



Finding Fuzzy Critical Path using Revised PILOT Method of Ranking

Research Article

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Abstract: In this paper a new ranking method called “Revised Point of Intersection of Legs of Trapezium (PILOT)” have been proposed used to find the fuzzy critical path, fuzzy earliest, fuzzy latest times and fuzzy floats in a project management problem. It is quiet interest to study in calculating critical path using generalized trapezoidal fuzzy number. A numerical example is given for justifying the proposed method.

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1. Introduction

The process of production management involves increase of profit by reducing the cost with the values imprecise, the fuzzy set theory could be applied appropriately. The Critical Path Method (CPM) is a technique of project modelling introduced by Morgan R. Walker and James E. Kelley in the year 1950. Critical path method plays a crucial role in project management. Many business firm are applying the concept of project management in order to maximizing the resource utilization and in reducing the overall cost properly. Fuzzy sets were introduced by Zadeh and Dieter Klaua in 1965 to denote, influence data and information holding nonstatistical uncertainties [15]. Since the parameters engaged in this process uncertainty, we can use fuzzy environment. Bellman and Zadeh proposed the concepts of decision making in fuzzy environments [1].

The process of production management involves maximizing of profit by minimizing the cost with the values imprecise; the fuzzy set theory could be applied appropriately. A fuzzy critical path problem is formulated with fuzzy quantities of activities and duration. Many of the existing methods are available to find fuzzy critical path. Yao and Lin [14] studied fuzzy critical path method based on signed distance ranking of fuzzy numbers in 2000. Chanas and Zelinski [2] proposed a method Critical path analysis in the network with fuzzy activity times in 2001. Liu [5] studied Fuzzy activity times in critical path and project crashing problems in 2003. Chen [3] discussed analysis of critical paths in a project network with fuzzy activity times in 2007. Chen and Hsueh [4] discussed A simple approach to fuzzy critical path analysis in project

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networks, Applied Mathematical Modelling in 2008. Shankar, Sireesha and Rao [6] proposed an analytical method for finding critical path in a fuzzy project network in 2010.

Since fuzzy numbers are provided by possibility distribution, it is tough to arrange clearly the ascending or descending order. A right method for ordering the fuzzy numbers is by the use of a ranking function. A ranking function is a function $\mathfrak{F} : R \rightarrow R$ which maps each fuzzy number into the real line, where a natural order exists. There are so many ranking methods available, nowadays. Among them, the notable procedures are Lexicographic screening procedure discovered by Wang, Wang, and Lung (2005) [12], Area between centroid and its original point method [13] by Wang and Lee (2008); SD of PILOT procedure [7]; Area method [8]; A Revised approach of PILOT ranking procedure [9]; A New Ranking Method based on Difference of Subinterval [10] and Sub Interval Average Method for Ranking of Linear Fuzzy Numbers [11] by Stephen Dinagar and Kamalanathan. We have proposed a new ranking method “Revised Point of Intersection of Legs of Trapezium (PILOT) method” [9] to defuzzify generalized trapezoidal fuzzy number. The rest of the article is organized as follows: the next section provides some basic concepts of fuzzy numbers; Section 3 explains the ranking procedure; the algorithm is explained in Section 4; Section 5 is provided with the numerical examples for solving fuzzy critical path problems.

2. Preliminaries

Definition 2.1 (Fuzzy set). A fuzzy set A in a nonempty set X is categorized by its membership function $\mu_A(x) \rightarrow [0, 1]$ and $\mu_A(x)$ is meant as the degree of membership of element x in fuzzy set A for each x belongs to X .

Definition 2.2. A fuzzy number A is a fuzzy set of the real line with a normal, convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by F .

Definition 2.3. A $GTrFN \tilde{A} = (a_1, b_1, c_1, d_1; w)$ is a fuzzy set of the real line R whose membership function $\mu_{\tilde{A}}(x) : R \rightarrow [0, w]$ is defined as

$$\mu_{\tilde{A}}^w(x) = \begin{cases} \mu_{L\tilde{A}}^w(x) = w \left(\frac{x-a_1}{b_1-a_1} \right), & \text{for } a_1 \leq x \leq b_1 \\ w, & \text{for } b_1 \leq x \leq c_1 \\ \mu_{R\tilde{A}}^w(x) = w \left(\frac{d_1-x}{d_1-c_1} \right), & \text{for } c_1 \leq x \leq d_1 \\ 0, & \text{Otherwise} \end{cases}$$

where $a_1 < b_1 < c_1 < d_1$ and $w \in (0, 1]$

Note 2.4. 1. \tilde{A} is a convex fuzzy set. It will be normalized for $w = 1$.

2. If $w = 1$, the generalized trapezoidal fuzzy number \tilde{A} is called Trapezoidal Fuzzy Number (TrFN) and is denoted as

$$\tilde{A} = (a_1, b_1, c_1, d_1).$$

3. If $a_1 = b_1$ and $c_1 = d_1$, then \tilde{A} is called crisp interval $[a_1, d_1]$.

4. If $b_1 = c_1$, then \tilde{A} is called a Generalized Triangular Fuzzy Number (GTFN) as $\tilde{A} = (a_1, b_1, c_1; w)$.

5. If $b_1 = c_1$, $w = 1$ then it is called Triangular Fuzzy Number (TFN) as $\tilde{A} = (a_1, b_1, c_1)$.

6. If $a_1 = b_1 = c_1 = d_1$ and $w = 1$, then \tilde{A} is called a real number.

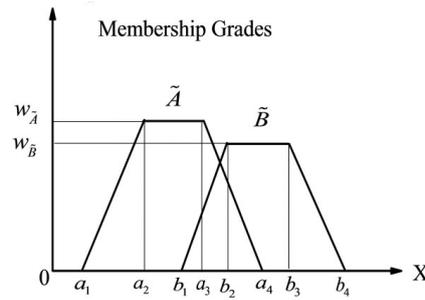


Figure 1. Two generalized trapezoidal fuzzy numbers \tilde{A} and \tilde{B} .

Fig. 1 shows two different generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1, b_1, c_1, d_1; w_{\tilde{A}})$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_{\tilde{B}})$ which denote two different decision maker’s opinions $w_{\tilde{A}}$ and $w_{\tilde{B}}$. Algebraic operations for *GTrFNs* are given by Property 2.5 to Property 2.8 where all the fuzzy numbers are positive [13].

Property 2.5. If $\tilde{A} = (a_1, b_1, c_1, d_1; w_{\tilde{A}})$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_{\tilde{B}})$, then

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_{\tilde{A}}, w_{\tilde{B}})).$$

Property 2.6. If $\tilde{A} = (a_1, b_1, c_1, d_1; w_{\tilde{A}})$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_{\tilde{B}})$, then

$$\tilde{A} \ominus \tilde{B} = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2; \min(w_{\tilde{A}}, w_{\tilde{B}})).$$

Property 2.7. If $\tilde{A} = (a_1, b_1, c_1, d_1; w_{\tilde{A}})$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_{\tilde{B}})$, then

$$\tilde{A} \otimes \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2; \min(w_{\tilde{A}}, w_{\tilde{B}})).$$

Property 2.8. If $\tilde{A} = (a_1, b_1, c_1, d_1; w_{\tilde{A}})$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_{\tilde{B}})$, then

$$\tilde{A} \div \tilde{B} = \left(\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}, \frac{d_1}{d_2}; \min(w_{\tilde{A}}, w_{\tilde{B}}) \right).$$

3. Ranking Procedure

In Fig. 2, the distance between the origin and the PILOT is shortest distance of PILOT [9], which is mentioned by line OP and it is calculated as follows:

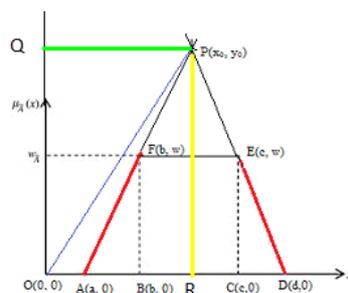


Figure 2. Area Covered by PILOT of a Generalized Trapezoidal Fuzzy Number $\tilde{A} = (a, b, c, d; w)$.

The area covered by the PILOT is derived as follows. The line OP makes 90° with y-axis when it meets at Q and with y-axis when it meets at R. It is shown in Figure 2.

Now the area of the rectangle ORPQ = OQ · OR

$$\begin{aligned} &= x_0 \cdot y_0 \\ &= \frac{(bd - ac)}{[(b + d) - (a + c)]} \cdot \frac{w(d - a)}{[(b + d) - (a + c)]} \\ \mathfrak{R}(A) &= \frac{(bd - ac)w(d - a)}{[(b + d) - (a + c)]^2} \end{aligned} \tag{1}$$

is the ranking function that area covered by PILOT $P(x_0, y_0)$.

4. Method of Finding Fuzzy Critical Path

A fuzzy project network is an acyclic digraph, where the vertices represent events, and the directed edges represent the activities, to be performed in a project. We denote this fuzzy project network by $\tilde{N} = (\tilde{V}, \tilde{A}, \tilde{T})$. Let $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, \dots, \tilde{v}_n\}$ be the set of fuzzy vertices (events), where \tilde{v}_1 and \tilde{v}_n are the tail and head events of the project, and each \tilde{v}_i belongs to some path from \tilde{v}_1 to \tilde{v}_n . Let $\tilde{A} \in (\tilde{V} \times \tilde{V})$ be the set of directed edges $A = \{\tilde{a}_{ij} = (\tilde{v}_i, \tilde{v}_j) / \forall \tilde{v}_i, \tilde{v}_j \in \tilde{V}\}$, that represents the activities to be performed in the project. Activity \tilde{a}_{ij} is then represented by one, and only one, arrow with a tail event \tilde{v}_i and a head event \tilde{v}_j . For each activity \tilde{a}_{ij} , a fuzzy number $\tilde{t}_{ij} \in \tilde{T}$ is defined as the fuzzy time required for the completion of \tilde{a}_{ij} . A critical path is a longest path from the initial event \tilde{v}_1 to the terminal event to \tilde{v}_n of the project network, and an activity \tilde{a}_{ij} on a critical path is called a critical activity.

Notations

- \tilde{t}_{ij} : The fuzzy activity time of activity \tilde{a}_{ij}
- $\tilde{E}S_j$: The earliest fuzzy time of event \tilde{v}_j
- $\tilde{L}S_i$: The latest fuzzy time of event \tilde{v}_i
- \tilde{T}_{ij} : The total float of fuzzy activity \tilde{a}_{ij}
- P_i : The i th path of the fuzzy project network
- P : The set of all paths in a fuzzy project network
- CPM (P_k): The fuzzy completion time of path P_k in a fuzzy project network.

4.1. Algorithm to Find the Critical Path of a Project

- Step 1** Identify Fuzzy activities in a fuzzy project.
- Step 2** Establish precedence relationships of all fuzzy activities by applying a fuzzy ranking function.
- Step 3** Construct the fuzzy project network with trapezoidal fuzzy numbers as fuzzy activity times.
- Step 4** Let $\tilde{E}S_i$ be the earliest fuzzy event time and $\tilde{L}S_i$ be the latest fuzzy event time for the initial event \tilde{v}_1 of the project network and assume that $\tilde{E}S_1 = \tilde{L}S_1 = \tilde{0}$. Compute the earliest fuzzy event time $\tilde{E}S_j$ of the event \tilde{v}_j by using the formula $\tilde{E}S_j = \max_{i \in N: i \rightarrow j} \{\tilde{E}S_i + \tilde{t}_{ij}\}$.
- Step 5** Let $\tilde{E}S_n$ be the earliest fuzzy event time and $\tilde{L}S_n$ be the latest fuzzy event time for the terminal event \tilde{v}_n of the project network and assume that $\tilde{E}S_n = \tilde{L}S_n$. Compute the latest fuzzy event $\tilde{L}S_i$ by using the following equation $\tilde{L}S_i = \min_{i \in N} \{\tilde{L}S_j - \tilde{t}_{ij}\}$.

Step 6 Compute the total float \tilde{T}_{ij} of each fuzzy activity \tilde{a}_{ij} by using the following equation $\tilde{T}_{ij} = \{\tilde{L}S_j - \tilde{E}S_i - \tilde{t}_{ij}\}$.

Step 7 If $\tilde{T}_{ij} = \tilde{0}$, then the activity \tilde{a}_{ij} is said to be a Fuzzy critical activity. That is activities with zero total float are called Fuzzy critical activities, and are always found on one or more Fuzzy critical paths.

Step 8 The length of the longest Fuzzy critical path from the start of the fuzzy project to its finish is the minimum time required to complete the Fuzzy Project. This (or these) Fuzzy critical path(s) determine the minimum fuzzy project duration.

5. Numerical Example

Suppose that there is a project network, as in Fig. 3, with the set of events $\tilde{V} = \{1, 2, 3, 4, 5, 6\}$, the fuzzy activity time for each activity as shown in Table 1. All the durations are in hours.

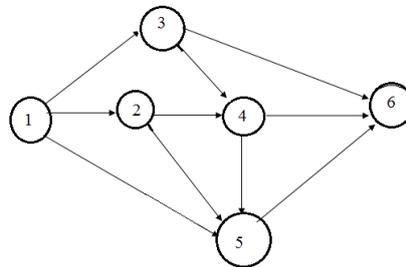


Figure 3.

Fuzzy activity	Fuzzy Activity Duration
(1-2)	(0, 5, 10, 15; 0.3)
(1-3)	(10, 15, 20, 30; 0.4)
(1-5)	(5, 15, 20, 25; 0.2)
(2-4)	(10, 20, 30, 40; 0.3)
(2-5)	(20, 30, 40, 50; 0.8)
(3-4)	(10, 15, 20, 25; 0.6)
(3-6)	(0, 5, 15, 20; 0.4)
(4-5)	(1, 4, 7, 10; 0.2)
(4-6)	(2, 6, 10, 14; 0.2)
(5-6)	(3, 8, 13, 17; 0.3)

Table 1. Fuzzy Activity with Duration.

To find Earliest Start, Earliest Finish, Latest Start, Latest Finish, Float.

To find earliest start fuzzy time

$$\begin{aligned}
 F\tilde{E}S_1 &= (0, 0, 0, 0; 0.1) \\
 F\tilde{E}S_2 &= F\tilde{E}S_1 + \tilde{a}_{12} = (0, 0, 0, 0; 0.1) + (0, 5, 10, 15; 0.3) = (0, 5, 10, 15; 0.1) \\
 F\tilde{E}S_3 &= F\tilde{E}S_1 + \tilde{a}_{13} = (0, 0, 0, 0; 0.1) + (10, 15, 20, 30; 0.4) = (10, 15, 20, 30; 0.1) \\
 F\tilde{E}S_4 &= \max\{[F\tilde{E}S_3 + \tilde{a}_{34}], [F\tilde{E}S_2 + \tilde{a}_{24}]\} \\
 &= \max\{[(10, 15, 20, 30; 0.1) + (10, 15, 20, 25; 0.6)], [(0, 5, 10, 15; 0.1) + (10, 20, 30, 40; 0.3)]\} \\
 &= \max\{(20, 30, 40, 55; 0.1) + (10, 25, 40, 55; 0.1)\}
 \end{aligned}$$

By using revised SD of PILOT Ranking Procedure in Eq. (1), we have

$$R\{(20, 30, 40, 55; 0.1)\} = 4.76$$

$$R\{(10, 25, 40, 55; 0.1)\} = 4.875$$

$$R\{(20, 30, 40, 55; 0.1)\} < R\{(10, 25, 40, 55; 0.1)\}$$

$$\Rightarrow F\tilde{E}S_4 = (10, 25, 40, 55; 0.1)$$

$$\begin{aligned} F\tilde{E}S_5 &= \max\{[F\tilde{E}S_1 + \tilde{a}_{15}], [F\tilde{E}S_2 + \tilde{a}_{25}], [F\tilde{E}S_4 + \tilde{a}_{45}]\} \\ &= \max\left\{[(0, 0, 0, 0; 0.1) + (5, 15, 20, 25; 0.2)], [(0, 5, 10, 15; 0.1) + (20, 30, 40, 50; 0.8)], \right. \\ &\quad \left. [(10, 25, 40, 55; 0.1) + (1, 4, 7, 10; 0.2)]\right\} \\ &= \max\{(5, 15, 20, 25; 0.2), (20, 35, 50, 65; 0.1), (11, 29, 47, 65; 0.1)\} \\ &= (20, 35, 50, 65; 0.1) \end{aligned}$$

$$\begin{aligned} F\tilde{E}S_6 &= \max\{[F\tilde{E}S_3 + \tilde{a}_{36}], [F\tilde{E}S_4 + \tilde{a}_{46}], [F\tilde{E}S_5 + \tilde{a}_{56}]\} \\ &= \max\left\{[(10, 15, 20, 30; 0.1) + (0, 5, 15, 20; 0.4)], [(10, 25, 40, 55; 0.1) \right. \\ &\quad \left. + (2, 6, 10, 14; 0.2)], [(20, 35, 50, 65; 0.1) + (3, 8, 13, 17; 0.3)]\right\} \\ &= \max\{(10, 20, 35, 50; 0.1), (12, 31, 50, 69; 0.1), (23, 43, 63, 82; 0.1)\} \\ &= (23, 43, 63, 82; 0.1) \end{aligned}$$

To find latest finish

$$F\tilde{L}F_6 = F\tilde{E}S_6 = (23, 43, 63, 82; 0.1)$$

$$F\tilde{L}F_5 = F\tilde{L}F_6 - \tilde{a}_{56} = (23, 43, 63, 82; 0.1) - (3, 8, 13, 17; 0.3) = (20, 35, 50, 65; 0.1)$$

$$\begin{aligned} F\tilde{L}F_4 &= \min\{F\tilde{L}F_6 - \tilde{a}_{46}, F\tilde{L}F_5 - \tilde{a}_{45}\} \\ &= \min\{(23, 43, 63, 82; 0.1) - (2, 6, 10, 14; 0.2), [(20, 35, 50, 65; 0.1) - (1, 4, 7, 10; 0.2)]\} \\ &= \min\{(21, 37, 53, 68; 0.1), (19, 31, 43, 55; 0.1)\} = (19, 31, 43, 55; 0.1) \end{aligned}$$

$$\begin{aligned} F\tilde{L}F_3 &= \min\{[F\tilde{L}F_6 - \tilde{a}_{36}], [F\tilde{L}F_4 - \tilde{a}_{34}]\} \\ &= \min\{[(23, 43, 63, 82; 0.1) - (0, 5, 15, 20; 0.4)], [(19, 31, 43, 55; 0.1) - (10, 15, 20, 25; 0.6)]\} \\ &= \min\{(23, 38, 48, 62; 0.1), (9, 16, 23, 30; 0.1)\} = (9, 16, 23, 30; 0.1) \end{aligned}$$

$$\begin{aligned} F\tilde{L}F_2 &= \min\{[F\tilde{L}F_5 - \tilde{a}_{25}], [F\tilde{L}F_4 - \tilde{a}_{24}]\} \\ &= \min\{[(20, 35, 50, 65; 0.1) - (20, 30, 40, 50; 0.8)], [(19, 31, 43, 55; 0.1) - (10, 20, 30, 40; 0.3)]\} \\ &= \min\{(0, 5, 10, 15; 0.1), (9, 11, 13, 15; 0.1)\} = (0, 5, 10, 15; 0.1) \end{aligned}$$

$$\begin{aligned} F\tilde{L}F_1 &= \min\{[F\tilde{L}F_3 - \tilde{a}_{13}], [F\tilde{L}F_2 - \tilde{a}_{12}], [F\tilde{L}F_5 - \tilde{a}_{15}]\} \\ &= \min\left\{[(0, 5, 10, 15; 0.1) - (0, 5, 10, 15; 0.3)], [(9, 16, 23, 30; 0.1) \right. \\ &\quad \left. - (10, 15, 20, 30; 0.4)], [(20, 35, 50, 65; 0.1) - (5, 15, 20, 25; 0.2)]\right\} \\ &= \min\{(0, 0, 0, 0; 0.1), (-1, 1, 3, 0; 0.1), (15, 20, 30, 40; 0.1)\} = (0, 0, 0, 0; 0.1) \end{aligned}$$

Earliest Start, Earliest Finish, Latest Start, Latest Finish, Float are represented in the Table 2. From the Table 2, it is known that the activities involved in the critical path are 1–2, 2–5, 5–6 according to the step 7 of the algorithm. Therefore the critical path is $\{1 \rightarrow 2 \rightarrow 5 \rightarrow 6\}$ and the duration is $(0, 5, 10, 15; 0.3) + (20, 30, 40, 50; 0.8) + (3, 8, 13, 17; 0.3) = (23, 43, 63, 82; 0.3)$.

Let us verify it from the Table 2.

Path	Total Duration	Defuzzify	Rank
$P_1 = \{1 \rightarrow 3 \rightarrow 6\}$	$(10,15,20,30;0.4) + (0,5,15,20;0.4) = (10,20,35,50;0.4)$	16.64	2
$P_2 = \{1 \rightarrow 3 \rightarrow 4 \rightarrow 6\}$	$(10,15,20,30;0.4) + (10,15,20,25;0.6) + (2,6,10,14;0.2) = (22,36,50,69;0.2)$	11.94	6
$P_3 = \{1 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6\}$	$(10,15,20,30;0.4) + (10,15,20,25;0.6) + (1,4,7,10;0.2) + (3,8,13,17;0.3) = (24,42,60,82;0.2)$	14.53	4
$P_4 = \{1 \rightarrow 2 \rightarrow 4 \rightarrow 6\}$	$(0,5,10,15;0.3) + (10,20,30,40;0.3) + (2,6,10,14;0.2) = (12,31,50,69;0.2)$	12.15	5
$P_5 = \{1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6\}$	$(0,5,10,15;0.3) + (10,20,30,40;0.3) + (1,4,7,10;0.2) + (3,8,13,17;0.3) = (14,37,60,82;0.2)$	14.73	3
$P_6 = \{1 \rightarrow 2 \rightarrow 5 \rightarrow 6\}$	$(0,5,10,15;0.3) + (20,30,40,50;0.8) + (3,8,13,17;0.3) = (23,43,63,82;0.3)$	24.17	1
$P_7 = \{1 \rightarrow 5 \rightarrow 6\}$	$(5,15,20,25;0.2) + (3,8,13,17;0.3) = (8,23,33,42;0.2)$	8.29	7

Table 2. Critical Path Calculation.

From Table 2, the fuzzy critical path is $P_6 = \{1 \rightarrow 2 \rightarrow 5 \rightarrow 6\}$. The maximum fuzzy duration is the length of the critical path. The fuzzy project duration is $(23, 43, 63, 82; 0.3)$.

Activity	(1-2)	(1-3)	(1-5)	(2-4)	(2-5)	(3-4)	(3-6)	(4-5)	(4-6)	(5-6)
Duration	(0, 5, 10, 15;0.3)	(10, 15, 20, 30;0.4)	(5, 15, 20, 25;0.2)	(10, 20, 30, 40;0.3)	(20, 30, 40, 50;0.8)	(10, 15, 20, 25;0.6)	(0, 5, 15, 20;0.4)	(1, 4, 7, 10;0.2)	(2, 6, 10, 14;0.2)	(3, 8, 13, 17;0.3)
Earliest Start	(0, 0, 0, 0; 0.1)	(0, 0, 0, 0; 0.1)	(0, 0, 0, 0; 0.1)	(0, 5, 10, 15;0.1)	(0, 5, 10, 15;0.1)	(10, 15, 20, 30;0.1)	(10, 15, 20, 30;0.1)	(10, 25, 40, 55;0.1)	(10, 25, 40, 55;0.1)	(20, 35, 50, 65;0.1)
Earliest Finish	(0, 5, 10, 15;0.1)	(10, 15, 20, 30;0.1)	(5, 15, 20, 25;0.1)	(10, 25, 40, 55;0.1)	(20, 35, 50, 65;0.1)	(20, 30, 40, 55;0.1)	(10, 20, 35, 50;0.1)	(11, 29, 47, 65;0.1)	(12, 31, 50, 69;0.1)	(23, 43, 63, 82;0.1)
Latest Start	(0, 0, 0, 0;0.1)	(-1, 1, 3, 0; 0.1)	(15, 20, 30, 40;0.1)	(9, 11, 13, 15;0.1)	(0, 5, 10, 15; 0.1)	(9, 16, 23, 30;0.1)	(23, 38, 48, 62;0.1)	(19, 31, 43, 55;0.1)	(19, 37, 53, 68; 0.1)	(20, 35, 50, 65;0.1)
Latest Finish	(0, 5, 10, 15;0.1)	(9, 16, 23, 30;0.1)	(20, 35, 50, 65;0.1)	(19, 31, 43, 55;0.1)	(20, 35, 50, 65;0.1)	(19, 31, 43, 55;0.1)	(23, 43, 63, 82;0.1)	(20, 35, 50, 65;0.1)	(23, 43, 63, 82; 0.1)	(23, 43, 63, 82;0.1)
Total Float	(0, 0, 0, 0;0.1)	(-1, 1, 3, 0; 0.1)	(15, 20, 30, 40;0.1)	(9, 6, 3, 0; 0.1)	(0, 0, 0, 0;0.1)	(-1, 1, 3, 0; 0.1)	(13, 23, 28, 32;0.1)	(9, 6, 3, 0; 0.1)	(11, 12, 13, 13; 0.1)	(0, 0, 0, 0;0.1)

Table 3. Calculation of Total Float for Each Activity in Fuzzy Project Network and Critical Path.

6. Conclusion

In this paper, the critical path, early start, early finish, latest start, latest finish and floats have been calculated using a new ranking method Revised SD of PILOT ranking procedure. In this method the critical path has been arrived in fuzzy nature not converting into crisp nature. It is easier for computation. The same method could be used for some other decision making problems with generalized trapezoidal fuzzy number.

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