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On Nano $g\beta$-closed Sets

Research Article

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Abstract: In this paper, we introduced nano $g\beta$-closed sets and use some properties relation between nano $g\beta$-closed sets.

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1. Introduction

Lellis Thivagar et al [4] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. K. Bhuvaneswari et.al [1] introduced nano generalized closed sets and A. Revathy et.al [6] introduced nano β-open sets. In this paper, we introduced nano $g\beta$-closed sets and use some properties relation between nano $g\beta$-closed sets.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1 ([5]). *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

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(2). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup\limits_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2 ([4]). *If (U, R) is an approximation space and $X, Y \subseteq U$; then*

(1). $L_R(X) \subseteq X \subseteq U_R(X)$;

(2). $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

(3). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6). $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7). $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;

(8). $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;

(9). $U_RU_R(X) = L_RU_R(X) = U_R(X)$;

(10). $L_RL_R(X) = U_RL_R(X) = L_R(X)$.

Definition 2.3 ([4]). *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $R(X)$ satisfies the following axioms:*

(1). U and $\phi \in \tau_R(X)$,

(2). The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 ([4]). *If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5 ([4]). *If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$. That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$. That is, $Ncl(H)$ is the smallest nano closed set containing H.*

Definition 2.6 ([4]). *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

(1). nano pre-open set if $H \subseteq Nint(Ncl(H))$.

(2). nano α-open set if $H \subseteq Nint(Ncl(Nint(H)))$.

(3). nano semi-open set if $H \subseteq Ncl(Nint(H))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.7 ([6]). *A subset H of a nano topological space $(U, \tau_R(X))$ is called nano β-open set if $H \subseteq Ncl(Nint(Ncl(H)))$. The complement of nano β-closed set is called nano β-open set.*

Definition 2.8. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

(1). nano g-closed [1] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(2). nano sg-closed set [2] if $Nscl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano semi-open.

(3). nano αg-closed set [7] if $N\alpha cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano open.

(4). nano gp-closed set [3] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

3. Nano $g\beta$-closed Sets

Definition 3.1. *A subset H of a nano topological space $(U, \tau_R(X))$ is called nano $g\beta$-closed if $N\beta cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano open.*

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$. Then $\wp(U) - \{b, c, d\}$ is nano $g\beta$-closed.*

Theorem 3.3. *In a space $(U, \tau_R(X))$, every nano closed set is nano $g\beta$-closed.*

Proof. Let H be a nano closed set in U. Let G be a nano open set such that $H \subseteq G$. Since H is nano closed, that is $Ncl(H) = H$, $Ncl(H) \subseteq G$ but $N\beta cl(H) \subseteq Ncl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is nano $g\beta$-closed set. □

Remark 3.4. *The converse of Theorem 3.3 are not true as seen from the following Example.*

Example 3.5. *In Example 3.2, then $\{b, d\}$ is nano $g\beta$-closed set but not nano closed.*

Theorem 3.6. *In space $(U, \tau_R(X))$, every nano α-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano α-closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano α-closed, $N\beta cl(H) \subseteq N\alpha cl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is nano $g\beta$-closed set. □

Remark 3.7. *The converse of Theorem 3.6 are not true as seen from the following Example.*

Example 3.8. *In Example 3.2, then $\{b, d\}$ is nano $g\beta$-closed set but not nano α-closed.*

Theorem 3.9. *In a space $(U, \tau_R(X))$, every nano pre-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano pre-closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano pre-closed, $N\beta cl(H) \subseteq Npcl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is a nano $g\beta$-closed set. □

Remark 3.10. *The converse of Theorem 3.9 are not true as seen from the following Example.*

Example 3.11. *In Example 3.2, then $\{b, c\}$ is nano $g\beta$-closed set but not nano pre-closed.*

Theorem 3.12. *In a space $(U, \tau_R(X))$, every nano semi-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano semi closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano semi closed, $N\beta cl(H) \subseteq Nscl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is a nano $g\beta$-closed set. □

Remark 3.13. *The converse of Theorem 3.12 are not true as seen from the following Example.*

Example 3.14. *In Example 3.2, then $\{c, d\}$ is nano $g\beta$-closed set but not nano semi-closed.*

Theorem 3.15. *In a space $(U, \tau_R(X))$, every nano g-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano g-closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano g-closed, $N\beta cl(H) \subseteq Ncl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is nano $g\beta$-closed set. □

Remark 3.16. *The converse of Theorem 3.15 are not true as seen from the following Example.*

Example 3.17. *In Example 3.2, then $\{b\}$ is nano $g\beta$-closed set but not nano g-closed.*

Theorem 3.18. *In a space $(U, \tau_R(X))$, every nano sg-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano sg-closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano sg-closed, $N\beta cl(H) \subseteq Nsgcl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is a nano $g\beta$-closed set. □

Remark 3.19. *The converse of Theorem 3.18 are not true as seen from the following Example.*

Example 3.20. *In Example 3.2, then $\{b, d\}$ is nano $g\beta$-closed set but not nano sg-closed.*

Theorem 3.21. *In space $(U, \tau_R(X))$, every nano αg-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano αg-closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano αg-closed, $N\beta cl(H) \subseteq N\alpha gcl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is a nano $g\beta$-closed set. □

Remark 3.22. *The converse of Theorem 3.21 are not true as seen from the following Example.*

Example 3.23. *In Example 3.2, then $\{b, c\}$ is nano $g\beta$-closed set but not nano αg-closed.*

Theorem 3.24. *In space $(U, \tau_R(X))$, every nano gp-closed set is nano $g\beta$-closed.*

Proof. Let H be a nano gp-closed set in U such that $H \subseteq G$, where G is nano open. Since H is nano gp-closed, $N\beta cl(H) \subseteq Ngpcl(H) \subseteq G$. Therefore $N\beta cl(H) \subseteq G$. Hence H is a nano $g\beta$-closed set. □

Remark 3.25. *The converse of Theorem 3.24 are not true as seen from the following Example.*

Example 3.26. *In Example 3.2, then $\{b, d\}$ is nano $g\beta$-closed set but not nano gp-closed.*

Theorem 3.27. *Let H be a nano $g\beta$-closed subset of $(U, \tau_R(X))$. Then $N\beta cl(H) - H$ does not contain any non-empty nano closed sets.*

Proof. Let P in nano closed such that $P \subseteq N\beta cl(H) - H$. Since $U - P$ is nano open, $H \subseteq U - P$ and P is nano $g\beta$-closed, it follows that $N\beta cl(H) \subseteq U - P$ and thus $P \subseteq U - N\beta cl(H)$. This implies that $P \subseteq (U - N\beta cl(H)) \cap (N\beta cl(H) - H) = \phi$ and hence $P = \phi$. □

Theorem 3.28. *Let H a nano $g\beta$-closed set. Then H is nano β-closed $\Longleftrightarrow$ $\beta cl(H) - H$ is nano closed.*

Proof. Let H be a nano $g\beta$-closed set. If H is nano β-closed, then we have $N\beta cl(H) - H = \phi$ which is nano closed set. Conversely, let $N\beta cl(H) - H$ be a nano closed. Then, by Theorem 3.27, $N\beta cl(H) - H$ does not contain any non-empty nano closed subset and since $N\beta cl(H) - H$ is nano closed subset of itself, then $N\beta cl(H) - H = \phi$. This implies that $H = N\beta cl(H)$ and so H is β-closed set. □

Theorem 3.29. *Let H be a nano open and nano $g\beta$-closed set. Then $H \cap P$ is nano $g\beta$-closed whenever P in nano β-closed.*

Proof. Since H is nano $g\beta$-closed and nano open, then $N\beta cl(H) \subseteq H$ and thus H is nano β-closed. Hence $H \cap P$ is nano β-closed in U which implies that $H \cap P$ is nano $g\beta$-closed in U. □

Theorem 3.30. *If H is a nano $g\beta$-closed set and Q is any set such that $H \subseteq Q \subseteq N\beta cl(H)$, then Q is a nano $g\beta$-closed set.*

Proof. Let $Q \subseteq G$ where G is nano open set. Since H is nano $g\beta$-closed and $H \subseteq G$, then $N\beta cl(H) \subseteq G$ and also $N\beta cl(G) = N\beta cl(Q)$. Therefore $N\beta cl(Q) \subseteq G$ and hence Q is a nano $g\beta$-closed set. □

Remark 3.31. *If H and Q are nano $g\beta$-closed, union of two subset need not be nano $g\beta$-closed.*

Example 3.32. *In Example 3.2, then $H = \{b, c\}$ and $Q = \{d\}$ is nano $g\beta$-closed sets. Clearly $H \cup Q = \{b, c, d\}$ is not nano $g\beta$-closed set.*

Theorem 3.33. *Let $H \subseteq V \subseteq U$ and suppose that H is nano $g\beta$-closed in U, then H is nano $g\beta$-closed relative to V.*

Proof. Given that $H \subseteq V \subseteq U$ and H is nano $g\beta$-closed in U. To show that H is nano $g\beta$-closed relative to V. Let $H \subseteq V \cap G$, where G is nano open in U. Since H is nano $g\beta$-closed, $H \subseteq G$, implies $N\beta cl(H) \subseteq G$. It follows that $V \cap N\beta cl(H) \subseteq V \cap G$. Thus H is nano $g\beta$-closed relative to V. □

Theorem 3.34. *Assume that $Q \subseteq H \subseteq U$, Q is nano $g\beta$-closed set relative to H and that H is both nano open and nano $g\beta$-closed subset of U, then Q is nano $g\beta$-closed set relative to U.*

Proof. Let $Q \subseteq G$ and G be a nano open set in U. But given that $Q \subseteq H \subseteq U$, therefore $Q \subseteq HandQ \subseteq G$. This implies $Q \subseteq H \cap G$. Since Q is nano $g\beta$-closed relative to H, $H \cap N\beta cl(Q) \subseteq H \cap G$. Implies $(H \cap N\beta cl(Q)) \subseteq G$. Thus $(H \cap N\beta cl(Q)) \subseteq (N\beta cl(Q))^c \subseteq G \subseteq (N\beta cl(Q))^c$. Implies $H \subseteq (N\beta cl(Q))^c \subseteq G \subseteq (N\beta cl(Q))^c$. Since H is nano $g\beta$-closed in U, we have $N\beta cl(H) \subseteq G \subseteq (N\beta cl(Q))^c$. Also $Q \subseteq H$ implies $N\beta cl(Q) \subseteq N\beta cl(H)$. Thus $N\beta cl(Q) \subseteq N\beta cl(H) \subseteq G \subseteq (N\beta cl(Q))^c$. Therefore $N\beta cl(Q) \subseteq G$, since $N\beta cl(Q)$ is not contained in $N\beta cl(Q)^c$. Thus Q is nano $g\beta$-closed set relative to U. □

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