

Every s-norm S *satisfies* $S(x, y) \geq max(x, y)\quad \forall\, x, y \in [0,\ 1]$.

Example 1.18. *The four basic t-conorm are:*

(1). Maximum given by $S_M(x, y) = max(x, y)$.

(2). Probabilistic sum given by $S_P(x, y) = x + y - xy$.

(3). The Lukasiewicz is given by $S_L(x, y) = min(x + y, 1)$.

(4). Strongest t-conorm given by

$$S_D(x, y) = \begin{cases} \max(x, y), & \textit{if } \max(x, y) = 1; \\ 0, & \textit{otherwise.} \end{cases}$$

Definition 1.19. *If for two t-norms* T_1 *and* T_2 *the inequality* $T_1(x, y) \leq T_2(x, y)$ *holds for all* $(x, y) \in [0\ 1] \times [0\ 1]$ *then* T_1 *is said to be weaker than* T_2*, and we write in this case* $T_1 \leq T_2$*. We write* $T_1 < T_2$*,whenever* $T_1 \leq T_2$ *and* $T_1 \neq T_2$.

Remark 1.20. *It is not hard to see that T_D is the weakest t-norm and T_M is the strongest t-norm, that is, for all t-norm T*

$$T_D \leq T \leq T_M$$

We get the following ordering of the four basic t-norms:

$$T_D < T_L < T_P < T_M$$

Lemma 1.21. *Let T be a t-norm. Then $T(T(x,\ y)\ T(z,\ t)) = T(T(x,\ z)\ T(y,\ t))$ for all $x,\ y,\ z$ and $t \in [0,\ 1]$.*

Definition 1.22. *Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two doubt intuitionistic fuzzy sets of X_1 and X_2, respectively. Then the direct product of DIFSs A and B with respect to triangular binorm (i.e., (T, S)-normed) is denoted by $A \times B = (\mu_{A\times B}, \nu_{A\times B})$ where $\mu_{A\times B} : X_1 \times X_2 \to [0,1]$ defined by $\mu_{A\times B}(x,y) = S\{\mu_A(x), \mu_B(y)\}$ and $\nu_{A\times B} : X_1 \times X_2 \to [0,1]$ defined by $\nu_{A\times B}(x,y) = T\{\nu_A(x), \nu_B(y)\}$ for all $(x,y) \in X_1 \times X_2$.*

Definition 1.23 ([26])**.** *An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of a BCK-algebra X is said to be a doubt intuitionistic fuzzy subalgebra with respect to triangular binorm of X if*

(1). $\mu_A(x * y) \leq S\{\mu_A(x), \mu_A(y)\}$

(2). $\nu_A(x * y) \geq T\{\nu_A(x), \nu_A(y)\}\ \forall\ x, y \in X.$

Definition 1.24 ([16, 26])**.** *An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of a BCK-algebra X is said to be a doubt intuitionistic fuzzy ideal with respect to triangular binorm of X if*

(1). $\mu_A(0) \leq \mu_A(x)$

(2). $\nu_A(0) \geq \nu_A(x)$

(3). $\mu_A(x) \leq S\{\mu_A(x * y), \mu_A(y)\}$

(4). $\nu_A(x) \geq T\{\nu_A(x * y), \nu_A(y)\}\quad \forall x, y \in X.$

2. Main Section

In this section, we define direct product of an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy sets with respect to triangular binorm and investigate some related properties.

Definition 2.1. *Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy sets of X_1 and X_2, respectively. Then the direct product of $(\in, \in \vee q_k)$-intuitionistic fuzzy sets A and B with respect to triangular binorm (i.e., (T, S)-normed) is denoted by $A \times B = (\mu_{A\times B}, \nu_{A\times B})$ where $\mu_{A\times B} : X_1 \times X_2 \to [0,1]$ defined by $\mu_{A\times B}(x,y) = S\{\mu_A(x), \mu_B(y), \frac{1-k}{2}\}$ and $\nu_{A\times B} : X_1 \times X_2 \to [0,1]$ defined by $\nu_{A\times B}(x,y) = T\{\nu_A(x), \nu_B(y), \frac{1-k}{2}\}$ for all $(x,y) \in X_1 \times X_2$.*

Definition 2.2. *An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of BCK-algebra X is said to be an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra with respect to triangular binorm of X if*

(1). $\mu_A(x * y) \leq S\left\{\mu_A(x), \mu_A(y), \frac{1-k}{2}\right\}$ *for all $x, y \in X$.*

(2). $\nu_A(x * y) \geq T\left\{\nu_A(x), \nu_A(y), \frac{1-k}{2}\right\}$ *for all $x, y \in X$.*

Definition 2.3. *An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of BCK-algebra X is said to be an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideal with respect to triangular binorm (i.e., (T, S)-normed) of X if*

(1). $\mu_A(0) \leq S\left\{\mu_A(x), \frac{1-k}{2}\right\}$ *for all* $x \in X$.

(2). $\nu_A(0) \geq T\left\{\nu_A(x), \frac{1-k}{2}\right\}$ *for all* $x \in X$.

(3). $\mu_A(x) \leq S\left\{\mu_A(x * y), \mu_A(y), \frac{1-k}{2}\right\}$ *for all* $x, y \in X$.

(4). $\nu_A(x) \geq T\left\{\nu_A(x * y), \nu_A(y), \frac{1-k}{2}\right\}$ *for all* $x, y \in X$.

Definition 2.4. *An intuitionistic fuzzy set $A \times B$ of BCK-algebra $X_1 \times X_2$ is said to be an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1 \times X_2$ with respect to triangular binorm if*

(1). $\mu_{A\times B}((x_1, y_1) * (x_2, y_2)) \leq S\left\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}$ *for all* $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$.

(2). $\nu_{A\times B}((x_1, y_1) * (x_2, y_2)) \geq T\left\{\nu_{A\times B}(x_1, y_1), \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}$ *for all* $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$.

Definition 2.5. *An intuitionistic fuzzy set $A \times B$ of BCK-algebra $X_1 \times X_2$ is said to be an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideal of $X_1 \times X_2$ with respect to triangular binorm if*

(1). $\mu_{A\times B}(0, 0) \leq S\left\{\mu_{A\times B}(x_1, y_1), \frac{1-k}{2}\right\}$ *for all* $(x_1, y_1) \in X_1 \times X_2$.

(2). $\nu_{A\times B}(0, 0) \geq T\left\{\nu_{A\times B}(x_1, y_1), \frac{1-k}{2}\right\}$ *for all* $(x_1, y_1) \in X_1 \times X_2$.

(3). $\mu_{A\times B}(x_1, y_1) \leq S\left\{\mu_{A\times B}((x_1, y_1) * (x_2, y_2)), \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}$ *for all* $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$.

(4). $\nu_A((x_1, y_1) \geq T\left\{\nu_A((x_1, y_1) * (x_2, y_2)), \nu_A(x_2, y_2), \frac{1-k}{2}\right\}$ *for all* $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$.

Theorem 2.6. *Let A and B be two $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebras of X_1 and X_2, respectively. Then the Direct product $A \times B$ is an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1 \times X_2$.*

Proof. Let A and B be two $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebras of X_1 and X_2, respectively. For any $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$. We have

$$\begin{aligned}
\mu_{A\times B}((x_1, y_1) * (x_2, y_2)) &= \mu_{A\times B}(x_1 * x_2, y_1 * y_2) \\
&= S\left\{\mu_A(x_1 * x_2), \mu_B(y_1 * y_2), \frac{1-k}{2}\right\} \\
&\leq S\left\{S\left\{\mu_A(x_1), \mu_A(x_2), \frac{1-k}{2}\right\}, S\left\{\mu_B(y_1), \mu_B(y_2), \frac{1-k}{2}\right\}, \frac{1-k}{2}\right\} \\
&= S\left\{S\left\{\mu_A(x_1), \mu_B(y_1), \frac{1-k}{2}\right\}, S\left\{\mu_A(x_2), \mu_B(y_2), \frac{1-k}{2}\right\}, \frac{1-k}{2}\right\} \\
&= S\left\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}
\end{aligned}$$

$$\begin{aligned}
\nu_{A\times B}((x_1, y_1) * (x_2, y_2)) &= \nu_{A\times B}(x_1 * x_2, y_1 * y_2) \\
&= T\left\{\nu_A(x_1 * x_2), \nu_B(y_1 * y_2), \frac{1-k}{2}\right\} \\
&\geq T\left\{T\left\{\nu_A(x_1), \nu_A(x_2), \frac{1-k}{2}\right\}, T\left\{\nu_B(y_1), \nu_B(y_2), \frac{1-k}{2}\right\}, \frac{1-k}{2}\right\} \\
&= T\left\{T\left\{\nu_A(x_1), \nu_B(y_1), \frac{1-k}{2}\right\}, T\left\{\nu_A(x_2), \nu_B(y_2), \frac{1-k}{2}\right\}, \frac{1-k}{2}\right\} \\
&= T\left\{\nu_{A\times B}(x_1, y_1), \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}
\end{aligned}$$

Hence $A \times B$ is an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1 \times X_2$. □

Theorem 2.7. *Let A and B be two $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideals of X_1 and X_2, respectively. Then the direct product $A \times B$ is an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideal of $X_1 \times X_2$.*

Theorem 2.8. *If $A \times B = (\mu_{A\times B}, \nu_{A\times B})$ be an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy ideal of $X_1 \times X_2$. Then for all any $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$ and $(x_1, y_1) * (x_2, y_2) \leq (x_3, y_3)$*

(1). $\mu_{A\times B}(x_1, y_1) \leq S\left\{\mu_{A\times B}(x_2, y_2), \mu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}$.

(2). $\nu_{A\times B}(x_1, y_1) \geq T\left\{\nu_{A\times B}(x_2, y_2), \nu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}$.

Proof. Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$ such that $(x_1, y_1) * (x_2, y_2) \leq (x_3, y_3)$ then $((x_1, y_1) * (x_2, y_2)) * (x_3, y_3) = 0$. Now

$$
\begin{aligned}
(1).\ \mu_{A\times B}(x_1, y_1) &\leq S\left\{\mu_{A\times B}((x_1, y_1) * (x_2, y_2)), \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&\leq S\left\{\mu_{A\times B}(((x_1, y_1) * (x_2, y_2)) * (x_3, y_3)), \mu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&= S\left\{\mu_{A\times B}(0, 0), \mu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&\leq S\left\{S\left\{\mu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \mu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&= S\left\{S\left\{\mu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&= S\left\{S\left\{\mu_{A\times B}(x_3, y_3), \mu_{A\times B}(x_2, y_2)\right\}, \frac{1-k}{2}\right\} \\
&= S\left\{\mu_{A\times B}(x_3, y_3), \mu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}
\end{aligned}
$$

$$
\begin{aligned}
(2).\ \nu_{A\times B}(x_1, y_1) &\geq T\left\{\nu_{A\times B}((x_1, y_1) * (x_2, y_2)), \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&\geq T\left\{\nu_{A\times B}(((x_1, y_1) * (x_2, y_2)) * (x_3, y_3)), \nu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&= T\left\{\nu_{A\times B}(0, 0), \nu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&\geq T\left\{T\left\{\nu_{A}(x_3, y_3), \frac{1-k}{2}\right\}, \nu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&= T\left\{T\left\{\nu_{A\times B}(x_3, y_3), \frac{1-k}{2}\right\}, \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\} \\
&= T\left\{T\left\{\nu_{A\times B}(x_3, y_3), \nu_{A\times B}(x_2, y_2)\right\}, \frac{1-k}{2}\right\} \\
&= T\left\{\nu_{A\times B}(x_3, y_3), \nu_{A\times B}(x_2, y_2), \frac{1-k}{2}\right\}
\end{aligned}
$$

□

Definition 2.9. *Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ are intuitionistic fuzzy sets of X_1 and X_2 respectively. Define the doubt intuitionistic level set for the $A \times B$ as $(A \times B)_{\alpha,\beta} = \{(x, y) \in X_1 \times X_2 | \mu_{A\times B}(x, y) \leq \alpha, \nu_{A\times B}(x, y) \geq \beta\}$, where $\beta \in (0, \frac{1-k}{2}], \alpha \in [\frac{1-k}{2}, 1)$.*

Theorem 2.10. *Let A and B be two $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebras of X_1 and X_2, respectively. Then the direct product $A \times B$ is an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1 \times X_2$ if and only if $(A \times B)_{\alpha,\beta} \neq \phi$ is an subalgebra of $X_1 \times X_2$.*

Proof. Assume $A \times B$ is an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1 \times X_2$. To prove $(A \times B)_{\alpha,\beta} \neq \phi$ is an subalgebra of $X_1 \times X_2$. where $\beta \in (0, \frac{1-k}{2}], \alpha \in [\frac{1-k}{2}, 1)$. Let $(x_1, y_1), (x_2, y_2) \in (A \times B)_{\alpha,\beta}$. Therefore we have

$\mu_{A\times B}(x_1,y_1)\leq\alpha,\nu_{A\times B}(x_1,y_1)\geq\beta\}$ and $\mu_{A\times B}(x_2,y_2)\geq\alpha,\nu_{A\times B}(x_2,y_2)\leq\beta\}$. Since $A\times B$ is an $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1\times X_2$. $\mu_{A\times B}((x_1,y_1)*(x_2,y_2))\leq S\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2),\frac{1-k}{2}\}\leq S\{\alpha,\alpha\}=\alpha$ and $\nu_{A\times B}((x_1,y_1)*(x_2,y_2))\geq T\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2),\frac{1-k}{2}\}\geq T\{\beta,\beta,\frac{1-k}{2}\}=\beta$ which shows that $(x_1,y_1)*(x_2,y_2)\in(A\times B)_{\alpha,\beta}$. Hence $(A\times B)_{\alpha,\beta}\neq\phi$ is an subalgebra of $X_1\times X_2$.

Conversely, let$(A\times B)_{\alpha,\beta}\neq\phi$ is an subalgebra of $X_1\times X_2$. Also let $A\times B$ is not $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1\times X_2$. Then there exist $(x_1,y_1),(x_2,y_2)\in(X_1\times X_2)$ such that $\mu_{A\times B}((x_1,y_1)*(x_2,y_2))>S\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}$ and $\nu_{A\times B}((x_1,y_1)*(x_2,y_2))<T\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}$. Now let $t_0=\frac{1}{2}[\mu_{A\times B}((x_1,y_1)*(x_2,y_2))+S\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}]$ and $s_0=\frac{1}{2}[\nu_{A\times B}((x_1,y_1)*(x_2,y_2))+T\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}]$. This implies $\mu_{A\times B}((x_1,y_1)*(x_2,y_2))>t_0>S\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}$ and $\nu_{A\times B}((x_1,y_1)*(x_2,y_2))<s_0<T\{\nu_{A\times B}(x_1,y_1),\nu_{A\times B}(x_2,y_2)\}$.And so $(x_1,y_1),(x_2,y_2)\notin(A\times B)_{t_0,s_0}$ But $(x_1,y_1),(x_2,y_2)\in(A\times B)_{t_0,s_0}$. That is a contradiction. This completes the proof. □

Theorem 2.11. *Let A and B be two $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy ideals of X_1 and X_2, respectively. Then the direct product $A\times B$ is an $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy ideal of $X_1\times X_2$ if and only if $(A\times B)_{\alpha,\beta}\neq\phi$ is an ideal of $X_1\times X_2$.*

Theorem 2.12. *If $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebras of BCK/BCI-algebras X_1 and X_2respectively with respect to triangular binorm. Then*

(1). $\mu_{A\times B}(0,0)\leq S\left\{\mu_{A\times B}(x,y),\frac{1-k}{2}\right\}$.

(2). $\nu_{A\times B}(0,0)\geq T\left\{\nu_{A\times B}(x,y),\frac{1-k}{2}\right\}\ \ \forall\,(x,y)\in X_1\times X_2$.

Proof. By definition, $\mu_{A\times B}(0,0)=\mu_{A\times B}((x,y)*(x,y))\leq S\{\mu_{A\times B}(x,y),\mu_{A\times B}(x,y),\frac{1-k}{2}\}=S\{\mu_{A\times B}(x,y),\frac{1-k}{2}\}$. Therefore,$\mu_{A\times B}(0,0)\leq S\{\mu_{A\times B}(x,y),\frac{1-k}{2}\}$ for all $(x,y)\in X_1\times X_2$. Again, $\nu_{A\times B}(0,0)=\nu_{A\times B}((x,y)*(x,y))\geq T\{\mu_{A\times B}(x,y),\nu_{A\times B}(x,y),\frac{1-k}{2}\}=T\{\nu_{A\times B}(x,y),\frac{1-k}{2}\}$. Therefore, $\nu_{A\times B}(0,0)\geq T\{\mu_{A\times B}(x,y),\frac{1-k}{2}\}$ for all $(x,y)\in X_1\times X_2$. □

Lemma 2.13. *Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebras of BCK/BCI-algebras X_1 and X_2 respectively. Then the following are true.*

(1). $\mu_A(0)\leq\mu_B(y)$ *and* $\mu_B(0)\leq\mu_A(x)$, *for all* $x\in X_1,y\in X_2$.

(2). $\nu_A(0)\geq\nu_B(y)$ *and* $\nu_B(0)\geq\nu_A(x)$, *for all* $x\in X_1,y\in X_2$.

Proof. Assume that $\mu_A(0)>\mu_B(y)$ and $\mu_B(0)>\mu_A(x)$, for some $x\in X_1,y\in X_2$. Then, $\mu_{A\times B}(x,y)=S\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}\leq S\{\mu_A(0),\mu_A(0),\frac{1-k}{2}\}=\mu_{A\times B}(0,0)$ That is a contradiction. Similarly, let $\nu_A(0)<\nu_B(y)$ and $\nu_B(0)<\nu_A(x)$, for some $x\in X_1,y\in X_2$. Then, $\nu_{A\times B}(x,y)=T\{\nu_A(x),\nu_A(y),\frac{1-k}{2}\}\geq T\{\nu_A(0),\nu_A(0),\frac{1-k}{2}\}=\nu_{A\times B}(0,0)$ That is a contradiction. Thus proving the result. □

Theorem 2.14. *If $A\times B$ is a $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1\times X_2$, then either A is an $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebra of X_1 or B is an $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebra of X_2.*

Proof. Since $A\times B$ is a $(\in,\in\vee q_k)$-doubt intuitionistic fuzzy subalgebra of $X_1\times X_2$ then for all $(x_1,y_1),(x_2,y_2)\in X_1\times X_2$,, we have $\mu_{A\times B}((x_1,y_1)*(x_2,y_2))\leq S\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2),\frac{1-k}{2}\}$

By putting $x_1 = x_2 = 0$, we get,

$$\begin{aligned}
\mu_{A\times B}((0,y_1)*(0,y_2)) &\leq S\left\{\mu_{A\times B}(0,y_1), \mu_{A\times B}(0,y_2), \frac{1-k}{2}\right\}\\
\Rightarrow \mu_{A\times B}((0*0),(y_1*y_2)) &\leq S\left\{\mu_B(y_1), \mu_B(y_2), \frac{1-k}{2}\right\} \quad \text{using Lemma2.13}\\
\Rightarrow S\{\mu_A(0*0), \mu_B(y_1*y_2)\} &\leq S\left\{\mu_B(y_1), \mu_B(y_2), \frac{1-k}{2}\right\}\\
\Rightarrow \mu_B(y_1*y_2) &\leq S\left\{\mu_B(y_1), \mu_B(y_2), \frac{1-k}{2}\right\}
\end{aligned}$$

Similar way we can prove, $\nu_B(y_1*y_2) \geq T\{\nu_B(y_1), \nu_B(y_2), \frac{1-k}{2}\}$. Hence B is an $(\in, \in \vee q_k)$-doubt intuitionistic fuzzy subalgebra of X_2. □

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