International Journal of Mathematics And its Applications
Volume 5, Issue 4–C (2017), 347–351.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

(1, N)-Arithmetic Labelling in Multimedia and Network Technology

Research Article

V. Ramachandran[1]*

1 PG and Research Department of Mathematics, Mannar Thirumalai Naicker College, Madurai, Tamil Nadu, India.

Abstract: A graph G is said to be $(1, N)$-Arithmetic (where N is a positive integer) if there is a function f from the vertex set of G to $\{0, 1, N, (N+1), 2N, (2N+1), \ldots, N(q-1), N(q-1)+1\}$ in such a way that (i) f is $1-1$ (ii) f induces a bijection f^* from the edge set of G to $\{1, N+1, 2N+1, \ldots, N(q-1)+1\}$ where $f^*(uv) = |f(u) - f(v)|$. In this paper we investigate the experimental results and point out that the new image scrambling method established on $(1, N)$-Arithmetic labelling of tree can deliver a high level security owing to the strong anomaly of sorting transformation.

MSC: 05C78.

Keywords: Graceful, $(1, N)$-Arithmetic, image scrambling.

1. Introduction

The field of Graph theory plays a vital role in various fields. One of the most important areas in graph theory is Graph Labelling which is used in many applications like coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, data base management [5, 6]. Information security plays a vital role in all communication fields where the data has to be secured from eavesdropper [10]. Information hiding is a technique to hide the data which is classified into steganography and watermarking. The main objective of steganography is un-delectability, robustness and ability to retrieve back the full original data. Now-a day's image scrambling technique draws a great attention as it is used to protect the data by scrambling the image in confusable or disordered format. This paper gives an overview of $(1, N)$-Arithmetic of graphs in heterogeneous fields to some extent but mainly focuses on the multimedia and communication networks.

A.Rosa [18] in 1967 introduced graceful labelling. A function f is called a graceful labelling of a graph G with q edges if f is an injection from the vertices of G to the set $\{0, 1, 2, \ldots, q\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct. In 1991, R. B. Gnanajothi introduced Odd graceful labelling [13]. A graph G with q edges is said to be an Odd graceful if there exists an injection $f : V(G) \to \{0, 1, 2, \ldots, 2q-1\}$ such that when each edge $xy \in E(G)$ is assigned the label $|f(x) - f(y)|$ the resulting edge labels are $\{1, 3, 5, \ldots, 2q-1\}$. In 2002, C. Sekar introduced one modulo three graceful labelling [21]. A graph G is said to be one modulo three graceful if there is a function f from the vertex set of G to $\{0, 1, 3, 4, \ldots, 3(q-1), 3q-2\}$ in such a way that (i) f is $1-1$ (ii) f induces a bijection f^* from the edge set

* *E-mail: me.ram111@gmail.com*

of G to $\{1, 4, 7, \ldots, 3q-2\}$ where $f^*(uv) = |f(u) - f(v)|$. V.Ramachandran and C.Sekar [19] generalize these three concepts as one modulo N graceful labelling of graphs where N is any positive integer. V.Ramachandran and C.Sekar introduced [20] $(1, N)$-Arithmetic graph. A graph G is said to be $(1, N)$-Arithmetic (where N is a positive integer) if there is a function f from the vertex set of G to $\{0, 1, N, (N+1), 2N, (2N+1), \ldots, N(q-1), N(q-1)+1\}$ in such a way that (i) f is $1-1$ (ii) f induces a bijection f^* from the edge set of G to $\{1, N+1, 2N+1, \ldots, N(q-1)+1\}$ where $f^*(uv) = |f(u) - f(v)|$. In the case $N = 2$, the labelling is odd graceful and in the case $N = 1$ the labelling is graceful.

Joseph A. Gallian [12] surveyed numerous graph labelling methods. B. D. Acharya and S. M. Hegde introduced (k, d)-arithmetic graphs. A (p, q)-graph G is said to be (k, d)-arithmetic if its vertices can be assigned distinct nonnegative integers so that the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression $k, k+d, k+2d, \ldots, k+(q-1)d$. Putting $k = 1$ and $d = N$, a (k, d)-arithmetic graphs can be called as $(1, N)$-Arithmetic graphs. Let us assign the values from $\{0, 1, N, (N+1), 2N, (2N+1), \ldots, N(q-1), N(q-1)+1\}$ to the vertices of G through the function $f : V(G) \to \{0, 1, N, (N+1), 2N, (2N+1), \ldots, N(q-1), N(q-1)+1\}$. In this situation the induced mapping f^* to the edges is given by $f^*(uv) = f(u) + f(v)$. If the values of $f(u) + f(v)$ are $1, 1+N, 1+2N, \ldots, 1+N(q-1)$ all distinct, then we call the labelling of vertices as $(1, N)$-Arithmetic labelling. In case if the induced mapping f^* is defined as $f^*(uv) = |f(u) - f(v)|$ and if the resulting edge labels are distinct and equal to $\{1, 1+N, 1+2N, \ldots, 1+N(q-1)\}$. We call it as one modulo N graceful.

2. Main Results

For the digital image I of size P*Q pixels, we make use of $(1, N)$-Arithmetic labelling of tree for the purpose of image scrambling. In order to enhance the safety concert of the image encryption process, positions of Pixels in the original image is shuffled based on $(1, N)$-Arithmetic labelling is the consignment of labels which is characterized by integers to the point of intersection. $(1, N)$-Arithmetic numbering of tree is known as $(1, N)$-Arithmetic labelling of tree. One modulo N tree can be gracefully labelled in which prompted labels are all dissimilar, which is shown in figure.

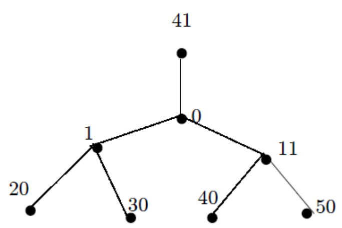


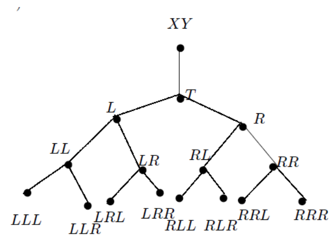
Figure 1. **(1, 5)-Arithmetic labelling**

By creating $(1, N)$-Arithmetic trees we can scramble the images in which the pixels are reflected as set of points.

V_0	V_1	V_2	V_3
V_4	V_5	V_6	V_7
V_8	V_9	V_{10}	V_{11}
V_{12}	V_{13}	V_{14}	V_{15}

Table 1. **Image of size 4*4 pixels**

The start node is a node 'XY' as V_0 and 'T' as V_1. The start node has 2 child nodes left [L] and right [R] and their corresponding sub-child respectively. Here we represent the pixels in an image as V_2 & V_3.



The pixel (V_4) is placed at node LL and the consecutive pixels (V_8) and (V_9) are placed at LLL and LLR. The pixel (V_5) is placed at LR and the consecutive pixels (V_{10}) and (V_{11}) are placed at LRL and LRR. The pixel (V_6) is placed at node RL and the consecutive pixels (V_{12}) and (V_{13}) are placed at RLL and RLR. The pixel (V_7) is placed at RR and the consecutive pixels (V_{14}) and (V_{15}) are placed at RRL and RRR. The shuffling of pixels is done in tree diagram.

7N+1	0	1	5N+1
N	2N	3N	4N
2N+1	3N+1	4N+1	10N+1
11N+1	8N+1	9N+1	6N+1

Table 2. **Final location of pixels**

Final scrambled sequence obtained as a result of the proposed method represents the 4*4 pixels. The above mentioned method of image scrambling may provide better scrambling results. This image scrambling method based on $(1, N)$-Arithmetic labelling of trees proves to be difficult for the eavesdropper to decode the sequence which in turn improves the security to higher level for any number of images. We can consider image I corresponds to graceful labels, image II corresponds to odd graceful labels, image III corresponds to one modulo 3 graceful labels and so on.

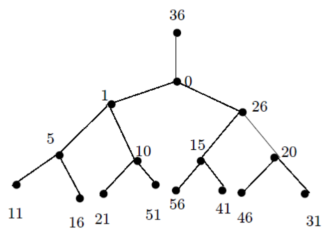


Figure 2. **(1, 5)-Arithmetic labelling**

Similarly by creating $(1, N)$-Arithmetic ladder we can scramble the images in which the pixels are reflected as set of points.

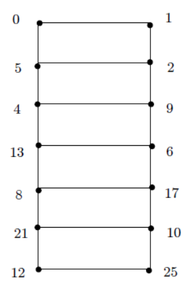


Figure 3. **(1, 2)-Arithmetic labelling of L_7**

3. Methodology

A worthy encryption method should be strong against all kinds of cryptanalytic, statistical, differential and brute-force attacks. Figure shows the experimental method of proposed image scrambling scheme established on $(1, N)$-Arithmetic labelling tree and ladder graph.

4. Conclusion

The results point out that the scrambled image cannot be decrypted correctly until the method has been known by eavesdropper, thus being awkward to decrypt the scramble image. This image scrambling method enhance the outcome of the scrambling which proves to be difficult for the eavesdropper to decode the sequence and making the system to afford a high level of security for many number of images. The experimental results show that the process is effective to scramble the image and can afford high security. Researches may get some information related to $(1, N)$-Arithmetic labelling and its applications in communication field and can get some ideas related to their field of research. In this paper, a new enhanced approach for image security using image scrambling scheme based on $(1, N)$-Arithmetic labelling of tree is proposed. The main aim of this proposal is to explore role of $(1, N)$-Arithmetic labelling in multimedia and network technology. $(1, N)$-Arithmetic labelling is powerful tool that makes things ease in various fields of networking as said above.

References

[1] Abir Awad and Abdelhakim Saadane, *Efficient Chaotic Permutations for Image Encryption Algorithms*, Proceedings of the World Congress on Engineering Vol I, (2010).

[2] M.Alfalayleh, L.Brankovic, H.Giggins and Md.Z.Islam, *Towards the graceful tree conjecture: a survey*, Proceedings of AWOCA 2004, 79 July, Ballina, Australia (2004).

[3] I.Arkut, R.Arkut and N.Ghani, *Graceful label numbering in optical MPLS networks*, Proc. SPIE, vol. 4233, pp. 18. OptiComm 2000: Optical Networking and Communications, Imrich Chlamtac; Ed (2000).

[4] J.-C.Bermond, *Graceful graphs, radio antennae and french windmills*, Graph Theory and Combinatorics, (1979), 1337.

[5] G.S.Bloom and D.F.Hsu, *On graceful digraphs and a problem in network addressing*, Congr. Numer., 35(1982), 91103.

[6] G.S.Bloom and D.F.Hsu, *On graceful directed graphs that are computational models of some algebraic systems*, Alavi, Y. (ed.) Graph Theory with Applications to Algorithms and Computers, Wiley, New York, (1985).

[7] S.E.Borujeni and M. Eshghi, *Chaotic Image Encryption Design Using Tompkins-Paige Algorithm*, Mathematical Problems in Engineering, (2009), 22 pages.

[8] S.Changgui and B.Bharat, *An efficient MPEG video encryption algorithm*, Proceedings of the symposium on reliable distributed systems, IEEE computer society Press, (1998), 381-386.

[9] M.Fujiyoshi, W.Saitou, O.Watanabe and H.Kiya, *Hierarchical encryption of multimedia contents for access control*, Proc. IEEE Int. Conf. Image Processing, (2006), 19771980.

[10] K.Eshghi and P.Azimi, *Applications of mathematical programming in graceful labeling of graphs*, J. Appl. Math., 1(2004), 18.

[11] J.A.Gallian, *A dynamic survey of graph labeling*, Electron. J. Combin., 17(2010), #DS6.

[12] R.B.Gnanajothi, *Topics in Graph theory*, Ph.D. Thesis, Madurai Kamaraj University, (1991).

[13] L.Goddyn, R.B.Richter and J.Siran, *Triangular embeddings of complete graphs from graceful labellings of paths*, J. Comb. Theory Ser. B 97(2007), 964970.

[14] S.W.Golomb, *How to number a graph*, Read, R.C. (eds.) Graph Theory and Computing. Academic Press, New York, (1972), 2337.

[15] K.Kuroiwa, M.Fujiyoshi and H.Kiya, *Codestream domain scrambling of moving objects based on DCT sign only correlation for motion JPEG movies*, Proc.IEEE Int. Conf. Image Processing, V(2007), 157160.

[16] G.Ringel, *Theory of Graphs and its Application*, Proc. Sympos. Smolenice 1963, Nakl. CSAV, Praha, 1964, vol. 162, (1963).

[17] A.Rosa, *On certain valuations of the vertices of a graph, Theory of graphs*, (International Symposium, Rome July 1966) Gordom and Breach, N.Y and Dunod paris, (1967), 349-355.

[18] V.Ramachandran and C.Sekar, *One modulo N gracefulness of Acyclic graphs*, Ultra Scientist of Physical Sciences, 25(3)(2013), 339352.

[19] V.Ramachandran and C.Sekar, *(1, N)-arithmetic graphs*, International Journal of Computers and Applications, 38(1)(2016), 5559.

[20] C.Sekar, *Studies in Graph theory*, Ph.D. Thesis, Madurai Kamaraj University, (2002).

[21] M.Sutton, *Sumable graphs labellings and their applications*, Ph.D. Thesis, Dept. Computer Science, The University of Newcastle, (2001).