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A Note on Invariant Submanifolds of LP-Sasakian Manifolds

Research Article

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Abstract: The object of this paper is to obtain some necessary and sufficient conditions for an invariant submanifold of a LP-Sasakian manifold to be totally geodesic.We consider the pseudo projective and Quasi conformal invariant submanifolds of Lorentzian para-sasakian manifolds.

Keywords: Invariant submanifold, LP-sasakian manifold, totally geodesic.

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1. Introduction

The theory of invariant submanifolds of an almost contact manifold has been an interesting area of research in differential geometry for a long time. In 1989, Matsumoto [14] introduced the notion of Lorentzian para-sasakian manifolds.Lorentzian para-sasakian manifold is called LP-Sasakian manifold. The study of geometry of invariant submanifolds was initiated by Bejancu and Papaghuic. The geometry of submanifolds have become an intresting subject in applied mathematics. LP-Sasakian manifolds have been studied by De and Shaikh [2], Ozgur [14], Shaikh and De [1] and also by several authors [13, 15] and many others. In this paper we investigate invariant submanifolds of a LP-sasakian manifolds satisfying $Q(S, P.h) = 0, Q(g, P.h) = 0$, where P denotes the Pseudo projective curvature tensor, and also search for the condition, $P(X, Y).h = fQ(g, h)$, $P(X, Y).h = fQ(S, h)$, $\tilde{C}(X, Y).h = fQ(S, h)$, $\tilde{C}(X, Y).h = fQ(g, h)$, where $\tilde{C}$ is the quasi conformal curvature tensor.

2. Preliminaries

Let (M, g) be an n dimensional Riemannian submanifold of an $(2n + 1)$-dimensional Riemannian manifold $(\tilde{M}, \tilde{g})$ endowed with an almost contact metric structure (ϕ, ξ, η, g), where ϕ is a $(1, 1)$ tensor field, ξ a vector field , η a one-form and g a compatible Riemannian metric on $\bar{M}$.That is,

$$\phi^2(X) = X + \eta(X)\xi, \ \ \eta(\xi) = -1, \ \ \tilde{g}(\phi X, \phi Y) = \tilde{g}(X, Y) + \eta(X)\eta(Y), \tag{1}$$

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \tilde{g}(X, \phi Y) = \tilde{g}(\phi X, Y). \tag{2}$$

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for all vector fields X, Y. Then such a structure $(\phi, \xi, \eta, \tilde{g})$ is termed as Lorentzian almost para contact structure and the manifold with the structure $(\phi, \xi, \eta, \tilde{g})$ is called a Lorentzian almost paracontact manifold.In the Lorentzian almost paracontact manifold $\bar{M}$ the following relation hold:

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \tag{3}$$

$$\tilde{g}(X, \phi Y) = \tilde{g}(\phi X, Y) \tag{4}$$

Let M be a submanifold of a $(2n+1)$-dimensional contact metric manifold $\bar{M}$. We denote by ∇ and $\overline{\nabla}$ the Levi-Cevita connections of M and $\overline{M}$, respectively. Then for any vector fields $X, Y \in \Gamma(TM)$, the second fundamental form h is given by

$$\overline{\nabla}_X Y = \nabla(X, Y) + h(X, Y).$$

Furthermore, for any section N of normal bundle $T^{\perp}M$ we have

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X N,$$

where $\nabla^{\perp}$ denotes the normal bundle connection of M. The second fundamental form h and shape operator A_N are related by

$$g(A_N X, Y) = g(h(X, Y), N). \tag{5}$$

A submanifold M is said to be totally geodesic if $h = 0$, which means that the geodesics in M are also geodesics in $\bar{M}$. On a Riemannian manifold M for a $(0, k)$-type tensor field $T(k \geq 1)$ and a $(0, 2)$-type tensor field E, we denote by $Q(E, T)$ a $(0, k+2)$-type tensor field defined as follows

$$\begin{aligned} Q(E,T)(X_1, X_2, \ldots, X_k; X, Y) = &-T((X \wedge_E Y)X_1, X_2, \ldots, X_k) - T(X_1, (X \wedge_E Y)X_2, \ldots, X_k) - \ldots \\ &\cdots - T(X_1, X_2, \ldots, X_{k-1}, (X \wedge_E Y)X_k). \end{aligned} \tag{6}$$

where $(X \wedge_E Y)Z = E(Y, Z)X - E(X, Z)Y$. Moreover, a submanifold M is said to be pseudo-parallel if

$$\overline{R}(X, Y).h = fQ(g, h). \tag{7}$$

A Lorentzian almost paracontact manifold $\bar{M}$ equipped with the structure (ϕ, ξ, η, g) is called an LP-Sasakian manifold [14] if

$$(\overline{\nabla}_X \phi)Y = \tilde{g}(\phi X, \phi Y)\xi + \eta(Y)\phi^2 X, \tag{8}$$

where $\overline{\nabla}$ denotes the operator of covariant differentiation with respect to the Lorentzian metric $\tilde{g}$. In an LP-Sasakian manifold $\bar{M}$ with the structure (ϕ, ξ, η, g), it is easily seen that

$$\tilde{\nabla}_X \xi = \phi X, \tag{9}$$

$$\tilde{R}(\xi, X)Y = \tilde{g}(X, Y)\xi - \eta(Y)X, \tag{10}$$

$$\tilde{R}(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{11}$$

$$\tilde{S}(X, \xi) = (n-1)\eta(X). \tag{12}$$

for all vector fields X, Y on $\bar{M}$ [14], where $\bar{S}$ denotes the Ricci tensor of $\bar{M}$ and $\bar{R}$ is the curvature tensor of $\bar{M}$. A submanifold M of an LP-Sasakian manifold $\bar{M}$ is called an invariant submanifold of $\bar{M}$ if $\phi(TM) \subset TM$. In an invariant submanifold of an LP-Sasakianmanifold

$$h(X, \xi) = 0, \tag{13}$$

for any vector field X tangent to M. In [7] Ozgur and Murathan proved the following lemma:

Lemma 2.1. *Let M be an n-dimensional invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then the following equations hold on M:*

$$\nabla_X \xi = \phi X,$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{14}$$

$$R(\xi, Y)\xi = \eta(Y)\xi + Y, \tag{15}$$

$$S(X, \xi) = (n-1)\eta(X), \tag{16}$$

$$h(X, \phi Y) = \phi h(X, Y),$$

$$S(\xi, \xi) = -(n-1), \ Q\xi = (n-1)\xi. \tag{17}$$

$$(\nabla_X \phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{18}$$

Let (M, g) be an n-dimensional Riemannian manifold. The Pseudo-projective curvature tensor and Quasi conformal curvature tensor respectively are defined by

$$P(X, Y)Z = aR(X, Y)Z + b[S(Y, Z)X - S(X, Z)Y] - \left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + b\right)[g(Y, Z)X - g(X, Z)Y], \tag{19}$$

$$\begin{aligned}\tilde{C}(X, Y)Z &= aR(X, Y)Z + b[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \\ &\quad - \left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + 2b\right)[g(Y, Z)X - g(X, Z)Y].\end{aligned} \tag{20}$$

where a and b are constants, S is the Ricci tensor, Q is the Ricci operator of M defined by $S(X, Y) = g(QX, Y)$.

3. Invariant Submanifolds of LP-Sasakian Manifolds Satisfying $P(X, Y).h = fQ(g, h)$

Let us consider M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$ satisfying

$$P(X, Y).h = fQ(g, h), \tag{21}$$

for all vector fields X, Y tangent to M, where f denotes the real valued function on M^n. The equation (21) can be written as

$$R^{\perp}(X, Y)h(U, V) - h(P(X, Y)U, V) - h(U, P(X, Y)V) = -f[h((X \wedge_g Y)U, V) + h(U, (X \wedge_g Y)V)], \tag{22}$$

we've $(X \wedge_g Y)Z = g(Y, Z)X - g(X, Z)Y$ substituting the above equation in (22), we have

$$\begin{aligned}&R^{\perp}(X, Y)h(U, V) - h(P(X, Y)U, V) - h(U, P(X, Y)V) \\ &\quad = -f[g(Y, U)h(X, V) - g(X, U)h(Y, V) + g(Y, V)h(U, X) - g(X, V)h(U, Y)],\end{aligned} \tag{23}$$

Putting $X = V = \xi$ in (23), we obtain

$$\begin{aligned} R^{\perp}(\xi, Y)h(U, \xi) - h(P(\xi, Y)U, \xi) - h(U, P(\xi, Y)\xi) \\ = -f[g(Y, U)h(\xi, \xi) - g(\xi, U)h(Y, \xi) + g(Y, \xi)h(U, \xi) - g(\xi, \xi)h(U, Y)], \end{aligned} \tag{24}$$

Using (13), (19) in (24), we get

$$-h(U, Y)\left(a - bS(\xi, \xi) - \left(\frac{r}{n}\right)\left(\frac{a}{n-1} + b\right)\right) = -f[h(U, Y)], \tag{25}$$

which implies

$$\left(f - a - b(n-1) + \left(\frac{r}{n}\right)\left(\frac{a}{n-1} + b\right)\right) h(U, Y) = 0. \tag{26}$$

that is, $h(U, Y) = 0$ which gives M^n is totally geodesic, provided $f \neq a + b(n-1) - \left(\frac{r}{n}\right)\left(\frac{a}{n-1} + b\right)$.
Conversely, let M^n be totally geodesic. Then, from (23) we get M^n satisfies $P(X, Y).h = fQ(g, h)$. Thus we can state the following:

Theorem 3.1. *Let M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then M^n satisfies $P(X, Y).h = fQ(g, h)$ iff M^n is totally geodesic, provided $f \neq \left(a + b(n-1) - \left(\frac{r}{n}\right)\left(\frac{a}{n-1} + b\right)\right)$.*

4. Invariant Submanifolds of LP-Sasakian manifolds satisfying $P(X, Y).h = fQ(S, h)$

Let us consider M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$ satisfying

$$P(X, Y).h = fQ(S, h), \tag{27}$$

for all vector fields X, Y tangent to M, where f denotes the real valued function on M^n. The equation (27) can be written as

$$R^{\perp}(X, Y)h(U, V) - h(P(X, Y)U, V) - h(U, P(X, Y)V) = -f[h((X \wedge_S Y)U, V) + h(U, (X \wedge_S Y)V)], \tag{28}$$

we've $(X \wedge_S Y)Z = S(Y, Z)X - S(X, Z)Y$. Substituting the above equation in (28), we have

$$\begin{aligned} R^{\perp}(X, Y)h(U, V) - h(P(X, Y)U, V) - h(U, P(X, Y)V) \\ = -f[S(Y, U)h(X, V) - S(X, U)h(Y, V) + S(Y, V)h(U, X) - S(X, V)h(U, Y)], \end{aligned} \tag{29}$$

Putting $X = V = \xi$ in (29), we obtain

$$R^{\perp}(\xi, Y)h(U, \xi) - h(P(\xi, Y)U, \xi) - h(U, P(\xi, Y)\xi) = -f[S(Y, U)h(\xi, \xi) - S(\xi, U)h(Y, \xi) + S(Y, \xi)h(U, \xi) - S(\xi, \xi)h(U, Y)], \tag{30}$$

Using (13), (17) ,(19) in (30), we get

$$-h(U, Y)\left(a + bS(\xi, \xi) - \left(\frac{r}{n}\right)\left(\frac{a}{n-1} + b\right)\right) = f[h(U, Y)S(\xi, \xi)], \tag{31}$$

which implies

$$\left(a + b(n-1) - \left(\frac{r}{n}\right)\left(\frac{a}{n-1} + b - f(n-1)\right)\right) h(U, Y) = 0. \tag{32}$$

that is, $h(U, Y) = 0$ which gives M^n is totally geodesic, provided $f \neq \left((\frac{a}{n-1}) + b\right)\left(1 - (\frac{r}{n(n-1)})\right)$.
Conversely, let M^n be totally geodesic. Then, from (29) we get M^n satisfies $P(X, Y).h = fQ(S, h)$. Thus we can state the following:

Theorem 4.1. *Let M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then M^n satisfies $P(X, Y).h = fQ(S, h)$ iff M^n is totally geodesic, provided $f \neq \left((\frac{a}{n-1}) + b\right)\left(1 - \left(\frac{r}{n(n-1)}\right)\right)$.*

5. Invariant Submanifolds of LP-Sasakian Manifolds Satisfying $\tilde{C}(X,Y).h = fQ(g,h)$

Let us consider M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$ satisfying

$$\tilde{C}(X,Y).h = fQ(g,h), \tag{33}$$

for all vector fields X,Y tangent to M, where f denotes the real valued function on M^n. The equation (33) can be written as

$$R^{\perp}(X,Y)h(U,V) - h(\tilde{C}(X,Y)U,V) - h(U,\tilde{C}(X,Y)V) = -f[h((X\wedge_g Y)U,V) + h(U,(X\wedge_g Y)V)], \tag{34}$$

we've $(X\wedge_g Y)Z = g(Y,Z)X - g(X,Z)Y$ substituting the above in (34), we have

$$\begin{aligned} &R^{\perp}(X,Y)h(U,V) - h(\tilde{C}(X,Y)U,V) - h(U,\tilde{C}(X,Y)V)\\ &\qquad = -f[g(Y,U)h(X,V) - g(X,U)h(Y,V) + g(Y,V)h(U,X) - g(X,V)h(U,Y)], \end{aligned} \tag{35}$$

Putting $X = V = \xi$ in (35), we obtain

$$\begin{aligned} &R^{\perp}(\xi,Y)h(U,\xi) - h(\tilde{C}(\xi,Y)U,\xi) - h(U,\tilde{C}(\xi,Y)\xi)\\ &\qquad = -f[g(Y,U)h(\xi,\xi) - g(\xi,U)h(Y,\xi) + g(Y,\xi)h(U,\xi) - g(\xi,\xi)h(U,Y)], \end{aligned} \tag{36}$$

Using (13), (20) in (36), we get

$$-h(U,Y)\left(a + 2b(n-1) - \left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + 2b\right)\right) = -f[h(U,Y)], \tag{37}$$

which implies

$$\left(a + 2b(n-1) - \left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + 2b\right) + f\right)h(U,Y) = 0. \tag{38}$$

that is, $h(U,Y) = 0$ which gives M^n is totally geodesic, provided $f \neq \left(\left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + 2b\right) - a - 2b(n-1)\right)$.
Conversely, let M^n be totally geodesic. Then, from (35) we get M^n satisfies $\tilde{C}(X,Y).h = fQ(g,h)$. Thus we can state the following:

Theorem 5.1. *Let M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then M^n satisfies $\tilde{C}(X,Y).h = fQ(g,h)$ iff M^n is totally geodesic, provided $f \neq \left(\left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + 2b\right) - a - 2b(n-1)\right)$.*

6. Invariant Submanifolds of LP-Sasakian Manifolds Satisfying $\tilde{C}(X,Y).h = fQ(S,h)$

Let us consider M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$ satisfying

$$\tilde{C}(X,Y).h = fQ(S,h), \tag{39}$$

for all vector fields X,Y tangent to M, where f denotes the real valued function on M^n. The equation (39) can be written as

$$R^{\perp}(X,Y)h(U,V) - h(\tilde{C}(X,Y)U,V) - h(U,\tilde{C}(X,Y)V) - f[h((X\wedge_S Y)U,V) + h(U,(X\wedge_S Y)V)], \tag{40}$$

we've $(X \wedge_S Y)Z = S(Y,Z)X - S(X,Z)Y$ substituting the above in (40), we have

$$\begin{aligned} R^{\perp}(X,Y)h(U,V) - h(\tilde{C}(X,Y)U,V) - h(U,\tilde{C}(X,Y)V) \\ = -f[S(Y,U)h(X,V) - S(X,U)h(Y,V) + S(Y,V)h(U,X) - S(X,V)h(U,Y)], \end{aligned} \tag{41}$$

Putting $X = V = \xi$ in (41), we obtain

$$\begin{aligned} R^{\perp}(\xi,Y)h(U,\xi) - h(\tilde{C}(\xi,Y)U,\xi) - h(U,\tilde{C}(\xi,Y)\xi) \\ = -f[S(Y,U)h(\xi,\xi) - S(\xi,U)h(Y,\xi) + S(Y,\xi)h(U,\xi) - S(\xi,\xi)h(U,Y)], \end{aligned} \tag{42}$$

Using (13), (20) in (42), we get

$$-h(U,Y)(a + (2b - ar))\left(\frac{1}{n(n-1)}\right) = -f(n-1)[h(U,Y)], \tag{43}$$

which implies

$$(a + (2b - ar)(frac1n(n-1)) - f(n-1))h(U,Y) = 0. \tag{44}$$

that is, $h(U,Y) = 0$ which gives M^n is totally geodesic, provided $f \neq \frac{an(n-1)+2b-ar}{n(n-1)^2}$.

Conversely, let M^n be totally geodesic. Then, from (41) we get M^n satisfies $\tilde{C}(X,Y).h = fQ(S,h)$. Thus we can state the following:

Theorem 6.1. *Let M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then M^n satisfies $\tilde{C}(X,Y).h = fQ(S,h)$ iff M^n is totally geodesic, provided $f \neq \left(\frac{an(n-1)+2b-ar}{n(n-1)^2}\right)$.*

7. Invariant Submanifolds of LP-Sasakian Manifolds Satisfying $Q(g, P.h) = 0$

Assuming that $Q(g, P.h) = 0$, then we get

$$0 = Q(g, P(X,Y).h)(W, K : U, V), \tag{45}$$

we also have

$$(P(X,Y).h)(U,V) = R \perp (X,Y)h(U,V) - h(P(X,Y)U,V) - h(U,P(X,Y)V), \tag{46}$$

for any vector fields $X, Y, W, K, U, V \in \Gamma(TM)$. We obtain directly from (20) and (46) that

$$\begin{aligned} 0 = &-g(V,W)(P(X,Y).h)(U,K) + g(U,W)(P(X,Y).h)(V,K) \\ &- g(V,K)(P(X,Y).h)(W,U) + g(U,K)(P(X,Y).h)(W,V), \end{aligned} \tag{47}$$

$$\begin{aligned} 0 = &-g(V,W)[R \perp (X,Y)h(U,K) - h(P(X,Y)U,K) - h(P(X,Y)K,U)] \\ &+ g(U,W)[R \perp (X,Y)h(U,K) - h(P(X,Y)U,K) - h(P(X,Y)K,V)] \\ &- g(V,K)[R \perp (X,Y)h(W,U) - h(P(X,Y)W,U) - h(P(X,Y)U,W)] \\ &+ g(U,K)[R \perp (X,Y)h(W,V) - h(P(X,Y)W,V) - h(P(X,Y)V,W)], \end{aligned} \tag{48}$$

putting $Y = K = W = U = \xi$ in the above equation and obtain $h(P(X,\xi)\xi, V) = 0$, which implies

$$h(X,V)\left[\frac{-n(n-1)b + ra + (n-1)b}{n(n-1)}\right] = 0. \tag{49}$$

that is $h(X,V) = 0$ which gives M^n is totally geodesic, provided $(n-1)^2 b - ra \neq 0$.

Theorem 7.1. *Let M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then M^n satisfies $Q(g, P.h) = 0$ iff M^n is totally geodesic, provided $(n-1)^2 b - ra \neq 0$.*

8. Invariant Submanifolds of LP-Sasakian Manifolds Satisfying $Q(S, P.h) = 0$

Assuming that $Q(S, P.h) = 0$, then we get

$$0 = Q(S, P(X,Y).h)(W, K : U, V), \tag{50}$$

for any vector fields $X, Y, W, K, U, V \in \Gamma(TM)$. We obtain directly from the above equation and (46) that

$$\begin{aligned} 0 = &-S(V,W)(P(X,Y).h)(U,K) + S(U,W)(P(X,Y).h)(V,K) \\ &- S(V,K)(P(X,Y).h)(W,U) + S(U,K)(P(X,Y).h)(W,V), \end{aligned} \tag{51}$$

$$\begin{aligned} 0 = &-S(V,W)[R \perp (X,Y)h(U,K) - h(P(X,Y)U,K) - h(P(X,Y)K,U)] \\ &+ S(U,W)[R \perp (X,Y)h(U,K) - h(P(X,Y)U,K) - h(P(X,Y)K,V)] \\ &- S(V,K)[R \perp (X,Y)h(W,U) - h(P(X,Y)W,U) - h(P(X,Y)U,W)] \\ &+ S(U,K)[R \perp (X,Y)h(W,V) - h(P(X,Y)W,V) - h(P(X,Y)V,W)]. \end{aligned} \tag{52}$$

putting $Y = K = W = U = \xi$ in the above equation and obtain $S(\xi,\xi)h(P(X,\xi)\xi, V) = 0$, which implies

$$h(X,V)\left((n-1)^2 b - \left(\frac{r}{n}\right)\left(\left(\frac{a}{n-1}\right) + b\right)\right) = 0. \tag{53}$$

that is $h(X,V) = 0$ which gives M^n is totally geodesic, provided $n(n-1)^3 b - ra - (n-1)rb \neq 0$.

Theorem 8.1. *Let M^n be an invariant submanifold of an LP-Sasakian manifold $\bar{M}$. Then M^n satisfies $Q(S, P.h) = 0$ iff M^n is totally geodesic, provided $n(n-1)^3 b - ra - (n-1)rb \neq 0$.*

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