

A Bargaining Cooperative and Non-cooperative Game Theory in Wireless Sensor Networks

Research Article

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Abstract: A Bargaining Cooperation and non-cooperation has been recognized as an important mechanism to enhance connectivity and throughput in multi-hop wireless networks, especially under varying channel conditions. One major problem of cooperation and non-cooperation is that it incurs energy and possibly delays costs. To a rational and selfish node these costs are worth incurring only if it receives at least comparable returns in the long term. In light of this, we propose a new incentive mechanism called bandwidth exchange (BE) where a node can delegate a portion of its bandwidth to another node in exchange for relay cooperation and non-cooperation. In an n-node wireless network over a fading channel and use a Cooperative and non-cooperative game Nash Bargaining Solution (CANGNBS) mechanism to study the benefits of BE in terms of rate and coverage gains. In this paper we specifically study BE in the simple form of exchanging orthogonal frequency bands to provide incentives for relaying in a wireless network. Other forms of exchanging bandwidth such as delegation of time-slots or using spreading codes of different lengths are also possible. Using a Nash Bargaining framework, we explore the advantage of BE in both static and fading channels.

Keywords: Bargaining cooperative game, Wireless Sensor Network, Power Control, Bargaining Non-Cooperative Game.

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1. Introduction

In recent years there has been a growing interest in applying game theory to study Wireless systems used game theory to investigate power control and rate control for Wireless data. Energy efficiency and achieving reliability is a key issue in wireless sensor networks. Battery capacity is limited and it is usually impossible to replace them. Any operation performed on a sensor consumes energy, involving discharge of battery power. Hence battery power efficiency is a critical factor while considering the energy efficiency of WSN. The three domains of energy Consumption in a sensor are sensing, data processing and data communication, out of which Communication is the main consumer of energy. Hence transmission at optimal power level is very essential. Optimal transmit power level implies the power level which reduces the interference, increases the successful packet transmission and provides the desired quality of service. Maintaining the transmit power under control is furthermore favourable to decrease the packet collision probability, which if not leads to more retransmitted packets wasting even more energy. Hitherto energy efficiency has been investigated extensively and various approaches to achieve an energy efficient network includes, scheduling sensor nodes to alternate between energy-conserving modes of operation, competent routing algorithms, clustering, incorporating astuteness and use of spatial localization at every node to lessen transmission of redundant data. Nodes specialization to different roles such as idle, sensing, routing and routing sensing to maximize the utility of the nodes has been proposed. An approach for optimizing transmit power for cooperative and non-cooperative

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game scenario where all the nodes use a uniform transmit power, and numerical results of transmit power sufficient to satisfy the network connectivity have been proposed.

2. Game Theory For Sensor Networks

Game theory is a theory of decision making under conditions of uncertainty and interdependence. In game theory, behaviour in strategic situations in a mathematic mode is captured. A strategic game consists of: a set of *players*, which may be a group of nodes or an individual node, a set of actions for each player to make a decision and preferences over the set of action profiles for each player. In any game utility represents the motivation of players. Applications of cooperative and non-cooperative game theory always attempt to find equilibriums. If there is a set of strategies with the property that no player can profit by changing his or her strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the Nash equilibrium. Game theory offers models for distributed allocation of resources and thus provides a way of exploring characteristics of wireless sensor networks. Energy harvesting technologies essential for independent sensor networks using a non-cooperative game theoretic technique is analysed. Cooperative and Non-cooperative Nash Equilibrium (CANNE) were projected as the solution of this game to attain the optimal Probabilities of sleep and wake up states that were used for energy conservation. The energy efficiency problem in wireless sensor networks as the maximum network lifetime routing problem is looked. Here the transmit power levels is adjusted to just reach the anticipated next hop receiver such that the energy consumption rate per unit information transmission can be reduced.

2.1. Nash Bargaining Solution

We discussed cooperative and non-cooperative game theory approaches of analysing wireless networks. We showed that, in general, the outcome of cooperative and non-cooperative games may be inefficient. In addition to efficiency, fairness is also a major concern that may not be addressed by cooperative and non-cooperative games. Additionally, the authors conclude that cooperative and non-cooperative spectrum sharing systems achieve many of the same long term benefits in terms of increasing spectrum utilization as cooperative and non-cooperative systems. Our research focuses on a cooperative and non-cooperative game theory model to address the issues of efficiency and fairness. This section focuses on works that formulate a cooperative and non-cooperative game model and use the NBS. The NBS addresses two of the major drawbacks of the NE of cooperative and non-cooperative games: inefficiency and fairness. In a fair scheme to allocate subcarrier, rate, and power for multiuser orthogonal frequency-division multiple-access systems is proposed. The problem is to maximize the overall system rate, under each user's maximal power and minimal rate constraints, while considering fairness among users. The authors propose a multiuser bargaining algorithm developed based on optimal coalition pairs among users, which achieves the CANNBS. The work in this document focuses on applying the CANNBS to the spectrum sharing game. Specifically, we formulate a game similar to the one studied in and develop a distributed scheme that aims to achieve the operating point dictated by the NBS. It differs from in that there are multiple receivers which are not co-located. Since the receivers are not co-located, a centralized solution is not feasible. We focus on a distributed solution that requires minimal information. Our work also differs from in that we study the wireless medium. In the wireless medium, interactions between players lead to non-convexity of the optimization problem, which prevents us from deriving an analytical solution similar to.

2.2. Cooperative and Non-cooperative Game Theory and Nash Bargaining

Game theory provides a set of mathematical tools that are useful in analysing complex decision problems with interactions between self-interested decision makers, called players. The basic component of game theory is a game, $G = \{M, A, (u_i)\}$.

$M = \{1 \dots N\}$ is the set of players, A_i is the set of actions for player i , $A = A_1 \times A_2 \times \dots \times A_n$, and u_i is the objective function, sometimes called utility function, which player i wishes to maximize. Define $U = \{(u_i(a)) | a \in A\}$ as the set of achievable utility for all players. In a non-cooperative game, players individually attempt to maximize their own utility without regard to the utility achieved by other players. In a cooperative game, players bargain with each other before the game is played. If an agreement is reached, players act according to the agreement reached. If the players cannot reach an agreement, players act in a non-cooperative way. Note that the agreements reached must be binding, so players are not allowed to deviate from what is agreed upon. That is, a mechanism must exist that prevents players from deviating from the action they agreed upon. John Nash wrote in his seminal paper on cooperative and non-cooperative games that to understand the outcome of a bargaining game, we should not focus on trying to model the bargaining process itself, but instead, we should list the properties, or axioms, that we expect the outcome of the bargaining process to exhibit. Once we define these axioms, we can eliminate all possible outcomes that do not satisfy the axioms. This narrows down the possible outcomes of the bargaining process, but we cannot distinguish which of all possible outcomes remaining is likely to occur. So, if we define the axioms such that only one possible outcome satisfies them, there is no ambiguity in the outcome of the bargaining process. This way of analysing cooperative and non-cooperative games is called axiomatic bargaining game theory. Before we proceed, we need to introduce some terminology and assumptions.

- **Agreement point**

- An action vector $a \in A$ that is a possible outcome of the bargaining process.

- **Disagreement point**

- An action vector $a \in A$ that is expected to be the result of non-cooperative play given a failure of the bargaining process (i.e., what will happen if players cannot come to an agreement). Clearly, the utility achieved by every player at any agreement point has to be at least as much as the utility achieved at the disagreement point.

- **Pareto optimal**

- $u \in U$ is Pareto optimal if $u \leq u_0$ implies $u_0 \notin U$. u is weak Pareto optimal if $u < u_0$ implies $u_0 \notin U$. Define POU as the set of Pareto optimal points of U and WPOU as the set of weak Pareto optimal points of U . Notice that POU contained in WPOU.

- **Bargaining Problem**

- A vector containing utility space and a disagreement point for a game.

- **Bargaining solution**

- A bargaining solution \emptyset , defined on a class of bargaining problems Σ , is a map that associates with each problem $(U, u_0) \in \Sigma$ a unique point in U , where $u_0 = u(a_0)$ is the utility achieved at the disagreement point a_0 .

To determine the bargaining solutions, we need to make some assumptions about the utility space U :

- $U \subset R^n$ is upper-bounded, closed and convex.
- There exists $u \in U$ such that $u_0 < u$.

2.3. Nash Proposed the Following Axioms

- **Individual Rationality (IR):** $\emptyset(U, u_0) > u_0$.
- **Pareto optimality (PO):** $\emptyset(U, u_0) \in PO(U)$.
- **Invariance to affine transformations:** If $\xi : R^n \rightarrow R^n$, $\xi(u) = u'$ with $u'_i = c_i u_i + d_i$, $c_i, d_i \in R$, $c_i > 0$, for all i , then $\emptyset(\xi(U), \xi(u_0)) = \xi(\emptyset(U, u_0))$.
- **Independence of irrelevant alternative:** If $u' \in V \subset U$ and $u' = \emptyset(U, u_0)$ then $\emptyset(V, u_0) = u'$.
- **Symmetry:** If U is symmetric with respect to i and j , $u_i^0 = u_j^0$, and $u' = \emptyset(U, u_0)$ then $u'_i = u'_j$.

3. The Efficient Operation Problem as a Bargaining Game

Background on bankruptcy problem and bargaining solutions can be referred in our earlier work. Here, we denote Bankruptcy problem as a pair (B, Q) , where B represents the amount of estate, and Q is the set of investments of n creditors $Q = q_1, q_2, \dots, q_n$, such that $0 \leq B \leq \sum_{i=1}^n q_i$. Consider an allocation $X_j(n) = [x_1^j, x_2^j, x_n^j] \subset X(n)$ of the estate among n creditors, where $X(n)$ is the set of all possible allocations. Then $X_j(n)$ should satisfy

$$0 \leq x_i^j \leq q_i \quad \text{and} \quad \sum_{i=1}^n x_i^j = B \quad (1)$$

A question here is how creditors come to a fair agreement of such allocation. In cooperative and non-cooperative game theory this process of players negotiating towards an agreement of mutual benefits is modelled as a bargaining game.

Definition 3.1 (Cooperative and Non-cooperative game Nash Bargaining Solution). $\emptyset(U, u^0)$ Is the CANGNBS if and only if it satisfies the axioms IR, PO. The unique Cooperative and Non-cooperative game Nash Bargaining Solution (CANGNBS) is the maximize of the Nash Product (NP),

$$\operatorname{argmax}_{u > u_0} \prod_{i=1}^N (u_i - u_i^0) \quad (2)$$

Theorem 3.2. If U has a unique NP maximize, u^* , which coincides with the NP maximize of U_c, u_c^* then u^* is unique Cooperative and Non-cooperative game Nash Bargaining solution for U .

Proof. We know that $U \subset U_c$. Since the unique maximize of the Cooperative and Non-cooperative Nash Bargaining solution for U and U_c are the same, this implies $u^* = u_c^* \in U$. Thus by the axiom u^* is the unique Cooperative and Non-cooperative Nash Bargaining solution for U . That is show that the Pareto optimal axiom is not guaranteed to be satisfied. This is because the author considers Pareto optimal versus all possible mixed strategies. So it is possible for a mixed strategy to obtain an expected utility that Pareto dominates the NP maximize of U . in the remainder of this manuscript, we refer to Pareto optimality with respect to only pure strategies. So in this sense, the NP maximize of U is Pareto optimal. \square

Definition 3.3 (Cooperative and Non-Cooperative Game Theory Bargaining Problem). Let $\{(i)/(i) = 1, 2, \dots, n\}$ be the set of nodes. Q denotes the set of nodes feasible payoff allocations, which is a nonempty set on R^n . $E^{\min} = E_1^{\min}, E_2^{\min}, \dots, E_n^{\min}$ Where E_i^{\min} denotes node i minimal payoff. Then (Q, E_i^{\min}) is a n -person cooperative and non-cooperative game theory bargaining problem. The cooperative and non-cooperative game theory bargaining problem (Q, E^{\min}) has many Pareto solutions denoted by $f(Q, E^{\min})$, and this paper considers the RBS. Assume that the maximal and minimal payoff is E_i^{\max} and E_i^{\min} respectively. Then the cooperative and non-cooperative game theory bargaining solution can be expressed as

$$E_i^* = \operatorname{arg} \max_{E_i \in Q} \prod_{i=1}^n V_i \quad (3)$$

Where V_i is denoted by

$$V_i = E_i - E_i^{\min} + \frac{1}{N-1} \sum_{j \neq i} (E_j^{\max} - E_j) \tag{4}$$

In the following, through solving the optimization problem, we can obtain the optimal bandwidth allocations.

Theorem 3.4. *The multinode cooperative and non-cooperative bandwidth allocation problem is a bargaining problem.*

Proof. According to the nodes utility, Q is closed on R^n . Assume that $U^a = [U_1^a \dots U_n^a]$, $U^b = [U_1^b \dots U_n^b]$, $0 \leq \theta \leq 1$ then,

$$\theta U^a + (1 - \theta) U^b = [\theta U_1^a + (1 - \theta) U_1^b \dots \theta U_n^a + (1 - \theta) U_n^b] \tag{5}$$

For all $i = 1, 2, \dots, n$.

$$\theta U_i^a + (1 - \theta) U_i^b = \theta \alpha_i^a + (1 - \theta) \alpha_i^b U_i^{\max} + \left(1 - \sum_{i=1}^n (\theta \alpha_i^a + (1 - \theta) \alpha_i^b) \right) U_i^{\min} \tag{6}$$

According to $U^a \in Q$, $U^b \in Q$, we have $0 \leq \alpha_i^a \leq 1$, $0 \leq \alpha_i^b \leq 1$, $\sum_{i=1}^n \alpha_i^a \leq 1$, $\sum_{i=1}^n \alpha_i^b \leq 1$, then

$$\begin{aligned} \theta \alpha_i^a + (1 - \theta) \alpha_i^b &\geq \theta \sum_{i=1}^n \alpha_i^a + (1 - \theta) \sum_{i=1}^n \alpha_i^b \\ &= \sum_{i=1}^n (\theta \alpha_i^a + (1 - \theta) \alpha_i^b) \end{aligned} \tag{7}$$

$$\sum_{i=1}^n \theta \alpha_i^a + (1 - \theta) \alpha_i^b = \theta \sum_{i=1}^n \alpha_i^a + (1 - \theta) \sum_{i=1}^n \alpha_i^b \leq 1 \tag{8}$$

Then, we can conclude that Q is closed and convex on R^n , i.e., the multimode cooperative and non-cooperative game bandwidth allocation is a bargaining problem, and the proof is completed. Considering the minimal and maximal utilities, each node can obtain the optimal bandwidth allocation by RBS. First, we introduce the notion of node i 's normalized utility (NU), which is defined as the surplus utility penalty relative to the maximum utility, and there is

$$U_i = \frac{U_i - U_i^{\min}}{U_i^{\max} - U_i^{\min}} \tag{9}$$

According to (9), node i 's maximal normalized utility and minimal normalized utility is 1 and 0, respectively based on the definition of RBS, we have

$$V_i = U_i + \frac{1}{n-1} \sum_{j \neq i} 1 - U_j \tag{10}$$

Then, the corresponding optimization problem (P1) can be expressed as

$$P_1 \max \prod_{i=1}^n V_i$$

Such that $\sum_{i=1}^n \alpha_i \leq 1$, $0 \leq \alpha_i \leq 1$, for all $i = 1, 2, \dots, n$. □

Theorem 3.5. *The Nash solution is the unique allocation rule for n -player bargaining problems that satisfies EFF, SYM, CAT and IIA.*

Proof. It can be easily checked that NA satisfies EFF, SYM, CAT and IIA. Let ϕ be an allocation rule for n-player bargaining problems that satisfies the four properties and let $(F, d) \in B^N$. Let $z = NA(F, d)$. We now show that $\phi(F, d) = z$. Let $\mu = \{x \in R^N : h(x) \leq h(z)\}$, where h is defined as Let $(F, d) \in B^N$ and let $z \in NA(F, d)$. For each $z \in R^N$, let $h(x) = \sum_{i \in N} \prod_{j \neq i} (z_j - d_j) z_i$. Then, for each $x \in F$, $h(x) \leq h(z)$ which, in turn, ensures that $F \subset \mu$. Let f^A be the positive affine transformation that associates, to each $x \in R^N$, the vector $(U_1^A(x), \dots, U_n^A(x))$ where, for each $i \in N$,

$$U_i^A(x) = \frac{1}{z_i - d_i} x_i - \frac{d_i}{z_i - d_i}.$$

Now, we compute $U^A(\mu)$.

$$\begin{aligned} U^A(\mu) &= \left\{ y \in \mathfrak{R}^N : (U^A)^{-1}(y) \in \mu \right\} \\ &= \left\{ y \in R^N : h\left((U^A)^{-1}(y)\right) \leq h(z) \right\} \\ &= \left\{ y \in R^N : h((z_1 - d_1)y_1 + d_1, \dots, (z_n - d_n)y_n + d_n) \leq h(z) \right\} \\ &= \left\{ y \in R^N : \sum_{i \in N} \prod_{j \neq i} (z_j - d_j) ((z_i - d_i)y_i + d_i) \leq \sum_{i \in N} \prod_{j \neq i} (z_j - d_j) z_i \right\} \end{aligned}$$

This, after straight forward algebra, leads to

$$\begin{aligned} U^A(\mu) &= \left\{ y \in R^N : \sum_{i \in N} \prod_{j \in N} (z_j - d_j) y_i \leq \sum_{i \in N} \prod_{j \in N} (z_j - d_j) \right\} \\ &= \left\{ y \in R^N : \prod_{j \in N} (z_j - d_j) \sum_{i \in N} y_i \leq \prod_{j \in N} (z_j - d_j) \sum_{i \in N} 1 \right\} \\ &= \left\{ y \in R^N : \sum_{i \in N} y_i \leq n \right\} \end{aligned} \quad (11)$$

Note that $U^A(d) = (0, \dots, 0)$ and, hence $(U^A(\mu), f^A(d))$ is a symmetric bargaining problem. Since ϕ satisfies EFF and SYM, $\phi(U^A(\mu), U^A(d)) = (1, \dots, 1)$. Since ϕ also satisfies CAT, $\phi(\mu, d) = (U^A)^{-1}((1, \dots, 1)) = z$. Finally, $z \in U$, $U \subset \mu$, and ϕ satisfies IIA, $\phi(U, d) = z$. After the characterization result above, it is natural to wonder if the result is tight or, in other words, if any of the axioms are superfluous. \square

Proposition 3.6. *None of the axioms used in the characterization of the Nash solution given by Theorem 3.5 is superfluous.*

Proof. We show that for each of the axioms in the characterization, there is an allocation rule different from the Nash solution that satisfies the remaining three.

Remove EFF: The allocation rule ϕ defined, for each bargaining problem (F, d) by $\phi(F, d) = d$ satisfies SYM, CAT and IIA.

Remove SYM: Let ϕ be the allocation rule that can be defined as follows.

Let (F, d) be a bargaining problem. Then

$$\phi_1(F, d) = \max_{x \in F_d} x_1 \quad (12)$$

For each $i > 1$, $\phi_i(F, d) = \max_{x \in F_d^i} x_i$, where $F_d^i = \{x \in F_d : \text{for each } j < i, x_j = \phi_j(F, d)\}$. These kind of solutions are known as serial **dictatorships**, since there is an ordering of the players and each player chooses the allocation he prefers among those that are left when he is given the turn to choose. This allocation rule satisfies EFF, CAT and IIA.

Remove CAT: Let ϕ be the allocation rule that, for each bargaining problem (F, d) , selects the allocation $\phi(F, d) = d + \bar{t}(1, \dots, 1)$, where $\bar{t} = \max\{t \in R : d + t(1, \dots, 1) \in F_d\}$ (the compactness of F_d ensures that \bar{t} is well defined). This allocation rule, known as the egalitarian solution (Kalai 1977), satisfies EFF, SYM, and IIA. \square

4. Simulation and Performance Analysis

For simulation, 50 nodes are uniformly distributed in a 100×50 meter area grid as shown in Figure 1. This is a five-layer network with ten nodes per layer and multiple sinks. The coverage range for each node has been set to 16m. So there are two lower layer nodes in the coverage of each upper layer edge node and three lower layer nodes in the coverage of the other upper layer nodes. Each node randomly chooses one of the lower hop neighbor layer nodes in its coverage to transmit its message.

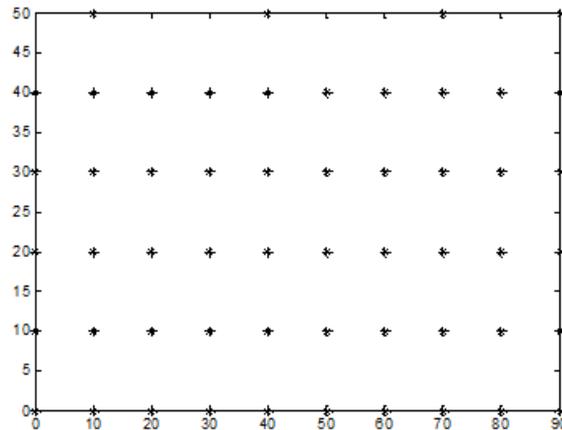


Figure 1. Sample Network Topology

Thus the messages flow in the correct direction, but do not use the same path every time. Multiple sinks are used to avoid congestion in the last row of nodes. The topology is designed such that there will be one sink node for a group of three nodes in the last row. In this phase each cluster head gets paired with other cluster head sensors and transmits data cooperatively together with the node which senses information and transmits the data to the Cluster Head within its cluster. Then the Cluster Head broadcasts the data to ‘J’ cluster head nodes which constitutes the distributed array. Each node in the transmitting cooperative group has data and encodes the transmission sequence according to Space Time Block Coding as if each node were a distinct transmit antenna element.

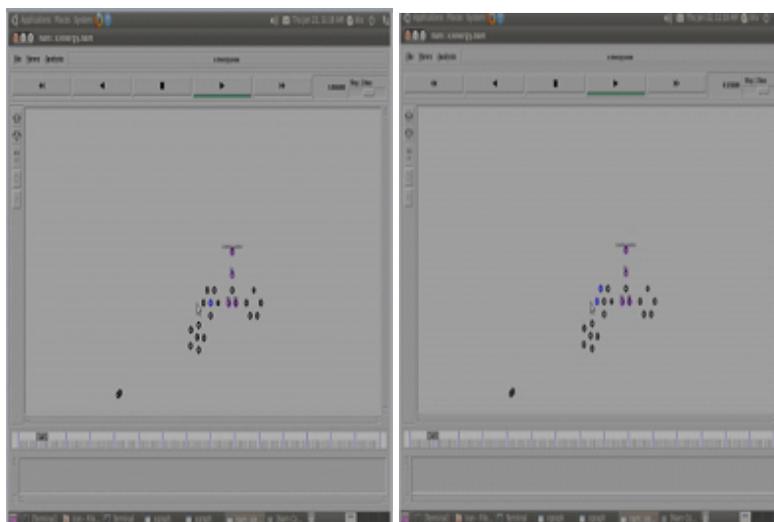


Figure 2. Initial state snap shot

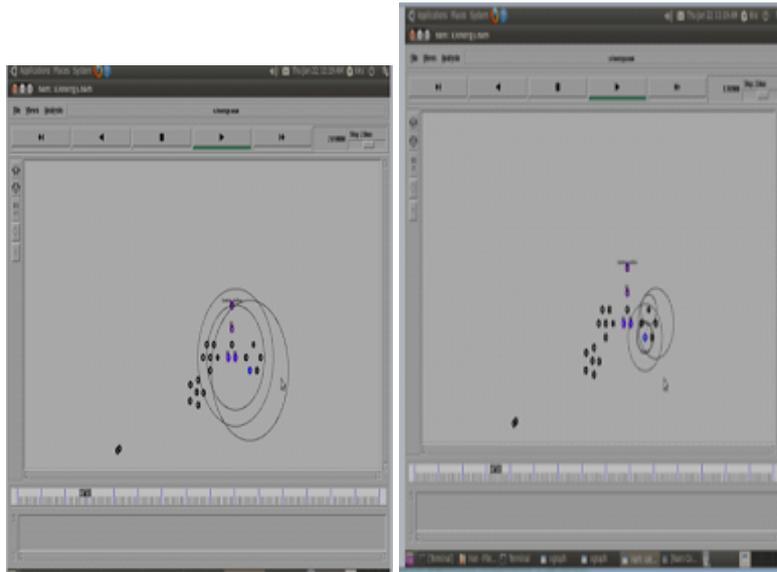


Figure 3. Middle state snap shot

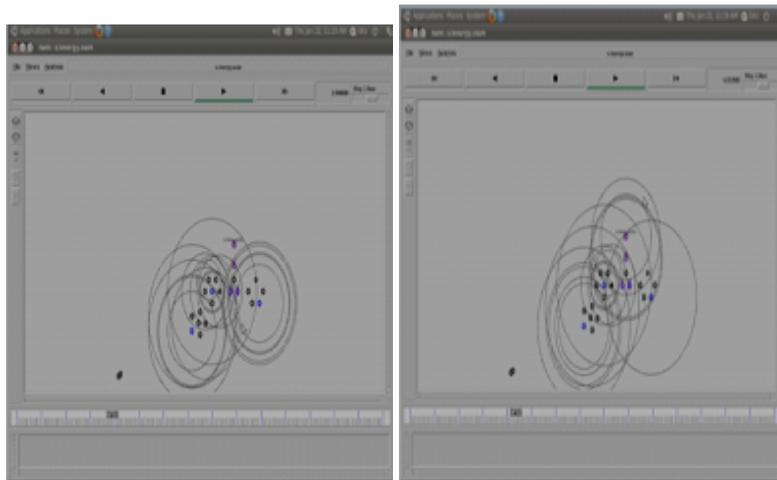


Figure 4. Final State Snap shot

The cluster sensor node sends its data to the selected cooperative node, and they encode the data using Space Time Block Coding and transmit the encoded data to the sink cooperatively bargaining. The Figure 2-5, shows the number of nodes alive for each and every round of data transmission. The performance of cluster head scheme is analyzed in terms of energy consumption.

4.1. Network Architecture after Node Failure

Figure 5 and 6 shows the network architecture after some nodes are failure and active due lack of battery power or by the environmental interference. The following plot shows the performance metric after some nodes failure. The red points indicate sensor nodes and a black point indicates failure nodes.

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