Ricci Solitons on \((\varepsilon, \delta)\) Trans-Sasakian Manifolds

K. Bhavya\(^1\)*, G. Somashekhara\(^2\) and G.S. Shivaprasanna\(^3\)

1 Department of Mathematics, Presidency University, Bengaluru, Karnataka, India.
2 Department of Mathematics, M.S.Ramaiah University of Applied Sciences, Bengaluru, Karnataka, India.
3 Department of Mathematics, Dr.Ambedkar Institute of Technology, Bengaluru, Karnataka, India.

Abstract: The object of this paper is to study the characterisation of Ricci Solitons in generalised ricci recurrent and \(\phi\) recurrent \((\varepsilon, \delta)\) trans-sasakian manifolds based on the 1-form.


Keywords: Ricci, \(\phi\)-recurrent, pseudo-projective, \((\varepsilon, \delta)\) Trans sasakian, Ricci recurrent, shrinking, expanding, steady.

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1. Introduction

In the year 1982, Hamilton [11] introduced solution to the Ricci flow known as Ricci Soliton. In Contact Riemannian geometry RameshSharma [10] initiated the study of Ricci Solitons. Later many authors [5, 7, 9] extensively studied Ricci Solitons in contact metric manifolds. Ricci Soliton in a Riemannian manifold \((M, g)\) is a natural generalisation of an Einstein metric and is defined as a triple \((g, V, \lambda)\) with \(g\) a Riemannian metric, \(V\) a vector field, \(\lambda\) a real scalar such that

\[
(L_V g)(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0,
\]

(1)

where \(L_V\) represents the Lie derivative operator along the vector field \(V\) and \(S\) is the Ricci tensor of \(M\). Depending on the \(\lambda\) value, whether it is negative, zero, and positive, the Ricci Soliton will be shrinking, steady, and expanding respectively. In 1985, Obulina [7] introduced a new class of contact manifold namely Trans-Sasakian manifold. Many authors [3, 4, 6] have studied this manifold and obtained many interesting results. In this paper, we study the conditions which characterise Ricci Solitons in Trans-sasakian manifolds. Section 2 contains a review of Trans-sasakian manifolds and Ricci solitons. In section 3-6 we prove characterizing conditions for ricci Solitons in generalised Ricci recurrent, generalised \(\phi\) -recurrent, generalised pseudo-projective \(\phi\) -recurrent, generalised concircular \(\phi\) -recurrent \((\varepsilon, \delta)\) Trans-sasakian Manifolds.

2. Preliminaries

A manifold \(M\) of dimension \(n\) is an almost contact manifold if it admits an almost contact metric structure \((\phi, \xi, \eta, g)\) consisting of a vector field \(\xi\), a 1-form \(\eta\), a tensor field \(\phi\) of type \((1,1)\) and a Riemannian metric \(g\) compatible with \((\phi, \xi, \eta)\)

* E-mail: bhavya.k6666@gmail.com
satisfying

$$\phi^2 X = -X + \eta(X)\xi, \quad (2)$$

$$\eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0. \quad (3)$$

An almost contact metric manifold $M$ is called an $(\varepsilon)$-almost contact metric manifold if

$$g(\xi, \xi) = \varepsilon, \quad (4)$$

$$\eta(X) = \varepsilon g(X, \xi), \quad (5)$$

$$g(\phi X, \phi Y) = g(X, Y) - \varepsilon \eta(X)\eta(Y), \quad (6)$$

for all vector fields $X, Y$ on $M$, where $\varepsilon = g(\xi, \xi) = \pm 1$. An $(\varepsilon)$-almost contact metric manifold is said to be $(\varepsilon, \delta)$-Trans-sasakian manifold if

$$(\nabla_X \phi)(Y) = \alpha [g(X, Y)\xi - \varepsilon \eta(Y)X] + \beta [g(\phi X, Y)\xi - \delta \eta(Y)\phi X], \quad (7)$$

holds for some smooth functions $\alpha$ and $\beta$ on $M$ and $\varepsilon = \pm 1$, $\delta = \pm 1$. For $\beta = 0$, $\alpha = 1$, $(\varepsilon, \delta)$-trans-sasakian manifold reduces to $(\varepsilon)$-sasakian and for $\alpha = 0$, $\beta = 1$ it reduces to a $(\delta)$-kenmotsu manifold. In an $(\varepsilon)$-almost contact metric manifold $M$ is an $(\varepsilon, \delta)$-Trans-sasakian manifold if and only if

$$\nabla_X \xi = -\varepsilon \alpha \phi X - \delta \beta \phi^2 X, \quad (8)$$

where $\nabla$ denotes the Riemannian connection of $g$. In an $(\varepsilon, \delta)$-Trans-sasakian manifold $M$, the following relation holds with $(\alpha, \beta)$ are constants

$$\nabla X \phi(Y) = \varepsilon g(\phi(\nabla X \xi), Y)\xi - \eta(Y)\phi(\nabla X \xi), \quad (9)$$

$$\nabla X \eta(Y) = \delta \beta [\varepsilon g(X, Y) - \eta(X)\eta(Y)] - \alpha g(\phi X, Y), \quad (10)$$

$$R(X, Y)\xi = (\beta^2 - \alpha^2) [\eta(X)Y - \eta(Y)X], \quad (11)$$

where $R$ is the Riemannian Curvature tensor

$$S(X, \xi) = (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)\eta(X), \quad (12)$$

Let $(g, V, \lambda)$ be a Ricci Soliton in Trans-sasakian manifold $M$. Put $V = \xi$ then from (8) and (1) we get

$$S(X, Y) = \beta \delta \varepsilon \eta(X)\eta(Y) - g(X, Y)(\beta \delta + \lambda). \quad (13)$$

The above equation gives

$$QX = \beta \delta \varepsilon \eta(X)\xi - (\beta \delta + \lambda) X, \quad (14)$$

$$S(X, \xi) = -\varepsilon \lambda \eta(X), \quad (15)$$

$$r = -[(n - \varepsilon)\beta \delta + n \lambda], \quad (16)$$

Also by the covariant derivative definition, we have

$$(\nabla w S)(Y, \xi) = \nabla w S(Y, \xi) - S(\nabla w Y, \xi) - S(Y, \nabla w \xi), \quad (17)$$

The following results will be used later.
Lemma 2.1. In a $\phi$-recurrent $(\varepsilon, \delta)$-Trans-sasakian manifold $(M^n, g)$ the characteristic vector field $\xi$ and the vector fields $\rho_1, \rho_2$ associated with two 1-forms $A$ and $B$ are co-directional and the 1-forms $A$ and $B$ are defined as follows

$$A(W) = \eta(\rho_1)\eta(W), \quad B(W) = \eta(\rho_2)\eta(W), \quad (18)$$

Replacing $W$ by $\xi$ in (18) it follows that

$$A(\xi) = \eta(\rho_1), \quad B(\xi) = \eta(\rho_2), \quad (19)$$

3. Generalised Ricci-Recurrent $(\varepsilon, \delta)$-Trans-Sasakian Manifold

A trans-sasakian manifold is said to be generalised Ricci-recurrent manifold if there exist two non-zero 1-forms $A$ and $B$ such that

$$(\nabla_W S)(Y, Z) = A(W)S(Y, Z) + (n - 1)B(W)g(Y, Z), \quad (20)$$

Replacing $Z$ by $\xi$ in (20) and using (12), We have

$$(\nabla_W S)(Y, \xi) = (n - 1)(\varepsilon\alpha^2 - \beta^2\delta)A(W)\eta(Y) + \varepsilon(n - 1)B(W)\eta(Y), \quad (21)$$

Using (8) and (12) we get

$$(\nabla_W S)(Y, \xi) = \beta\delta\varepsilon(n - 1)(\varepsilon\alpha^2 - \beta^2\delta)g(W, Y) - \alpha(n - 1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W, Y) + \varepsilon\alpha S(Y, \phi W) - \beta\delta S(Y, W), \quad (22)$$

On comparing (21) with (22) we have

$$\beta\delta S(Y, W) = \beta\delta\varepsilon(n - 1)(\varepsilon\alpha^2 - \beta^2\delta)g(W, Y) - \alpha(n - 1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W, Y) + \varepsilon\alpha S(Y, \phi W) - \varepsilon(n - 1)B(W)\eta(Y), \quad (23)$$

Substituting $Y = \xi$ in (23) we get,

$$S(\xi, W) = (n - 1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) - \frac{(n - 1)(\varepsilon\alpha^2 - \beta^2\delta)}{\beta\delta}A(W) - \frac{\varepsilon}{\beta\delta}B(W)(n - 1), \quad (24)$$

Using Lemma (18), (24) reduces to

$$S(\xi, W) = (n - 1)\eta(W)[(\varepsilon\alpha^2 - \beta^2\delta) - \frac{(\varepsilon\alpha^2 - \beta^2\delta)}{\beta\delta}]\eta(\rho_1) - \frac{\varepsilon}{\beta\delta}\eta(\rho_2)], \quad (25)$$

Substitute (15) and (19) in (25), we obtain

$$\lambda = -\frac{(n - 1)(\varepsilon\alpha^2 - \beta^2\delta)}{\varepsilon}[1 - \frac{1}{\beta\delta}A(\xi)] + \frac{(n - 1)}{\beta\delta}B(\xi), \quad (26)$$

Theorem 3.1. Ricci Soliton in Generalised Ricci-Recurrent $(\varepsilon, \delta)$-Trans-sasakian manifold $(M, g)$ with two 1-forms $A$ and $B$ is

- **Expanding**, if $A(\xi) > 1, B(\xi) > 1$.
- **Steady**, if $(\varepsilon\alpha^2 - \beta^2\delta) = \frac{\varepsilon B(\xi)}{(\beta\delta - A(\xi))}$.
- **Shrinking**, if $A(\xi) < 1, B(\xi) < 1$. 

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4. Generalised $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-Sasakian Manifold

A $(\varepsilon, \delta)$-Trans-sasakian Manifold is said to be generalised $\phi$-Recurrent manifold [2] if its curvature tensor $R$ satisfies the condition

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

for arbitrary vector fields $X, Y, Z, W$. Let us consider a generalised $\phi$-Recurrent Trans-sasakian Manifold. By virtue of (2) and (27) we have,

$$-((\nabla_W R)(X,Y)Z) + \eta((\nabla_W R)(X,Y)Z)\xi = A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

Contracting (28) with respect to $U$, we obtain

$$-g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\varepsilon(U) = A(W)g(R(X,Y)Z,U) + B(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)],$$

Let $e_i (i = 1, 2, 3, \ldots, n)$ be an orthonormal basis of the tangent space at any point of the manifold. Put $X = U = e_i$ in (29) and take summation over $i, 1 \leq i \leq n$, we get

$$- (\nabla_W S)(Y,Z) = A(W)S(Y,Z) + B(W)g(Y,Z)(n-1),$$

Replacing $Z$ by $\xi$ in (30) and using (12) we have

$$(\nabla_W S)(Y,\xi) = -(n-1)\eta(Y)[(\varepsilon\alpha^2 - \beta^2\delta)A(W) + \varepsilon B(W)],$$

Substitute (8), (12) in (17), we get

$$(\nabla_W S)(Y,\xi) = \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W,Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W,Y) + \varepsilon\alpha S(Y,\phi W) - \beta\delta S(Y,W),$$

On comparison of (31) with (32), we have

$$\beta\delta S(Y,W) = \beta\delta\varepsilon(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(W,Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2\delta)g(\phi W,Y) + \varepsilon\alpha S(Y,\phi W) + (n-1)\eta(Y)[(\varepsilon\alpha^2 - \beta^2\delta)A(W) + \varepsilon B(W)],$$

Put $Y = \xi$ in (33), we get

$$S(\xi,W) = (n-1)(\varepsilon\alpha^2 - \beta^2\delta)\eta(W) + \frac{(n-1)}{\beta\delta}[((\varepsilon\alpha^2 - \beta^2\delta)A(W) + \varepsilon B(W)],$$

Applying Lemma (18), (34) reduces to

$$S(\xi,W) = (n-1)\eta(W)[(\varepsilon\alpha^2 - \beta^2\delta) + \frac{1}{\beta\delta}(\varepsilon\alpha^2 - \beta^2\delta)\eta(\rho_1) + \varepsilon\eta(\rho_2)],$$

Make use of (15) and (19) in (35), we obtain

$$\lambda = \frac{(n-1)}{\varepsilon}(\varepsilon\alpha^2 - \beta^2\delta)[1 + \frac{1}{\beta\delta}A(\xi)] - \frac{(n-1)}{\beta\delta}B(\xi).$$

**Theorem 4.1.** Ricci Soliton in Generalised $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-sasakian manifold $(M, g)$ with two 1-forms $A$ and $B$ is

- **Expanding** if, $A(\xi) < 1, B(\xi) < 1$.
- **Steady** if, $(\varepsilon\alpha^2 - \beta^2\delta) = \frac{-\varepsilon B(\xi)}{(\beta\delta + A(\xi))}$.
- **Shrinking** if, $A(\xi) > 1, B(\xi) > 1$. 


5. Generalised Pseudo-Projective $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-Sasakian Manifold

In a $(\varepsilon, \delta)$-Trans-sasakian Manifold $M$, the Pseudo- Projective curvature tensor $\overline{P}$ is given by [1]

$$\overline{P}(X,Y)Z = a R(X,Y)Z + b [S(Y,Z)X - S(X,Z)Y] - \frac{r}{n} \left( \frac{a}{n-1} + b \right) [g(Y,Z)X - g(X,Z)Y],$$

(37)

where $a$ and $b$ are constants such that $a, b \neq 0$. A $(\varepsilon, \delta)$-Trans-sasakian manifold is said to be Pseudo-Projective $\phi$-Recurrent manifold if there exists two non-zero 1-forms $A$ and $B$ such that

$$\phi^2((\nabla_{w}\overline{P})(X,Y)Z) = A(W)\overline{P}(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

(38)

for arbitrary vector fields $X, Y, Z, W$. Let us consider a Pseudo-Projective $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-sasakian Manifold. By virtue of (2) and (38), we have

$$-((\nabla_{w}\overline{P})(X,Y)Z) + \eta((\nabla_{w}\overline{P})(X,Y)Z) = A(W)\overline{P}(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

(39)

Contracting (39) with respect to $U$, we obtain

$$-g((\nabla_{w}\overline{P})(X,Y)Z, U) + \eta((\nabla_{w}\overline{P})(X,Y)Z) = A(W)g(\overline{P}(X,Y,Z), U) + B(W)[g(Y,Z)g(Y, U) - g(X,Z)g(Y, U)],$$

(40)

Let $e_i (i = 1, 2, 3, \ldots, n)$, be an orthonormal basis of the tangent space at any point of the manifold. Put $X = U = e_i$ in (40) and take summation over $i, 1 \leq i \leq n$, we get

$$-g((\nabla_{w}S)(Y, Z) = A(W)[S(Y, Z) - \frac{r}{n} g(Y, Z)] + \frac{(n - 1)\varepsilon}{a + b(n - 1)} B(W)g(Y, Z),$$

(41)

Replacing $Z$ by $\xi$ in (41) and using (2) and (12), we have

$$(\nabla_{w}S)(Y, \xi) = A(W) \left( \frac{\varepsilon r}{n} - (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta) \right) - B(W) \left( \frac{(n - 1)\varepsilon}{a + b(n - 1)} \right) \eta(Y).$$

(42)

Substitute (8), (12) in (17), we get

$$(\nabla_{w}S)(Y, \xi) = \beta \delta \varepsilon (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)g(\phi W, Y) - \alpha (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)g(\phi W, Y) + \varepsilon \alpha S(Y, \phi W) - \beta \delta S(Y, W),$$

(43)

On comparing (42) with (43) we have

$$\beta \delta S(Y, W) = \beta \delta \varepsilon (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)g(\phi W, Y) - \alpha (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)g(\phi W, Y) + \varepsilon \alpha S(Y, \phi W) + A(W) \left( \frac{\varepsilon r}{n} + (n - 1) \right) (\varepsilon \alpha^2 - \beta^2 \delta) \eta(Y) + B(W) \left[ \frac{(n - 1)\varepsilon}{a + b(n - 1)} \right] \eta(Y),$$

(44)

Put $Y = \xi$ in (44), we get

$$S(\xi, W) = (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)\eta(W) + \frac{A(W)}{\beta \delta} \left[ (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta) - \frac{\varepsilon r}{n} \right] + \frac{B(W)}{\beta \delta} \left[ \frac{(n - 1)\varepsilon}{a + b(n - 1)} \right].$$

(45)

Applying Lemma (18), (45) reduces to

$$S(\xi, W) = (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)\eta(W) + \frac{\eta(P1)\eta(w)}{\beta \delta} \left[ (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta) - \frac{\varepsilon r}{n} \right] + \frac{\eta(P2)\eta(w)}{\beta \delta} \left[ \frac{(n - 1)\varepsilon}{a + b(n - 1)} \right].$$

(46)

Make use of (15), (16) and (19) in (46), we obtain

$$\lambda = -\frac{(n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)}{\varepsilon} - \left[ \frac{(n - \varepsilon)}{n} \frac{\beta \delta A(\xi)}{\beta \delta + A(\xi)} + \frac{B(\xi)}{\beta \delta + A(\xi)} \right] \left[ \frac{(n - 1)\varepsilon}{a + b(n - 1)} \right].$$

(47)

Theorem 5.1. Ricci Soliton in a Generalised Pseudo-Projective $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-sasakian manifold $(M, g)$ with two 1-forms $A$ and $B$ is shrinking provided $A(\xi)$ and $B(\xi)$ is non-negative.
6. Generalised Concircular $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-sasakian Manifold

The concircular curvature tensor of $(M, g)$ is given by [8]

$$\overline{\nabla}(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]. \quad (48)$$

A $(\varepsilon, \delta)$-Trans-sasakian manifold is said to be Concircular $\phi$-Recurrent manifold if there exists two non-zero 1-forms $A$ and $B$ such that

$$\phi^2((\nabla_{w} \overline{\nabla})(X, Y)Z) = A(W)\overline{\nabla}(X, Y)Z + B(W)[g(Y, Z)X - g(X, Z)Y], \quad (49)$$

for arbitrary vector fields $X, Y, Z, W$. Let us consider $\phi$-Recurrent $(\varepsilon, \delta)$-Trans-sasakian Manifold. By virtue of (2) and (49), we have

$$-(\nabla_{w} \overline{\nabla})(X, Y)Z + \eta((\nabla_{w} \overline{\nabla})(X, Y)Z)\xi = A(W)\overline{\nabla}(X, Y)Z + B(W)[g(Y, Z)X - g(X, Z)Y], \quad (50)$$

Contracting (50) with respect to $U$, we have

$$-g((\nabla_{w} \overline{\nabla})(X, Y)Z) + \eta((\nabla_{w} \overline{\nabla})(X, Y)Z)\xi(U) = A(W)g((\overline{\nabla}(X, Y)Z, U) + B(W)[g(Y, Z)g(X, U) - g(X, Z)g(Y, U)]. \quad (51)$$

Let $\epsilon_{i}(i = 1, 2, 3, ..., n)$, be an orthonormal basis of the tangent space at any point of the manifold. Then put $X = U = \epsilon_{i}$ in (51) and take summation over $i$, $1 \leq i \leq n$, we get

$$(\nabla_{w} S)(Y, Z) = \frac{\nabla_{w} r}{n} g(Y, Z) - A(W)S(Y, Z) + g(Y, Z) \left[ \frac{A(W)r}{n} - B(W)(n-1) \right], \quad (52)$$

Replacing $Z$ by $\xi$ in (52) and using (2) and (12), we have

$$(\nabla_{w} S)(Y, \xi) = \frac{\nabla_{w} r}{n} \eta(Y) - A(W)(n-1)(\varepsilon\alpha^2 - \beta^2 \delta)\eta(Y) + \varepsilon\eta(Y) \left[ \frac{A(W)r}{n} - B(W)(n-1) \right], \quad (53)$$

For a constant $r$ (53) reduces to

$$(\nabla_{w} S)(Y, \xi) = -A(W)(n-1)(\varepsilon\alpha^2 - \beta^2 \delta)\eta(Y) + \varepsilon\eta(Y) \left[ \frac{A(W)r}{n} - B(W)(n-1) \right], \quad (54)$$

Substitute (8) and (12) in (17) we get

$$(\nabla_{w} S)(Y, \xi) = \beta \delta \varepsilon (n-1)(\varepsilon\alpha^2 - \beta^2 \delta)g(W, Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2 \delta)g(\phi W, Y) + \varepsilon \alpha S(Y, \phi W) - \beta \delta S(Y, W), \quad (55)$$

On comparison of (54) with (55), we have

$$\beta \delta S(Y, W) = \beta \delta \varepsilon (n-1)(\varepsilon\alpha^2 - \beta^2 \delta)g(W, Y) - \alpha(n-1)(\varepsilon\alpha^2 - \beta^2 \delta)g(\phi W, Y)$$

$$+ \varepsilon \alpha S(Y, \phi W) + A(W)(n-1)(\varepsilon\alpha^2 - \beta^2 \delta)\eta(Y) - \varepsilon \eta(Y) \left[ \frac{A(W)r}{n} - B(W)(n-1) \right], \quad (56)$$

Take $Y = \xi$ in (56) we get

$$\beta \delta S(\xi, W) = \beta \delta (n-1)(\varepsilon\alpha^2 - \beta^2 \delta)\eta(W) + A(W)(n-1)(\varepsilon\alpha^2 - \beta^2 \delta) - \varepsilon \left[ \frac{A(W)r}{n} - B(W)(n-1) \right] \quad (57)$$
Applying Lemma (18), (57) reduces to

\[ S(\xi, W) = (n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)\eta(W) + \frac{(n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)}{\beta \delta} \eta(p_1)\eta(W) - \frac{\varepsilon}{\beta \delta} \eta(W) \left[ \frac{\eta(p_1) r}{n} - \eta(p_2)(n - 1) \right], \quad (58) \]

Make use of (15), (16) and (19) in (58), we obtain

\[ \lambda = -\frac{(n - 1)(\varepsilon \alpha^2 - \beta^2 \delta)}{\varepsilon} - \frac{(n - \varepsilon)}{n} \frac{\beta \delta A(\xi)}{(\beta \delta + A(\xi))} + \frac{B(\xi)(n - 1)}{(\beta \delta + A(\xi))}. \quad (59) \]

**Theorem 6.1.** Ricci Soliton in a Generalised Concircular φ-Recurrent (ε, δ)-Trans-sasakian manifold \((M, g)\) with two 1-forms \(A\) and \(B\) and a constant scalar curvature \(r\) is shrinking for non-negative \(A(\xi)\) and \(B(\xi)\).

### 7. Conclusion

Based on the nature of two one forms associated with the curvature conditions, Ricci solitons in generalised Ricci recurrent, φ-recurrent, pseudo-projective φ-recurrent and concircular φ-recurrent curvatures under \((\varepsilon, \delta)\)-Trans-sasakian manifolds is classified into expanding, shrinking and steady. This study may be extended to η -ricci solitons in real hyper surfaces of complex space forms.

### References