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Alternating Group in Symmetric Group of Degree Five

Research Article

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Abstract: Let S_5 be a finite non abelian group of symmetric group. Then S_5 containing $5! = 120$ elements. First, we observe the maltiplication table of S_5. In this paper find out elements of alternating group of symmetric group S_5.

Keywords: Symmetric Group-perumutation.

1. Introduction and Preliminary

If A is a finite set having n distinct elements, then we shall have $n!$ distinct arrangements of the elements of A. Therefore there will be $n!$ distinct permutation of degree n. If P_n be the set consisting of all permutations of degree n, then the set P_n will have $n!$ distinct elements. This set P_n is called the *symmetric set* of permutations of degree n. It is denoted by S_n. We are converted by sets to group with mapping conditions. If S_5 be a finite non abelian group of symmetric group, then S_5 containing $5! = 120$ elements. In this paper find out elements of alternating group of symmetric group S_5.

Definition 1.1. *If A_n is is the set of all even permutation of degree n then $A_n \subset P_n$ and A_n contains $\frac{n1}{2}$ elements. The set A_n is called an Alternating set of permutations of degree n.*

Definition 1.2. *The group P_n of all permutations of degree n is called symmetric group of degree n or symmetric group of order $n!$.*

Proposition 1.3. *The set P_n of all permutations on n symbols is a finite non-abelian group of order $n!$ with respect to composite of mappings as the operation.*

Definition 1.4. *Let A be a finite set. A bijection from A to itself is called a permutation of A.*

2. Elements of Symmetric Group

Let $A = \{1, 2, 3, 4, 5\}$ Then S_5 consists of

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, P_{01} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}, P_{02} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 3 & 5 \end{pmatrix}, P_{03} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & 3 \end{pmatrix}, P_{04} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 4 & 3 \end{pmatrix},$$

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$$P_{05}=\begin{pmatrix}1&2&3&4&5\\1&2&5&3&4\end{pmatrix}, P_{06}=\begin{pmatrix}1&2&3&4&5\\1&3&2&4&5\end{pmatrix}, P_{07}=\begin{pmatrix}1&2&3&4&5\\1&3&2&5&4\end{pmatrix}, P_{08}=\begin{pmatrix}1&2&3&4&5\\1&3&4&2&5\end{pmatrix}, P_{09}=\begin{pmatrix}1&2&3&4&5\\1&3&4&5&2\end{pmatrix},$$

$$P_{10}=\begin{pmatrix}1&2&3&4&5\\1&3&5&4&2\end{pmatrix}, P_{11}=\begin{pmatrix}1&2&3&4&5\\1&3&5&2&4\end{pmatrix}, P_{12}=\begin{pmatrix}1&2&3&4&5\\1&4&2&3&5\end{pmatrix}, P_{13}=\begin{pmatrix}1&2&3&4&5\\1&4&2&5&3\end{pmatrix}, P_{14}=\begin{pmatrix}1&2&3&4&5\\1&4&3&2&5\end{pmatrix},$$

$$P_{15}=\begin{pmatrix}1&2&3&4&5\\1&4&3&5&2\end{pmatrix}, P_{16}=\begin{pmatrix}1&2&3&4&5\\1&4&5&3&2\end{pmatrix}, P_{17}=\begin{pmatrix}1&2&3&4&5\\1&4&5&2&3\end{pmatrix}, P_{18}=\begin{pmatrix}1&2&3&4&5\\1&5&2&3&4\end{pmatrix}, P_{19}=\begin{pmatrix}1&2&3&4&5\\1&5&2&4&3\end{pmatrix},$$

$$P_{20}=\begin{pmatrix}1&2&3&4&5\\1&5&3&2&4\end{pmatrix}, P_{21}=\begin{pmatrix}1&2&3&4&5\\1&5&3&4&2\end{pmatrix}, P_{22}=\begin{pmatrix}1&2&3&4&5\\1&5&4&3&2\end{pmatrix}, P_{23}=\begin{pmatrix}1&2&3&4&5\\1&5&4&2&3\end{pmatrix}, P_{24}=\begin{pmatrix}1&2&3&4&5\\2&1&3&4&5\end{pmatrix},$$

$$P_{25}=\begin{pmatrix}1&2&3&4&5\\2&1&3&5&4\end{pmatrix}, P_{26}=\begin{pmatrix}1&2&3&4&5\\2&1&4&3&5\end{pmatrix}, P_{27}=\begin{pmatrix}1&2&3&4&5\\2&1&4&5&3\end{pmatrix}, P_{28}=\begin{pmatrix}1&2&3&4&5\\2&1&5&4&3\end{pmatrix}, P_{29}=\begin{pmatrix}1&2&3&4&5\\2&1&5&3&4\end{pmatrix},$$

$$P_{30}=\begin{pmatrix}1&2&3&4&5\\2&3&1&4&5\end{pmatrix}, P_{31}=\begin{pmatrix}1&2&3&4&5\\2&3&1&5&4\end{pmatrix}, P_{32}=\begin{pmatrix}1&2&3&4&5\\2&3&4&1&5\end{pmatrix}, P_{33}=\begin{pmatrix}1&2&3&4&5\\2&3&4&5&1\end{pmatrix}, P_{34}=\begin{pmatrix}1&2&3&4&5\\2&3&5&4&1\end{pmatrix},$$

$$P_{35}=\begin{pmatrix}1&2&3&4&5\\2&3&5&1&4\end{pmatrix}, P_{36}=\begin{pmatrix}1&2&3&4&5\\2&4&1&3&5\end{pmatrix}, P_{37}=\begin{pmatrix}1&2&3&4&5\\2&4&1&5&3\end{pmatrix}, P_{38}=\begin{pmatrix}1&2&3&4&5\\2&4&3&1&5\end{pmatrix}, P_{39}=\begin{pmatrix}1&2&3&4&5\\2&4&3&5&1\end{pmatrix},$$

$$P_{40}=\begin{pmatrix}1&2&3&4&5\\2&4&5&3&1\end{pmatrix}, P_{41}=\begin{pmatrix}1&2&3&4&5\\2&4&5&1&3\end{pmatrix}, P_{42}=\begin{pmatrix}1&2&3&4&5\\2&5&1&3&4\end{pmatrix}, P_{43}=\begin{pmatrix}1&2&3&4&5\\2&5&1&4&3\end{pmatrix}, P_{44}=\begin{pmatrix}1&2&3&4&5\\2&5&3&1&4\end{pmatrix},$$

$$P_{45}=\begin{pmatrix}1&2&3&4&5\\2&5&3&4&1\end{pmatrix}, P_{46}=\begin{pmatrix}1&2&3&4&5\\2&5&4&3&1\end{pmatrix}, P_{47}=\begin{pmatrix}1&2&3&4&5\\2&5&4&1&3\end{pmatrix}, P_{48}=\begin{pmatrix}1&2&3&4&5\\3&1&2&4&5\end{pmatrix}, P_{49}=\begin{pmatrix}1&2&3&4&5\\3&1&4&2&5\end{pmatrix},$$

$$P_{50}=\begin{pmatrix}1&2&3&4&5\\3&1&5&4&2\end{pmatrix}, P_{51}=\begin{pmatrix}1&2&3&4&5\\3&1&2&5&4\end{pmatrix}, P_{52}=\begin{pmatrix}1&2&3&4&5\\3&1&4&5&2\end{pmatrix}, P_{53}=\begin{pmatrix}1&2&3&4&5\\3&1&5&2&4\end{pmatrix}, P_{54}=\begin{pmatrix}1&2&3&4&5\\3&2&1&4&5\end{pmatrix},$$

$$P_{55}=\begin{pmatrix}1&2&3&4&5\\3&2&4&1&5\end{pmatrix}, P_{56}=\begin{pmatrix}1&2&3&4&5\\3&2&5&4&1\end{pmatrix}, P_{57}=\begin{pmatrix}1&2&3&4&5\\3&2&1&5&4\end{pmatrix}, P_{58}=\begin{pmatrix}1&2&3&4&5\\3&2&4&5&1\end{pmatrix}, P_{59}=\begin{pmatrix}1&2&3&4&5\\3&2&5&1&4\end{pmatrix},$$

$$P_{60}=\begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix}, P_{61}=\begin{pmatrix}1&2&3&4&5\\3&4&1&5&2\end{pmatrix}, P_{62}=\begin{pmatrix}1&2&3&4&5\\3&4&2&1&5\end{pmatrix}, P_{63}=\begin{pmatrix}1&2&3&4&5\\3&4&2&5&1\end{pmatrix}, P_{64}=\begin{pmatrix}1&2&3&4&5\\3&4&5&2&1\end{pmatrix},$$

$$P_{65}=\begin{pmatrix}1&2&3&4&5\\3&4&5&1&2\end{pmatrix}, P_{66}=\begin{pmatrix}1&2&3&4&5\\3&5&1&2&4\end{pmatrix}, P_{67}=\begin{pmatrix}1&2&3&4&5\\3&5&1&4&2\end{pmatrix}, P_{68}=\begin{pmatrix}1&2&3&4&5\\3&5&2&1&4\end{pmatrix}, P_{69}=\begin{pmatrix}1&2&3&4&5\\3&5&2&4&1\end{pmatrix},$$

$$P_{70}=\begin{pmatrix}1&2&3&4&5\\3&5&4&2&1\end{pmatrix}, P_{71}=\begin{pmatrix}1&2&3&4&5\\3&5&4&1&2\end{pmatrix}, P_{72}=\begin{pmatrix}1&2&3&4&5\\4&1&2&3&5\end{pmatrix}, P_{73}=\begin{pmatrix}1&2&3&4&5\\4&1&2&5&3\end{pmatrix}, P_{74}=\begin{pmatrix}1&2&3&4&5\\4&1&3&2&5\end{pmatrix},$$

$$P_{75}=\begin{pmatrix}1&2&3&4&5\\4&1&3&5&2\end{pmatrix}, P_{76}=\begin{pmatrix}1&2&3&4&5\\4&1&5&3&2\end{pmatrix}, P_{77}=\begin{pmatrix}1&2&3&4&5\\4&1&5&2&3\end{pmatrix}, P_{78}=\begin{pmatrix}1&2&3&4&5\\4&2&1&3&5\end{pmatrix}, P_{79}=\begin{pmatrix}1&2&3&4&5\\4&2&1&5&3\end{pmatrix},$$

$$P_{80}=\begin{pmatrix}1&2&3&4&5\\4&2&3&1&5\end{pmatrix}, P_{81}=\begin{pmatrix}1&2&3&4&5\\4&2&3&5&1\end{pmatrix}, P_{82}=\begin{pmatrix}1&2&3&4&5\\4&2&5&3&1\end{pmatrix}, P_{83}=\begin{pmatrix}1&2&3&4&5\\4&2&5&1&3\end{pmatrix}, P_{84}=\begin{pmatrix}1&2&3&4&5\\4&3&1&2&5\end{pmatrix},$$

$$P_{85}=\begin{pmatrix}1&2&3&4&5\\4&3&1&5&2\end{pmatrix}, P_{86}=\begin{pmatrix}1&2&3&4&5\\4&3&2&1&5\end{pmatrix}, P_{87}=\begin{pmatrix}1&2&3&4&5\\4&3&2&5&1\end{pmatrix}, P_{88}=\begin{pmatrix}1&2&3&4&5\\4&3&5&2&1\end{pmatrix}, P_{89}=\begin{pmatrix}1&2&3&4&5\\4&3&5&1&2\end{pmatrix},$$

$$P_{90}=\begin{pmatrix}1&2&3&4&5\\4&5&1&2&3\end{pmatrix}, P_{91}=\begin{pmatrix}1&2&3&4&5\\4&5&1&3&2\end{pmatrix}, P_{92}=\begin{pmatrix}1&2&3&4&5\\4&5&2&1&3\end{pmatrix}, P_{93}=\begin{pmatrix}1&2&3&4&5\\4&5&2&3&1\end{pmatrix}, P_{94}=\begin{pmatrix}1&2&3&4&5\\4&5&3&2&1\end{pmatrix},$$

$$P_{95}=\begin{pmatrix}1&2&3&4&5\\4&5&3&1&2\end{pmatrix}, P_{96}=\begin{pmatrix}1&2&3&4&5\\5&1&2&3&4\end{pmatrix}, P_{97}=\begin{pmatrix}1&2&3&4&5\\5&1&2&4&3\end{pmatrix}, P_{98}=\begin{pmatrix}1&2&3&4&5\\5&1&3&2&4\end{pmatrix}, P_{99}=\begin{pmatrix}1&2&3&4&5\\5&1&3&4&2\end{pmatrix},$$

$$P_{100} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 4 & 3 & 2 \end{pmatrix}, P_{101} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 4 & 2 & 3 \end{pmatrix}, P_{102} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 3 & 4 \end{pmatrix}, P_{103} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{pmatrix}, P_{104} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 3 & 1 & 4 \end{pmatrix},$$

$$P_{105} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 3 & 4 & 1 \end{pmatrix}, P_{106} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 3 & 1 \end{pmatrix}, P_{107} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 1 & 3 \end{pmatrix}, P_{108} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 1 & 2 & 4 \end{pmatrix}, P_{109} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 1 & 4 & 2 \end{pmatrix},$$

$$P_{110} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 1 & 4 \end{pmatrix}, P_{111} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 4 & 1 \end{pmatrix}, P_{112} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 2 & 1 \end{pmatrix}, P_{113} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix}, P_{114} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix},$$

$$P_{115} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 3 & 2 \end{pmatrix}, P_{116} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 1 & 3 \end{pmatrix}, P_{117} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 3 & 1 \end{pmatrix}, P_{118} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix}, P_{119} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}.$$

In this group e is the identity element. Thus S_5 is a group containing $5! = 120$ elements.

3. Main Result

Definition 3.1. *The subgroup of S_n consisting of the even permutations of n letters is the alternating group A_n on n letters.*

Proposition 3.2. *The set A_n of all even permutations of degree n forms a finite group order $\frac{n!}{2}$ with respect to permutation multiplication.*

Result 3.3. *The alternating group is* $A_5 = \{e, P_3, P_5, P_7, P_8, P_{10}, P_{12}, P_{15}, P_{17}, P_{19}, P_{20}, P_{22}, P_{25}, P_{26}, P_{28}, P_{30}, P_{33}, P_{35}, P_{37}, P_{38}, P_{40}, P_{42}, P_{45}, P_{47}, P_{48}, P_{52}, P_{53}, P_{55}, P_{56}, P_{57}, P_{60}, P_{63}, P_{65}, P_{67}, P_{68}, P_{70}, P_{73}, P_{74}, P_{76}, P_{78}, P_{81}, P_{83}, P_{85}, P_{86}, P_{88}, P_{90}, P_{93}, P_{95}, P_{96}, P_{99}, P_{101}, P_{103}, P_{104}, P_{106}, P_{108}, P_{111}, P_{113}, P_{115}, P_{116}, P_{119}\}$ *is a subgroup of S_5 of order 60.*

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