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On Special Types of Numbers

Research Article

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Abstract: A positive integer n is said to be multiply perfect number if there is a k such that $\sigma(n) = kn$, where $k \geq 1$. In this paper we survey some results of interest on perfect numbers, multiply perfect numbers, k-hyperperfect numbers, superperfect numbers and k-hyper super perfect numbers. To state some results established earlier we have (1). If $n = 3^{k-1}\left(3^k - 2\right)$ where $3^k - 2$ is prime, then n is a 2-hyperperfect number. (2). If $n = 3^{p-1}$ where p and $\frac{3^p-1}{2}$ are primes, then n is a super-hyperperfect number.

Keywords: k-perfect numbers, hyperperfect numbers.

1. Introduction

A positive integer n is said to be multiply perfect if there is a k such that $\sigma(n) = kn$, where $k \geq 1$. These are also called k-multiply perfect numbers.

For $k = 2$; $(6, 28, 496, 8128, \ldots)$,

For $k = 3$; $(120, 672, 523776, 459818240, \ldots)$,

For $k = 4$; $(30240, 32760, 2178540, \ldots)$,

For $k = 5$; $(14182439040, 459818240, \ldots)$,

For $k = 6$; $(154345556085770649600, \ldots)$.

2. Hyperperfect Number

A positive integer n is called k-hyperperfect number if $n = 1 + k[\sigma(n) - n - 1]$ or

$$\sigma(n) = \frac{k+1}{k}\,n + \frac{k-1}{k}$$

Example 2.1.

(1). If k is 1 then $\sigma(n) = 2n$. Therefore the numbers are $6, 28, 496, 8128, \ldots$.

(2). If k is 2 then $\sigma(n) = \frac{3}{2}n + \frac{1}{2}$. Therefore the numbers are $21, 2133, 19521, 176661, \ldots$.

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k	k-Hyperperfect number
1	$6, 28, 496, 8128, \ldots$
2	$21, 2133, 19521, 176661, \ldots$
3	$325, \ldots$
4	$1950625, 1220640625, \ldots$
6	$301, 16513, 60110701, \ldots$
10	$159841, \ldots$
12	$697, 2041, 1570153, 62722153, \ldots$

Table 1. **k-hyperperfect numbers for different k values**

Observations

- If a number is perfect iff it is 1-hyperperfect number.

Theorem 2.2. *If $n = 3^{k-1}\left(3^k - 2\right)$ where $3^k - 2$ is prime, then n is a 2-hyperperfect number.*

Proof. Since the divisor function σ is multiplicative and for a prime p and prime power, we have $\sigma(p) = p + 1$ and $\sigma(p^\alpha) = \frac{p^{\alpha+1}-1}{p-1}$. Then

$$\begin{aligned}
\sigma(n) &= \sigma(3^{k-1}(3^k-2)) \\
&= \sigma\left(3^{k-1}\right).\sigma\left(3^k-2\right) \\
&= \left(\frac{3^{(k-1)+1}-1}{3-1}\right).\left(3^k-2+1\right) \\
&= \frac{3^k-1}{2}.\left(3^k-1\right) \\
&= \frac{1}{2}\left(3^k-1\right).\left(3^k-1\right) \\
&= \frac{1}{2}(3^{2k}-2.3^k+1) \\
&= \frac{1}{2}\left(3^{2k}-2.3^k\right)+\frac{1}{2} \\
&= \frac{3^k}{2}\left(3^k-2\right)+\frac{1}{2} \\
&= \frac{3}{2}3^{k-1}\left(3^k-2\right)+\frac{1}{2}
\end{aligned}$$

$\because$ A positive integer n is called 2-Hyperperfect number if $n = \frac{3}{2}n + \frac{1}{2}$. Therefore Given n is a 2-Hyperperfect number. □

3. Super Perfect Number

A positive integer n is called super-perfect number if $\sigma(\sigma(n)) = 2n$.

Example 3.1. *The first few Super-perfect numbers are* $2, 4, 16, 64, 4096, 65536, 262144, \ldots$. *Since*

$$\sigma(2) = 1 + 2 = 3$$

$$\sigma(\sigma(2)) = \sigma(3) = 1 + 3 = 2(2)$$

Observations

- If n is an even superperfect number then n must be a power of 2, that is 2^{k-1}, where 2^{k-1} is a prime.
- If any odd superperfect numbers exist, they are square numbers.

Theorem 3.2. *If n is an even superperfect number, then $\phi(\phi(n)) = \frac{n}{4}$.*

Proof. Here ϕ is Euler's totient function. If n is an even superperfect number, then n is of the form 2^{p-1}. So,

$$\begin{aligned}
\phi(n) &= \phi\left(2^{p-1}\right) \\
&= 2^{p-1}\left(1-\frac{1}{2}\right) \\
&= 2^{p-1}\left(\frac{1}{2}\right) \\
\phi(\phi(n)) &= \phi\left(2^{p-2}\right) \\
&= 2^{p-2}\left(1-\frac{1}{2}\right) \\
\phi(\phi(n)) &= 2^{p-1}\frac{1}{2}\left(\frac{1}{2}\right) \\
\phi(\phi(n)) &= \frac{n}{4}
\end{aligned}$$

□

4. Super-hyper Perfect Number

If $\sigma(\sigma(n)) = \frac{1}{2}(3n+1)$, then n is called Super-hyperperfect number.

Example 4.1. *The first few Super-hyperperfect numbers are $9, 729, 531441, \ldots$. Since*

$$\begin{aligned}
\sigma(9) &= 1+3+9 = 13 \\
\sigma(\sigma(9)) &= \sigma(13) = 1+13 = 14 \\
\frac{1}{2}(3(9)+1) &= \frac{1}{2}(27+1) = 14
\end{aligned}$$

Therefore 9 is a Super-hyperperfect number.

Theorem 4.2. *If $n = 3^{p-1}$ where p and $\frac{3^p-1}{2}$ are primes, then n is a super-hyperperfect number.*

Proof. Given that $n = 3^{p-1}$

$$\begin{aligned}
\sigma(\sigma(n)) &= \sigma\left(\sigma\left(3^{p-1}\right)\right) \\
&= \sigma\left(\frac{3^p-1}{2}\right) \\
&= \frac{3^p-1}{2}+1, \quad \text{since } \frac{3^p-1}{2} \text{ is prime} \\
&= \frac{3^p}{2}-\frac{1}{2}+1 \\
&= \frac{3^p}{2}+\frac{1}{2} \\
&= \frac{3}{2}3^{p-1}+\frac{1}{2} \\
&= \frac{3}{2}n+\frac{1}{2}
\end{aligned}$$

Therefore $n = 3^{p-1}$ is a Super-hyperperfect number. □

References

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