A Note on L-cordial Labeling of Graphs

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Abstract: We discuss L-cordial labeling of some families of graph. We show that Book graph $\theta(C_4, n)$, Book graph $\theta(C_5, n)$, One point union of $C_3$ i.e., $(C_3)^n$, a double triangular snake $2S(C_3, n)$ are L-cordial.

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1. Introduction and Preliminaries

Prof. Cahit was the first person to use the word cordial [3]. In any cordial labeling of graph the values assigned to vertex are restricted to 0 and 1. Even in edge cordial labeling the similar observation is followed. After [3] a number of papers on graph cordial labeling are published. We have explained L-Cordial labeling in [1]. A graph whose L-cordial labeling is available is called as L-cordial. Not much work has been done in this sort of labeling. We show that Book graph $\theta(C_4, n)$, Book graph $\theta(C_5, n)$, One point union of $C_3$ i.e., $(C_3)^n$ a double triangular snake $2S(C_3, n)$ are L-cordial.

Definition 1.1 (Fusion of vertex). Let $v \in V(G_1)$, $v' \in V(G_2)$ where $G_1$ and $G_2$ are two graphs. We fuse $v$ and $v'$ by replacing them with a single vertex say $w$ and all edges incident with $v$ in $G_1$ and that with $v'$ in $G_2$ are incident with $w$ in the new graph $G = G_1 FG_2$.

\[ \deg G u = \deg G_1(v) + \deg G_2(v') \text{ and } |V(G)| = |V(G_1)| + |V(G_2)| - 1, |E(G)| = |E(G_1)| + |E(G_2)| \]

Definition 1.2. Book graph $\theta(G, n)$ is having $n$ copies $n$ of graph with a common edge. The common edge is same and fixed one in all copies of $G$. It has $1 + 3n$ edges and $2n + 2$ vertices. Let the fixed edge be $e = (uv)$.

Definition 1.3. One point union of $G$ i.e $(G)^n$ At fixed point on $G_n$ copies of $G$ are fused. It has $n|V(G)| - n + 1$ vertices and $n|E(G)|$ edges.

Definition 1.4. Double snake on $C_3$ i.e., $2S(C_3, n)$ a double triangular snake consists of two triangular snakes that have a common path. That is, a double triangular snake is obtained from a path $v_1, v_2, \ldots, v_n, v_{n+1}$ by joining $v_i$ and $v_{i+1}$ to a new vertex $w_i$ for $i = 1, 2, \ldots, n - 1$ and to a new vertex $u_i$ for $i = 1, 2, \ldots, n$.

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2. Main Results

**Theorem 2.1.** Book graph $\theta(C_4, n)$ is L-cordial.

**Proof.** There are $n$ copies of $G = \theta(C_4, n)$ with fixed edge $e = (uv)$ common to all pages. $C^i$ be the $i^{th}$ page of book and is given by $(u, e_1^i, w_1^i, e_2^i, w_2^i, e_3^i, v)$ where $i = 1, 2, \ldots, n$. We define a function $f : E(G) \rightarrow \{1, 2, 3, \ldots, n\}$ a bijective function as follows:

- $f(e) = 1$
- $f(e_1^i) = 3i$
- $f(e_2^i) = 3(i - 1) + 2$
- $f(e_3^i) = 3i + 1$

It follows that Every time we go on adding a page to book the label of $u$ and $v$ alternates. For a book with $n$ pages we have $v_f(1) = v_f(0)$ and $v_f(1) = v_f(0) + 1$ for $n$ is odd.

**Theorem 2.2.** Book graph $\theta(C_5, n)$ is L-cordial.

**Proof.** The common edge $e = (uv)$ and $i^{th}$ page of the book is given by $(u, e_1^i, w_1^i, e_2^i, w_2^i, e_3^i, w_3^i, e_4^i, v)$.

![Figure 1: Book graph $\theta(C_5, 4)$. The vertex labels are very close to vertex, the other numbers are edge labels](image)

Define a function $f : E(G) \rightarrow \{1, 2, 3, \ldots, |E|\}$ given by

- $f(e_1^i) = 4(i - 1) + 2$
- $f(e_2^i) = 4(i - 1) + 3$
- $f(e_3^i) = 4(i - 1) + 4$
- $f(e_4^i) = 4(i - 1) + 5$

for $i = 1, 3, 5, 7, \ldots$

- $f(e_1^i) = 4(i - 1) + 2$
- $f(e_2^i) = 4(i - 1) + 4$
- $f(e_3^i) = 4(i - 1) + 3$
- $f(e_4^i) = 4(i - 1) + 5$

for $i = 2, 4, 6, 8, \ldots$

$v_f(0) = \frac{(3n+2)}{2} = v_f(1)$ for $n$ is even $v_f(0) = (3n+1)$, $v_f(1) = v_f(0) + 1$ for $n$ is odd.

**Theorem 2.3.** One point union of $C_3$ i.e., $(C_3)^n$ is L-cordial.

**Proof.** The $i^{th}$ copy on $(C_3)^n$ be given by $C^n = (v, e_1^i, u_1^i, e_2^i, u_2^i, e_3^i, v)$. Define a function $f : E(G) \rightarrow \{1, 2, 3, \ldots, q - 1\}$ given by $f(e_j^i) = 3(i - 1) + j$ for $j = 1, 2, 3, \ldots$ and for all $i$.

$v_f(0) = n$, $v_f(1) = n + 1$ for odd $n$

$v_f(0) = n + 1$, $v_f(1) = n$ for even $n$

**Theorem 2.4.** A double triangular snake $2S(C_3, n)$ is L-cordial.

**Proof.** To obtain a double snake on $C_3$ i.e $DS(C_3, n)$ we start with a path
\[ P_{n+1} = (v_1, e_1, v_2, e_2, \ldots, e_n, v_{n+1}) \]. Between every two vertices \( v_i \) and \( v_{i+1} \) of \( P_{n+1} \) two vertices \( w_i \) and \( u_i \) are taken. Each \( w_i \) and \( u_i \) are joined to \( v_i \) and \( v_{i+1} \) giving edge \( p_i, p_{i+1} \) and \( q_i, q_{i+1} \) respectively. Define a function \( f : E(G) \to \{1, 2, 3, \ldots, |E|\} \) as follows:

For \( n = 1, n = 2 \) we have shown the labelling in Figure 2 (a), (b) and (c) above. For the snakes of length greater than 3 we use the labelling above for first three blocks.

For \( i \equiv 0, 2 \pmod{4} \) we have,

\[
\begin{align*}
    f(p_i^j) &= 20 + (i - 4)5 \text{ for } j = 1 \\
    f(p_i^j) &= 18 + (i - 4)5 \text{ for } j = 2 \\
    f(q_i^j) &= 17 + (i - 4)5 \text{ for } j = 1 \\
    f(p_i^j) &= 19 + (i - 4)5 \text{ for } j = 1 \\
    f(e_i) &= 16 + (i - 4)5 \text{ for } j = 1
\end{align*}
\]

For \( i \equiv 1, 3 \pmod{4} \) we get,

\[
\begin{align*}
    f(p_i^j) &= 25 + (i - 5)5 \text{ for } j = 1 \\
    f(p_i^j) &= 23 + (i - 5)5 \text{ for } j = 2 \\
    f(q_i^j) &= 24 + (i - 5)5 \text{ for } j = 1 \\
    f(p_i^j) &= 22 + (i - 5)5 \text{ for } j = 2 \\
    f(e_i) &= 21 + (i - 5)5
\end{align*}
\]

This produces vertex numbers as follows:

For \( n = 4 \) we have \( v_f(0) = 7 \) and \( v_f(1) = 6 \)

For \( n = 5 \) we have \( v_f(0) = 8 \) and \( v_f(1) = 8 \).

\[
\begin{align*}
    v_f(0) &= 7 + \left(\frac{n-6}{2} + 1\right)3, \quad v_f(1) = v_f(0) - 1 \text{ for } n = 2x, x = 3, 4, 5, \ldots \\
    v_f(0) &= 7 + \left(\frac{n-6}{2} + 1\right)3 + 1 = v_f(1) \text{ for } n = 2x + 1, x = 3, 4, 5, \ldots
\end{align*}
\]

It follows that \( f \) is L-cordial function.

References


