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Regional Maximum Annual Rainfall Estimates Using TL-moment and LQ-moment: A Comparative Case Study for North East India

Research Article

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Abstract: Rainfall data of the North East region of India has been considered for selecting the best fit model for rainfall frequency analysis. The five extreme probability distributions, namely Generalized extreme value (GEV), Generalized Logistic (GLO), Pearson type 3 (PE3), Generalized Log normal (GLO) and Generalized Pareto (GPA) distributions have been fitted using LQ-moment. Also three probability distributions namely Generalized extreme value (GEV), Generalized Logistic(GLO) and Generalized Pareto (GPA) distributions have been fitted using TL-moment. Both TL-moment and LQ-moment analysis show that GPA distribution is the best fitting distribution for the North Eastern Region. Relative root mean square error (RRMSE) and RBIAS are employed to compare between the results found from TL-moment and LQ-moment analysis. It is found that the TL-moment method is significantly more efficient than LQ-moment for maximum rainfall estimates of North East India. The rainfall frequency model for the region has been developed by using the identified robust distribution for the region.

Keywords: TL-moments, LQ-moments, Mone curlo simulation.

1. Introduction

The economy of the North East India depends on agriculture. Most common natural disaster in this area is flood caused by heavy rainfall which causes destruction of agriculture and properties. So proper analysis of extreme rainfall is an important task. It is also important for construction of dam, bridge, road etc. There are several methods for maximum rainfall frequency analysis. To develop a suitable model for maximum rainfall for a certain return period for a particular region, it is necessary to make a comparative study among the methods.

In this study regional rainfall frequency analysis of North East Region has been considered for development of frequency analysis model. For this study the TL-moment and LQ-moment method has been used for estimation of parameters of the probability distributions. The five probability distributions, namely generalized extreme value (GEV), generalized Logistic(GLO), Pearson type 3 (PE3), 3 parameter Log normal (GNO) and generalized Pareto (GPA) distributions have been considered for this study. The homogeneity of the study region has been carried out by using heterogeneity measure proposed by Hosking and Wallis [8]. Two goodness of fitness measures namely Z-statistics and LQ-moment ratio diagram have been employed for identification of the best fitting distribution for our study region. Also RRMSE and RBIAS is used to make a comparison between the two best fitting distribution getting from TL-moment and LQ-moment analysis.

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Application of extreme value distribution to rainfall data have been investigated by several authors from different parts of the world. Bora, D.J. et al. [3] analysed annual maximum rainfall data of 12 gauged stations of the North East India using L-moment and LH-moment. It is found that GPA distribution designated by LH-moment of order 1 is the most suitable distribution for maximum rainfall analysis of North East India. Shabri, A. B. et al. [14] used L-moment and TL-moment to analyse the maximum rainfall data of 40 stations of Selangor Malaysia. Comparison between the two approaches showed that the L-moments and TL-moments produced equivalent results. GLO and GEV distributions were identified as the most suitable distributions for representing the statistical properties of extreme rainfall in Selangor. Deka, S. et al. [6] fitted three extreme value distributions using LH moment of order zero to four and found that GPA distribution is the best fitting distribution for the majority of the stations in North East Region of India. Norbiato et al. [12] tried to characterize the severity of a flash flood generating storm on 29^{th} August 2003 in the eastern Italian Alps which was characterized by extra ordinary rainfall. Regional frequency analysis based on the index variable method and L-moments are utilized to analyse annual maximum rainfall data for the region of north eastern Italy. It was found that the regional growth curves based on Kappa distribution may be useful for the region. Trefry et al. [15] used L-moments method to analyse annual maximum rainfall and partial duration rainfall data of 152 stations of the state of Michigan. It was found that GEV distribution is the best fit distribution for annual maximum rainfall data and GPA distribution is the best fit distribution for partial duration rainfall data. Ogunlela [13] studied the stochastic analysis of rainfall event in Ilorin using probability distribution functions. He found that the log Pearson type III distribution is the best for describing peak daily rainfall data of Ilorin. Adamowski et al. [1] used L-moments method for regional rainfall frequency analysis of Canada and found that GEV distribution is the best fit distribution for rainfall frequency analysis of Canada.

1.1. Study Region and Data Collection

For this study 12 distantly situated gauged stations of the North East India viz. Imphal, Agartala, Shillong, Guwahati, Silchar, Jorhat, Dhubri, Lengpui, Lakhimpur, Pasighat, Mohanbari and Itanagar are considered. Annual daily maximum rainfall data of these stations for a period of 30 years from 1984 to 2013 are considered for this study. Data are collected from Regional Meteorological centre, Guwahati.

1.2. Method of TL-Moment

Let $X_1, X_2, \ldots, X_n$ be a sample from a continuous distribution function $F(.)$ with quantile function $Q(F)$ and let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ be the order statistics. Then the rth L-moment λ_r is given by

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} E\left(X_{r-k:r}\right), \qquad r = 1, 2, \ldots \tag{1}$$

In TL-moment defined by Elamir et al. [7], the term $E(X_{r-k:r})$ in the above equation (1) is replaced by $(X_{r+t_1-k:r+t_1+t_2})$. That is for each r, the conceptual sample size will be increased from r to $r + t_1 + t_2$ and work only with the expectation of r ordered statistics $Y_{t_1+1:r+t_1+t_2}, \ldots, Y_{t_1+r:r+t_1+t_2}$ by trimming the t_1 smallest and t_2 largest from the conceptual sample. Thus the rth TL-moment is defined as

$$\lambda_r^{(t_1,\ t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} E\left(X_{r+t_1-k:r+t_1+t_2}\right), \quad r = 1, 2 \ldots \tag{2}$$

For $t_1 = t_2 = 0$, the TL-moment yields the original L-moment. When $t_1 = t_2 = t$, then the rth TL-moment is defined as

$$\lambda_r^{(t)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} E\left(X_{r+t-k:r+2t}\right), \quad r = 1, 2 \ldots \tag{3}$$

Taking $t = 1$ the first four TL-moments can be expressed as

$$\lambda_1^{(1)} = E\left[X_{2:3}\right] = 6\beta_1 - 6\beta_2 \tag{4}$$

$$\lambda_2^{(1)} = \frac{1}{2}E\left[X_{3:4} - X_{2:4}\right] = 6\left(-2\beta_3 + 3\beta_2 - \beta_1\right) \tag{5}$$

$$\lambda_3^{(1)} = \frac{1}{3}E\left[X_{4:5} - 2X_{3:5} + X_{2:5}\right] = \frac{20}{3}\left(-5\beta_4 + 10\beta_3 - 6\beta_2 + \beta_1\right) \tag{6}$$

$$\lambda_4^{(1)} = \frac{1}{4}E\left[X_{5:6} - 3X_{4:6} + 3X_{3:6} - X_{2:6}\right] = \frac{15}{2}\left(-14\beta_5 + 35\beta_4 - 30\beta_3 + 10\beta_2 - \beta_1\right) \tag{7}$$

The TL-co-efficient of variation, TL-co-efficient of skewness and TL-co-efficient of kurtosis are defined as

$$\tau_2^{(1)} = \frac{\lambda_2^{(1)}}{\lambda_1^{(1)}}, \quad \tau_3^{(1)} = \frac{\lambda_3^{(1)}}{\lambda_2^{(1)}} \quad \text{and} \quad \tau_4^{(1)} = \frac{\lambda_4^{(1)}}{\lambda_2^{(1)}} \tag{8}$$

The r^{th} sample TL-moment is given by

$$l_r^{(t)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} \hat{E}\left(X_{r+t-k:r+2t}\right), \quad r = 1, 2 \ldots \tag{9}$$

where unbiased estimator is given by

$$\hat{E}\left(X_{r+t-k:r+2t}\right) = \frac{1}{\begin{pmatrix} n \\ r+2t \end{pmatrix}} \sum_{i=1}^{n} \begin{pmatrix} i-1 \\ r+t-k-1 \end{pmatrix} \begin{pmatrix} n-i \\ t+k \end{pmatrix} X_{i:n} \tag{10}$$

1.3. Method of LQ-Moment

Let $X_1, X_2, X_3, \ldots, X_n$ be a sample from a continuous distribution function $F_x(.)$ with quantile function $Q_x\left(u\right) = F_x^{-1}(u)$. If $X_{1:n} \leq X_{2:n} \leq X_{3:n} \leq \cdots \leq X_n$ denote the order statistics, then the r^{th} LQ-moments ζ_r of X proposed by Mudholkar et al. [11] are given by

$$\zeta_r = r^{-1}\sum_{k=0}^{k=r-1}\left(-1\right)^k \begin{pmatrix} r-1 \\ k \end{pmatrix} t_{p,a\left(X_{r-k:r}\right)}, \quad r = 1, 2, \ldots. \tag{11}$$

where $0 \leq \alpha \leq \frac{1}{2}$, $0 \leq p \leq \frac{1}{2}$, and

$$t_{p,\alpha}\left(X_{r-k:r}\right) = pQ_{x_{r-k:r}}\left(\alpha\right) + \left(1-2p\right)Q_{x_{r-k:r}}\left(\alpha\right) + pQ_{x_{r-k:r}}(1-\alpha) \tag{12}$$

The linear combination $t_{p,\alpha}$ is a quick measure of the location of the sampling distribution of order statistic $X_{r-k:r}$. With appropriate combinations of a and p, estimators for $t_{p,\alpha}\left(.\right)$ can be found which are functions of commonly used estimators such as median, trimean and Gastwirth. The trimean-based estimator is defined as

$$\frac{Q_{x_{r-k:r}}\left(\frac{1}{4}\right)}{4} + \frac{Q_{x_{r-k:r}}\left(\frac{1}{2}\right)}{2} + \frac{Q_{x_{r-k:r}}\left(\frac{3}{4}\right)}{4}$$

The first four LQ-moments of the random variable X are given by:

$$\zeta_1 = \tau_{p,\alpha}(X), \tag{13}$$

$$\zeta_2 = \frac{1}{2}\left[\tau_{p,\alpha}(X_{2:2}) - \tau_{p,\alpha}(X_{1:2})\right], \tag{14}$$

$$\zeta_3 = \frac{1}{3}\left[\tau_{p,\alpha}(X_{3:3}) - 2\tau_{p,\alpha}(X_{2:3}) + \tau_{p,\alpha}(X_{1:3})\right], \tag{15}$$

$$\zeta_4 = \frac{1}{4}[t_{p,\alpha}(X_{4:4}) - 3t_{p,\alpha}(X_{3:4}) + 3t_{p,\alpha}(X_{2:4}) - t_{p,\alpha}(X_{1:4})] \tag{16}$$

The LQ-CV, LQ-skewness and LQ-kurtosis are defined by

$$\eta = \frac{\zeta_2}{\zeta_1},\ \eta_3 = \frac{\zeta_3}{\zeta_2} \text{ and } \eta_4 = \frac{\zeta_4}{\zeta_2} \tag{17}$$

2. Regional Rainfall Frequency Analysis Using TL-moment

Screening of data: For L-moment method the Discordancy test D_i, proposed by Hosking and Wallis [8] is given by

$$D_i = \frac{1}{3}N(u_i - \overline{u})^T S^{-1}(u_i - \overline{u}) \tag{18}$$

where $S = \sum_{i=1}^{N}(u_i - \overline{u})(u_i - \overline{u})^T$ and $u_i = \left[t_2^i, t_3^i, t_4^i\right]^T$ for i-th station, N is the number of stations, S is covariance matrix of u_i and $\overline{u}$ is the mean of vector, u_i. Critical values of discordancy statistics are tabulated by Hosking and Wallis [8], for $N = 12$, the critical value is 2.757. If the D-statistics of a station exceeds 2.757, its data is discordant from the rest of the regional data. Similar procedure has been used for TL moment also. For discordancy test L-cv, L-skewness and L-kurtosis are replaced by TL-cv, TL-skewness and TL-kurtosis respectively. Calculated $D_i$values are given in Table 1. From Table 1 it is observed that the D_i values of 12 stations of the study region less than the critical value 2.757. Hence all the data of 12 stations can be considered for the study.

S.No.	Site name	No. of observation	TL-CV	TL-Skewness	TL-Kurtosis	D_i
1.	Imphal	30	0.0914	0.1555	0.0630	0.04
2.	Agartala	30	0.1027	0.1190	-0.0356	1.75
3.	Silchar	28	0.0768	0.0844	0.0491	0.52
4.	Lengpui	13	0.0622	0.2307	0.0974	1.14
5.	Dhubri	22	0.1306	0.1365	0.0649	2.23
6.	Itanagar	26	0.0826	0.3286	0.2127	1.25
7.	Jorhat	25	0.0773	-0.0738	-0.1000	1.50
8.	Passighat	30	0.0954	0.1695	0.1244	0.52
9.	Guwahati	30	0.0793	0.2080	0.0972	0.26
10	Mohanbari	30	0.0804	-0.0329	-0.0762	0.95
11.	Lakhimpur	30	0.0778	0.1572	0.0158	0.71
12.	Shilling	30	0.0947	0.1416	0.1298	1.13

Table 1. Discordancy measures of each sites of the NE region using TL-moment.

Heterogeneity Measure: Hosking and Wallis [8] suggested the heterogeneity test, H, where L- moments are used to assess whether a group of stations may reasonably be treated as belonging to a homogeneous region. The proposed heterogeneity tests are based on: the L-co-efficient of variation (L-Cv), L-skewness (L-Sk) and L-kurtosis (L-Ck). These tests are defined respectively as

$$V_1 = \sqrt{\sum_{i=1}^{N} n_i \left(t_2^{(i)} - t_2^R\right)^2 / \sum_{1}^{N} n_i} \tag{19}$$

$$V_2 = \sum_{i=1}^{N} \left\{ n_i [\left(t_2^{(i)} - t_2^R \right)^2 + \left(t_3^{(i)} - t_3^R \right)^2]^{\frac{1}{2}} \right\} / \sum_{i=1}^{N} n_i \tag{20}$$

$$V_3 = \sum_{i=1}^{N} \left\{ n_i [\left(t_3^{(i)} - t_3^R \right)^2 + \left(t_4^{(i)} - t_4^R \right)^2]^{\frac{1}{2}} \right\} / \sum_{i=1}^{N} n_i \tag{21}$$

The regional average L-moment ratios are calculated using the following formula

$$\begin{aligned} t_2^R &= \sum_{i=1}^{N} n_i t_2^i / \sum_{i=1}^{N} n_i, \\ t_3^R &= \sum_{i=1}^{N} n_i t_3^i / \sum_{1}^{N} n_i, \\ t_4^R &= \sum_{i=1}^{N} n_i t_4^i / \sum_{1}^{N} n_i \end{aligned} \tag{22}$$

where N is the number of stations and n_i is the record length at i-th station. The heterogeneity test is then defined as

$$H_j = \frac{V_j - \mu_{V_j}}{\sigma_{V_j}} \quad j = 1, 2, 3 \tag{23}$$

where μ_{V_j} and σ_{V_j} are the mean and standard deviation of simulated V_j values, respectively. The region is acceptably homogeneous, possibly homogeneous and definitely heterogeneous with a corresponding order of L-moments according as $H < 1$, $1 \leq H < 2$ and $H \geq 2$. Same procedure has been applied for TL-moment also. From the heterogeneity measures it is found that the values of $H_1 = 0.69$, $H_2 = 0.49$ and $H_3 = 1.04$. Hence our study region can be considered as a homogeneous one.

Goodness of Fit Measures:

(a). Z-statistics criteria: The Z-test judges how well the simulated L-Skewness and L-kurtosis of a fitted distribution matches the regional average L-skewness and L-kurtosis values. According to Hosking and Wallis [9] for each selected distribution, the Z-test is calculated as follows

$$Z^{DIST} = \left(\tau_4^{Dist} - t_4^R \right) / \sigma_4 \tag{24}$$

where DIST refers to a particular distribution, τ_4^{DIST} is the L-kurtosis of the fitted distribution while the standard deviation of t_4^R is given by

$$\sigma_4 = \left[(N_{sim})^{-1} \sum_{m=1}^{N_{sim}} \left(t_4^{(m)} - t_4^R \right)^2 \right]^{1/2}$$

t_4^m is the average regional L-kurtosis and has to be calculated for the m^{th} simulated region. This is obtained by simulating a large number of kappa distribution using Monte Carlo simulations. The value of the Z-statistics is considered to be acceptable at the 90% confidence level if $\left| Z^{DIST} \right| \leq 1.64$. If more than one candidate distribution is acceptable, the one with the lowest $\left| Z^{DIST} \right|$ is regarded as the best fit distribution.

Using the same procedure for TL-moment the Z-statistics values of three distributions used for our study are given in Table 2. It has been observed that the Z-statistic value of GPA distribution is less than 1.64. Therefore, GPA distribution is identified as the best fitting distribution for rainfall frequency analysis of North-East India.

S. No.	Probability distribution	Z-Statistic
1	GLO	2.94
2	GEV	2.16
3	GPA	-0.51

Table 2. **Z-statistics values of the distribution**

(b). TL-moment ratio diagram: It is a graph of the L-skewness and L-kurtosis which compares the fit of several distributions on the same graph. According to Hosking and Wallis [9], the expression of τ_4 in term of τ_3 for an assumed distribution is given by

$$\tau_4 = \sum_{k=0}^{8} A_k \tau_3^k \tag{25}$$

where the coefficients A_k polynomial approximation. For TL-moment ratio diagram equation (25) may be used replacing L-skewness and L-kurtosis by TL-skewness and TL-kurtosis respectively. The coefficients $A_k$are calculated by Shabri et al. [14]. The L-moment ratio diagram of our study region is shown in Figure 1. It has been observed from Figure 1 that the regional average values of TL-skewness and TL-kurtosis lies nearer to the GPA distribution curve. Hence, the TL-moment ratio diagram also shows that the GPA distribution is the best fit distribution to our study area.

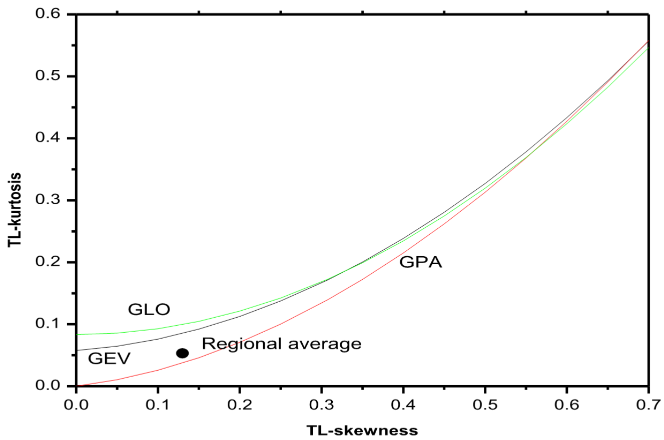

Figure 1. **TL-moment ratio diagram for NE region**

Quantile Estimation: The quantile function of the best fitting distribution GPA is given by

$$Q(F) = \left[+ \frac{\alpha}{k} \left\{ 1 - (1 - F)^k \right\} \right] \tag{26}$$

where $Q(F)$ is the quantile estimate or growth factor at return period T, $F = 1 - 1/T$, ξ, a, k are the parameters. The parameters of the GPA distribution are given in Table 3. Substituting parameters in the quantile function of GPA distribution regional growth factors are calculated. Calculated growth factors are given in Table 4.

Best fit distribution	Parameters		
	Location(ξ)	Scale(a)	Shape(k)
GPA	0.656	0.510	0.365

Table 3. **Parameters of best fitting distribution**

Best fit distribution	Return period (in years)				
	2	10	20	100	1000
GPA	0.968	1.451	1.586	1.794	1.942

Table 4. **Quantile estimates by using best fitting distribution**

3. Regional Rainfall Frequency Analysis Using LQ-moment

The procedure discussed in section 2.1 is also employed for LQ-moment in the same manner.

Screening of data: For discordancy test L-cv, L-skewness and L-kurtosis are replaced by LQ-cv, LQ-skewness and LQ-kurtosis respectively. Calculated D_i values are given in Table 5. From Table 1 it is observed that the D_i values of 12 stations of the study region are less than the critical value 2.757. Hence all the data of 12 stations can be considered for the study.

S.No	Name of sites	No. of observation	LQ-CV	LQ-Skewness	LQ-Kurtosis	D_i
1.	Guwahati	30	0.1492	0.3960	0.1093	1.12
2.	Mohanbari	30	0.1565	-0.0545	-0.0625	0.83
3.	Silchar	28	0.1534	0.0931	0.2052	0.69
4.	Lakhimpur	30	0.1518	0.2525	-0.0085	0.75
5.	Passighat	30	0.1893	0.2302	0.2472	0.58
6.	Agartala	30	0.2032	0.2278	-0.1384	2.22
7.	Imphal	30	0.1744	0.2548	0.2233	0.14
8.	Shillong	30	0.1779	0.2374	0.3275	0.62
9.	Itanagar	26	0.1546	0.5586	0.5756	1.77
10.	Dhubri	22	0.2042	0.0123	0.0151	1.25
11.	Jorhat	25	0.1530	-0.1019	-0.1672	1.26
12.	Lengpui	13	0.1339	0.2634	0.1863	0.77

Table 5. **Discordancy measures of each sites of the NE region using LQ-moment.**

Heterogeneity Measure: The heterogeneity measures of our study region have been found to be $H_1 = -1.45$, $H_2 = 0.87$ and $H_3 = 1.77$. It has been observed from heterogeneity measures that, our study region can be considered as a homogeneous one.

Goodness of Fit Measures:

1. Z-statistics criteria: The procedure is similar as discussed in section 1.2. The Z-statistics values of five distribution used for our study are given in Table 6 It has been observed that the Z-statistic values of GEV, GNO, PE3 and GPA distributions are less than 1.64. But that of GPA distribution is the lowest. Therefore, the GPA distribution is identified as the best fitting distribution for rainfall frequency analysis of North East India.

S.No.	Probability distribution	Z-Statistics values
1	GLO	2.12
2	GEV	1.50
3	LN3	1.25
4	PE3	0.91
5	GPA	0.43

Table 6. **Z-statistics values of the distribution**

2. LQ-moment ratio diagram: The procedure is similar as discussed in section 1.2 where the coefficients A_k are tabulated by Bhuyan and Borah [2]. The LQ-moment ratio diagram of our study region is shown in Figure 2. It has been observed from Fig. 2 that the regional average values of LQ-skewness and LQ-kurtosis also lies nearer to the GPA distribution

curve. Hence, the LQ-moment ratio diagram also shows that the GPA distribution is the best fit distribution to our study area.

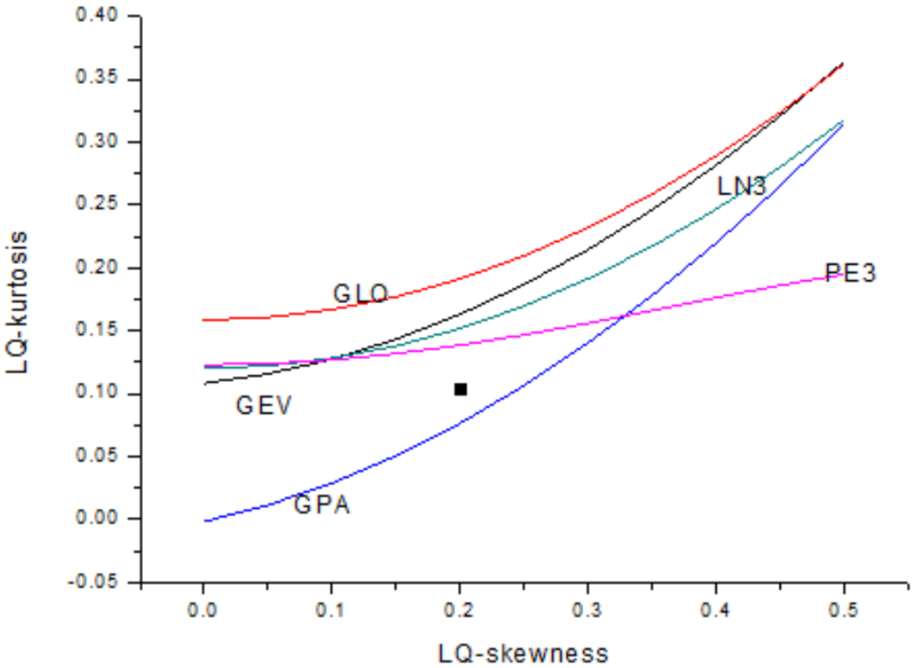

Figure 2. **LQ-moment ratio diagram for NE region**

Quantile Estimation: The parameters of the GPA distribution are given in Table 7. Substituting the parameters in the quantile function of GPA distribution given by equation (26), the growth factors are calculated. The estimated growth factors are given in Table 8.

Best fit distribution	Parameters		
	Location (ξ)	Scale (a)	Shape (k)
GPA	0.668	0.511	0.357

Table 7. **Parameters of the best fitting distribution**

Best fit Distribution	Return Periods (in year)				
	2	10	20	100	1000
GPA	0.982	1.471	1.609	1.824	1.979

Table 8. **Quantile estimates by using best fitting distribution**

4. Results and Discussion

For both TL-moment and LQ-moment methods it is observed from Table 1 and Table 5 that the D_i values of all the twelve stations are less than critical value 2.757. Therefore, all the data of twelve stations are considered for the development of regional frequency analysis. It has been observed from heterogeneity measures that for both TL-moment and LQ-moment methods, our study region can be considered as a possibly homogeneous one. Z-statistics criteria for TL-moment and TL-moment ratio diagram shows that the GPA distribution is the best fitting distribution for our study region. On the other hand, Z-statistics criteria for LQ-moment and LQ-moment ratio diagram shows that GPA distribution is the best fitting distribution for our study region. A comparative study has been done between the two methods TL-moment and LQ-moment. For this purpose, Monte Curlo simulation proposed by Meshgi and Khalili [10] are used to evaluate error between simulated and calculated growth factors at different return periods. Commonly used two error functions are relative root mean square error (RRMSE) and relative bias (RBIAS) are given by

$$RRMSE = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(\frac{Q_T^m - Q_T^c}{Q_T^c}\right)^2}$$

$$RBIAS = \frac{1}{M}\sum_{m=1}^{M}\left(\frac{Q_T^m - Q_T^c}{Q_T^c}\right)$$

where M is the total number of samples, Q_T^m and Q_T^c are the simulated quantiles of mth sample and calculated quantiles from observed data respectively. The minimum RRMSE and RBIAS values and their associated variability are used to select the most suitable probability distribution function. For this purpose, boxplots, a graphical tool introduced by Tukey [16] are used. Box plot is a simple plot of five quantities, namely, the minimum value, the 1^{st} quantile, the median, the 3^{rd} quantile, and maximum value.

This provides the location of the median and associated dispersion of the data at specific probability levels. The probability distribution with the minimum achieved median RRMSE or RBIAS values, as well as the minimum dispersion in the median RRMSE or RBIAS values, indicated by both ends of the box plot are selected as the suitable distribution. RRMSE and RBIAS values are given in Table 9 and Table 10 respectively. Table 9 and Table 10 it is observed that the RRMSE and RBIAS values of GPA distribution designated by TL-moment are less than or equal to the respective RRMSE and RBIAS values of GPA distribution designated by LQ-moment. Fig. 3 and Fig. 4 represent the boxplot of RRMSE and RBIAS values respectively.

From Figure 3 and Figure 4 it is observed that GPA distribution designated by TL-moment has the minimum median RRMSE and RBIAS values as well as minimum dispersion. Hence GPA distribution is selected as suitable and the best fitting distribution for rainfall frequency analysis of North East India. Also the TL-moment method is significantly more efficient than LQ-moment for rainfall frequency analysis of North east India.

Methods	Best fit Distributions	Return period (in years)				
		2	10	20	100	1000
LQ-moment	GPA	0.067	0.109	0.187	0.568	3.387
TL-moment	GPA	0.067	0.077	0.115	0.260	0.665

Table 9. **RRMSE values GPA distribution for TL-moment and LQ-moment method respectively.**

Methods	Best fit Distributions	Return period (in years)				
		2	10	20	100	1000
LQ-moment	GPA	0.004	0.028	0.058	0.188	0.676
TL-moment	GPA	0.001	-0.001	0.009	0.056	0.184

Table 10. **RBIAS values GPA distribution for TL-moment and LQ-moment method respectively.**

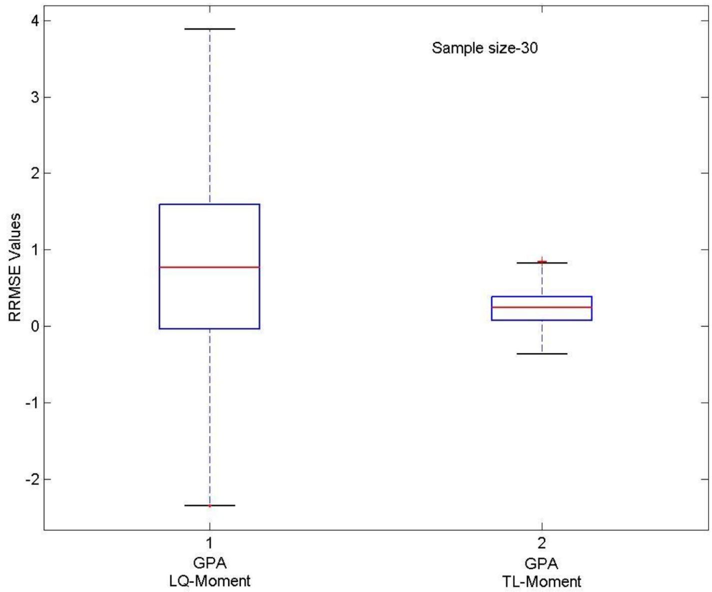

Figure 3. **Boxplot of RRMSE values**

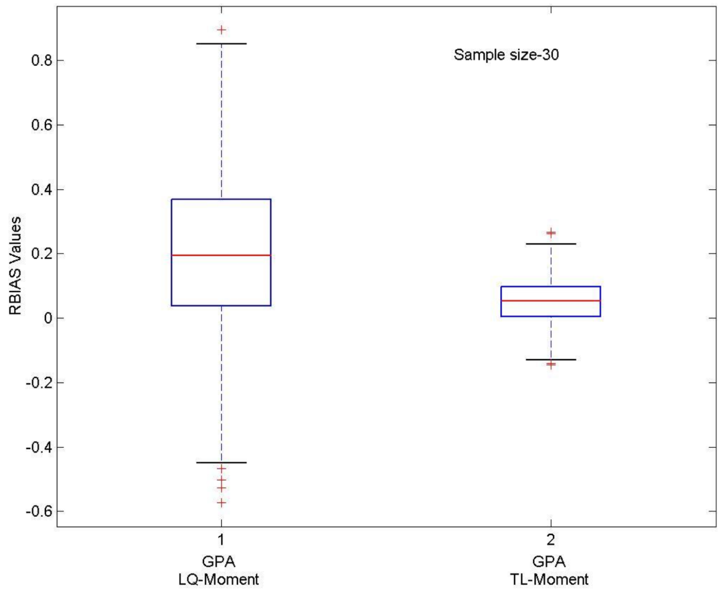

Figure 4. **Boxplot of RBIAS values**

5. Development of Model

The index flood procedure of Dalrymple [5] is used to develop regional rainfall frequency relationship. The form of regional rainfall frequency relationship or growth factor for the best fit distributions GPA can be expressed as

$$Q_T = \left[\xi + \frac{\alpha}{k}\left\{1 - (1-F)^k\right\}\right] * \mu_i \tag{27}$$

where Q_T is the maximum rainfall for return period T, $F = 1 - 1/T$, μ_i is the mean annual maximum rainfall of the ith site, ξ, α and k are the parameters of the respective distributions. Substituting the regional values of the best fit distribution based on the data of 12 gauged sites the regional rainfall frequency relationship for gauged sites of study area is expressed as:

$$Q_T = \left[0.656 + 1.397\left\{1 - (1 - \mathrm{F})^{0.365}\right\}\right] * \mu_i \quad (28)$$

6. Conclusion

For both the methods, TL-moment and LQ-moment Discordancy measure shows that data of all gauging sites of our study area are suitable for using regional frequency analysis. By using the TL-moment and LQ-moment based homogeneity test, the region has been found to be homogeneous. Using TL-moment ratio diagram and Z-statistic it is found that GPA distribution is the best fitting distribution for rainfall frequency analysis of North East India. Also using LQ-moment ratio diagram and Z-statistic it is found that GPA is the best fitting distribution for rainfall frequency analysis of North East India. Using RRMSE and RBIAS values it can be concluded that GPA distribution for TL-moment is more suitable distribution for rainfall frequency analysis of North East India. Also the TL-moment method is significantly more efficient than LQ-moment for rainfall frequency analysis of North east India. The regional rainfall frequency relationship for gauged stations has been developed for the region and can be used for estimation of rainfalls of desired return periods.

References

[1] K.Adamowski, Y.Alia and P.J.Pilin, *Regional rainfall distribution for Canada*, Atmospheric Research, 42(1-4)(1996), 75-88.

[2] A.Bhuyan and M.Borah, *LQ-moments for regional flood frequency analysis: a case study for the North-bank Region of the Brahmaputra river, India*, Journal of Modern Applied Statistical Methods, 10(2)(2011), 730-740.

[3] D.J.Bora, M.Borah and A.Bhuyan, *Regional analysis of maximum rainfall using L-moment and LH-moment: a comparative case study for the North East India*, Mausam, 68(3)(2017), 451-462.

[4] D.J.Bora, M.Borah and A.Bhuyan, *Regional analysis of maximum rainfall using L-moment and LQ-moment: a comparative case study for the North East India*, International Journal of Applied Mathematics & Statistical Sciences, 5(6)(2016), 79-90.

[5] T.Dalrymple, *Flood frequency analysis*, Water Supply Paper, no.1543 A U.S. Geological Survey, Reston, Va, (1960).

[6] S.Deka, M.Borah and S.C.Kakaty, *Statistical analysis of annual maximum rainfall in North-East India: an application of LH-moments*, Theoretical and Applied Climatology, 104(1)(2011), 111-122.

[7] E.A.H.Elamir and A.H.Seheult, *Trimmed L-moments*, Computational Statistics and Data Analysis, 43(2003), 299314.

[8] J.R.M.Hosking and J.R.Wallis, *Some statistics useful in regional frequency analysis*, Water Resources Research, 29(2)(1993(1993), 271-281.

[9] J.R.M.Hosking and J.R.Wallis, *Regional frequency analysis- An approach based on L-moments*, Cambridge University Press, New York, (1997).

[10] A.Meshgi and D.Khalili, *Comprehensive evaluation of regional flood frequency analysis by L- and LH-moments. I. A re-visit to regional homogeneity*, Stochastic Environmental Research and Risk Assessment, 23(2009), 119135.

[11] G.S.Mudholkar and A.D.Hutson, *LQ moments: analogs of L-Moments*, Journal of Statistical Planning and Inference, 71(1998), 191-208.

[12] D.Norbiato, M.Borga, M.Sangati and F.Zanon, *Regional frequency analysis of extreme precipitation in the eastern Italian Alps and the August 29, 2003 flash flood*, Journal of Hydrology, 345(3-4)(2007), 149-166.

[13] A.O.Ogunlela, *Stochastic analysis of rainfall events in Ilorin, Nigeria*, Journal of Agricultural Research and Development, 1(2001), 39-50.

[14] A.B.Shabri, Z.M.Doud and N.M.Ariff, *Regional analysis of annual maximum rainfall using TL-moments method*, Theoretical and Applied Climatology, 104(2011), 561570.

[15] C.Trefry, D.Watkins and D.Johnson, *Regional rainfall frequency analysis for the state of Michigan*, Journal of Hydrologic Engineering, 10(6)(2005), 437-449.

[16] J.W.Tukey, *Exploratory data analysis*, Addision-Wesley, Reading, (1977).