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Modelling of an Inventory-Transportation Problem with Non Instantaneous Deteriorating Items Incorporating Partial Backlogging

Research Article

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Abstract: An optimal replacement policy of non-instantaneous deteriorating inventory has been developed to determine the transportation policy. The model has been studied considering exponentially increasing demand. The partial backlogging has been taken in to consideration which is depended on waiting time for next replacement. The total average cost of the system has been derived for various cases. We have also carried out the sensitivity analysis to identify the sensitiveness of the various parameter used in this model. Some interesting numerical examples have been presented to illustrate the model.

Keywords: Partial backlogging, non-instantaneous, transportation, deterioration.

1. Introduction

Inventory model play an important role in operation research and management science. The past two decades have seen growing interest in modelling the inventory system. As revenue is the primary goal of any organisation or industry and also since market is full of competitive environment so it become necessity to get more and more profit without increasing the price of the product. Consecutively researchers are compelled to find some other alternative to get more and more profit and so we need to optimise the cost and to maintain a proper planning. Most of the physical goods decay over the time and their utility is decreases with time such an item is considered as deteriorating item. Ghare and Schrader [6] found that the inventory consumption of deteriorating items was related with the negative exponential function of time. Further covert and Philip [2] extended the work of Ghare and Schrader considering deterioration rate as two parameter weibull distribution. Dave and Patel [5] developed the deteriorating inventory with linear demand. Many of the researcher work on the assumption that the demand is dependent upon the current stock level of the system.

Gupta and Vrat [7], Mondal and Phujdar [8], Singh et al. [13] are some of them who works with stock dependent demand but all the literature discussed above are assumed that at the time of shortage all the demand are either lost or completely backlogged but practically there are some customer willing to wait for next replacement while other go for other alternative. This is the case for partial backlogging. Park [9] and Wee [14] assumed constant partially backlogging rate during the shortage period. Chang and Lin [4] also consider the partial backlogging with stock dependent demand in which partial backlogging is

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function of waiting time. Wu et al. [15] considered the partial backlogging which is decreases with increasing waiting time for next replenishment. Chang and Dye [3] developed a model with partial backlogging. Prasad and Mukherjee [11] determined optimal price discount policy for two warehouse inventory problem. Qu et al. [12] have discussed integrated transportation system with modified periodic policy for multiple products. Bhunia and Shaikh [1] have incorporated transportation cost to inventory with inventory level depended demand. Prasad and Mukherjee [10] developed an inventory model under stock and non-linear time dependent demand in which deterioration follows two parameter Weibull distributions.

It is also observed that some item which are not start to deteriorate as soon as it comes to the retailer while after certain period of time it start to decay this is the case of non instantaneous deterioration. Wu et al. [15] established an optimal policy for non instantaneous deteriorating items with partial backlogging considering stock dependent demand. Yang et al. [16] considered non instantaneous deteriorating item with price dependent demand and partial backlogging. Chang and Lin [4] have studied non instantaneous deteriorating item with stock dependent consumption rate under inflation. In this article we consider a problem on non instantaneous deteriorating items with exponentially increasing demand and partially backlogging which is dependent on waiting time of next replenishment. In this model we have incorporated transportation cost and then we optimized the average cost of the system. We have also carried out the sensitivity analysis and some special case which include model with no shortage, completely backlogged, instantaneous deterioration etc. Two numerical examples also have been considered which are solved with the help of Mathematica software using Newton Rapshon method to solve the system of non linear equation.

2. Notations and Assumptions

The following fundamental notations and assumptions are used to derive the model.

t_d : Period of time during which there is no deterioration

A : Ordering cost per period

c_h : Holding cost per unit per unit time

c_d : Cost of deteriorated unit

c_s : Shortage cost per unit per unit time

c_l : Lost sale cost per unit

t_1 : Time when inventory level reaches to zero

T : The replenishment cycle

$C(T, t_1)$: Average cost of the system

I_0 : Maximum inventory level

$I_1(t)$: Inventory level at any time t, $0 < t < t_1$

$I_2(t)$: Inventory level at any time t, $t_1 \leq t \leq T$

θ : The constant rate of deterioration, where $0 < \theta << 1$

S : Shortage level

Q : Ordering quantity

c_{tl} : Transportation cost per load

c_{tp} : Transportation cost per item

(a). A constant fraction of on hand inventory deteriorates per unit time and deteriorated amount of item are not replaced.

(b). The replenishment is infinite and lead time is zero.

(c). The demand rate $D\left(t\right)$defined as

$$\mathrm{D(t)} = \begin{cases} \mathrm{a}e^{bt}, & 0 < \mathrm{t} < \mathrm{t}_1 \\ \alpha, & t_1 \le \mathrm{t} \le \mathrm{T} \end{cases}$$

(d). Shortage are allowed and partially backlogged. Backlogging rate is inversely proportional to waiting time that is longer the waiting time, smaller the backlogging rate and vice versa and it is defined as $B(T-t) = \frac{1}{1+\delta(T-t)}$, where $\delta > 0$ and $T - t$ is waiting time up to next replenishment.

(e). It is assumed that the item starts to deteriorate after a certain period of time t_d that is items are non instantaneous deteriorating item.

3. Mathematical Model and its Solution

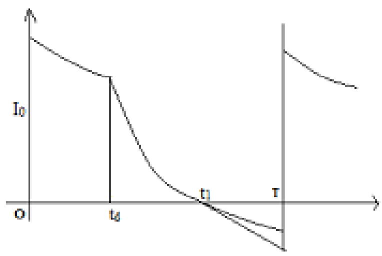

Figure 1: Geometry of the problem

We consider the non instantaneous deteriorating item with exponentially increasing demand with partial backlogging. The variation of inventory level during the given cycle is depicted in Figure 1. Initially I_0 amount of item are arrived at time $t = 0$ which is the maximum level of inventory during each cycle. Inventory level start to decreases up to t_d due to demand only and during the interval $[t_d, t_1]$ inventory level depleted to zero due to demand as well as deterioration. Shortage are allowed to occur during the time interval $[t_1, T]$ and all the demand during this shortage period are partially backlogged. On the basis of above discussion the dynamic of inventory level represented by following differential equations.

$$\frac{dI_1(t)}{dt} = -D(t), \;\; 0 \le t \le t_d \tag{1}$$

$$\text{or} \;\; \frac{dI_1(t)}{dt} = -ae^{bt}, \;\; 0 \le t \le t_d \tag{2}$$

With boundary condition $I_1(0) = I_0$, the solution of equation (2) can be given as

$$I_1(t) = -\frac{a}{b}(e^{bt} - 1) + I_0, \;\; 0 \le t \le t_d \tag{3}$$

During the time interval $[t_d, t_1]$ the inventory level depleted due to demand as well as deterioration and it is governed by the following differential equation

$$\frac{dI_2(t)}{dt} = -\theta I_2(t) - ae^{bt}, \;\; t_d \le t \le t_1 \tag{4}$$

With boundary condition $I_2(t_1) = 0$, the solution of equation (4) is given by

$$I_2(t) = \frac{a}{(b+\theta)} e^{-\theta t} \left[e^{(b+\theta)t_1} - e^{(b+\theta)t} \right], \;\; t_d \le t \le t_1 \tag{5}$$

Considering the continuity of $I(t)$ at $t = t_d$, then from equation (3) and (5)

$$I_1(t_d) = I_2(t_d)$$

$$\text{or} \quad -\frac{a}{b}(e^{bt_d} - 1) + I_0 = \frac{a}{(b+\theta)}e^{-\theta t}\left[e^{(b+\theta)t_1} - e^{(b+\theta)t_d}\right],$$

then the maximum inventory level is given by

$$I_0 = \frac{a}{(b+\theta)}e^{-\theta t_d}\left[e^{(b+\theta)t_1} - e^{(b+\theta)t_d}\right] + \frac{a}{b}(e^{bt_d} - 1) \tag{6}$$

Using equation (6) in equation (3) we get,

$$I_1(t) = -\frac{a}{b}(e^{bt} - 1) + \frac{a}{(b+\theta)}e^{-\theta t_d}\left[e^{(b+\theta)t_1} - e^{(b+\theta)t_d}\right] + \frac{a}{b}(e^{bt_d} - 1), \quad 0 \le t \le t_d \tag{7}$$

The behavior of shortage level at time t during the time interval $[t_1, T]$ is described by the following differential equation.

$$\frac{dI_3(t)}{dt} = -\frac{\alpha}{1 + \delta(T - t)}, \; t_1 \le t \le T \tag{8}$$

With the boundary condition $I_3(t_1) = 0$, then the solution of the equation (8) is represented by

$$I_3(t) = -\frac{\alpha}{\delta}\log\frac{1 + \delta(T - t_1)}{1 + \delta(T - t)}, \; t_1 \le t \le T \tag{9}$$

Now at time $t = T$ we get the maximum amount of demand backlogged per period is given by

$$S \equiv |I_3(T)| = \left|-\frac{\alpha}{\delta}\log[1 + \delta(T - t_1)]\right| \tag{10}$$

The order quantity can be obtained from equation (6) and(10) that is represented by $Q = I_0 + S$ or

$$Q = \frac{a}{(b+\theta)}e^{-\theta t_d}\left[e^{(b+\theta)t_1} - e^{(b+\theta)t_d}\right] + \frac{a}{b}(e^{b\,t_d} - 1) + \frac{\alpha}{\delta}\log[1 + \delta(T - t_1)] \tag{11}$$

The total relevant inventory cost per cycle involves following factors.

(a). Ordering cost per cycle is A.

(b). Inventory holding/storage cost of the system is given by

$$\begin{aligned} HC &= c_h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt\right] \\ &= c_h\left[\int_0^{t_d}\left\{-\frac{a}{b}(e^{bt} - 1) + I_0\right\}dt + \int_{t_d}^{t_1}\frac{a}{(b+\theta)}e^{-\theta t}\left\{e^{(b+\theta)t_1} - e^{(b+\theta)t}\right\}dt\right] \\ &= c_h\left[\frac{a}{b^2}(1 - e^{bt_d}) + \frac{a}{b}t_d + I_0 t_d - \frac{a}{b(b+\theta)}\left(e^{bt_1} - e^{bt_d}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{(t_1 - t_d)\theta}\right)\right] \end{aligned} \tag{12}$$

(c). The deterioration cost per cycle is represented by

$$\begin{aligned} DC &= c_d\left[I_2(t_d) - \int_{t_d}^{t_1} D(t)dt\right] \\ &= c_d\left[-\frac{a}{(b+\theta)}e^{-\theta t_d}\left(e^{(b+\theta)t_d} - e^{(b+\theta)t_1}\right) - \int_{t_d}^{t_1} ae^{bt}dt\right] \\ &= c_d\left[\frac{a}{(b+\theta)}\left(e^{(b+\theta)t_1 - \theta t_d} - e^{bt_d}\right) - \frac{a}{b}\left(e^{bt_1} - e^{bt_d}\right)\right] \end{aligned} \tag{13}$$

(d). The shortage cost in the entire cycle is described by

$$\begin{aligned} SC &= -c_s \int_{t_1}^{T} I_3(t)dt \\ &= \frac{c_s\alpha}{\delta}\int_{t_1}^{T} \left[\log\left(1+\delta(T-t_1)\right) - \log\left(1+\delta(T-t)\right)\right] dt \\ &= \frac{c_s\alpha}{\delta}\left[(T-t_1) - \frac{1}{\delta}\log\left(1+\delta(T-t_1)\right)\right] \end{aligned} \tag{14}$$

(e). The cost due to lost sales is given by

$$\begin{aligned} LC &= c_l \int_{t_1}^{T} \alpha\left(1 - \frac{1}{1+\delta(T-t)}\right) dt \\ &= c_l\alpha\left[(T-t_1) - \frac{1}{\delta}\log\left(1+\delta(T-t_1)\right)\right] \end{aligned} \tag{15}$$

(f). Transportation cost of the system is defined as

$$TC = \begin{cases} nc_{tl} + (Q-nk)\,c_{tp}, & nk < Q \le nk+R \\ (n+1)\,c_{tl}, & nk+R < Q < (n+1)k \end{cases} \tag{16}$$

Where, $n =$ Number of truck fully loaded and it is computed as $n = \left[\frac{Q}{k}\right]$; $k =$Number of item per load and $R = \left[\frac{c_{tl}}{c_{tp}}\right] < k$, where [ ] indicates greatest integer function. Total average cost of the system per cycle is given by

$$C\left(t_1, T\right) = \frac{1}{T}\left[A + HC + DC + SC + LC + TC\right]$$

Case (a): If $nk < Q \le nk+R$

$$C(t_1,T) = \frac{1}{T}\left[\begin{array}{l} A + c_h\left\{\frac{a}{b^2}\left(1-e^{b\,t_d}\right) + \frac{a}{b}t_d + I_0\,t_d - \frac{a}{b(b+\theta)}\left(e^{b\,t_1} - e^{b\,t_d}\right) - \frac{ae^{b\,t_1}}{\theta(b+\theta)}\left(1-e^{(t_1-t_d)\,\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)\,t_1-\theta\,t_d} - e^{b\,t_d}\right) - \frac{a}{b}\left(e^{b\,t_1} - e^{b\,t_d}\right)\right\} \\ + \frac{\alpha(c_s+c_l\delta)}{\delta}\left\{(T-t_1) - \frac{1}{\delta}\log\left(1+\delta(T-t_1)\right)\right\} + n\,c_{tl} + (Q-n\,k)c_{tp} \end{array}\right] \tag{17}$$

Case (b): If $nk+R < Q < (n+1)k$

$$C(t_1,T) = \left[\begin{array}{l} A + c_h\left\{\frac{a}{b^2}\left(1-e^{b\,t_d}\right) + \frac{a}{b}t_d + I_0 t_d - \frac{a}{b(b+\theta)}\left(e^{b\,t_1} - e^{b\,t_d}\right) - \frac{a\,e^{b\,t_1}}{\theta(b+\theta)}\left(1-e^{(t_1-t_d)\,\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)\,t_1-\theta\,t_d} - e^{b\,t_d}\right) - \frac{a}{b}\left(e^{b\,t_1} - e^{b\,t_d}\right)\right\} \\ + \frac{\alpha(c_s+c_l\delta)}{\delta}\left\{(T-t_1) - \frac{1}{\delta}\log\left(1+\delta(T-t_1)\right)\right\} + (n+1)c_{tl} \end{array}\right] \tag{18}$$

The necessary condition for minimization of cost are given by

$$\frac{\partial C}{\partial t_1} = 0 \text{ and } \frac{\partial C}{\partial T} = 0 \tag{19}$$

Provided that the optimal value for t_1 and T obtained from equation (19) satisfy the sufficient condition

$$\frac{\partial^2 C}{\partial t_1^2}\frac{\partial^2 C}{\partial T^2} - \left(\frac{\partial^2 C}{\partial t_1 \partial T}\right)^2 > 0 \tag{20}$$

3.1. Cost function for some special cases

Case 1: Model with completely backlogging, for this case $\delta = 0$ or $B(T-t) = 1$ then $LC = 0$ and $Q = I_0 + \alpha(T - t_1)$, the relevant cost per unit time is represented as

$$C_1(t_1,T) = \begin{bmatrix} A + c_h\left\{\frac{a}{b^2}(1-e^{bt_d}) + \frac{a}{b}t_d + I_0 t_d - \frac{a}{b(b+\theta)}\left(e^{bt_1} - e^{bt_d}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{(t_1-t_d)\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)t_1-\theta t_d} - e^{bt_d}\right) - \frac{a}{b}\left(e^{bt_1} - e^{bt_d}\right)\right\} + \frac{c_s\alpha}{2}(T-t_1)^2 \\ +nc_{tl} + (Q-nk)\,c_{tp} \end{bmatrix}, nk < Q \le nk + R \quad (21)$$

$$C_1(t_1,T) = \begin{bmatrix} A + c_h\left\{\frac{a}{b^2}(1-e^{bt_d}) + \frac{a}{b}t_d + I_0 t_d - \frac{a}{b(b+\theta)}\left(e^{bt_1} - e^{bt_d}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{(t_1-t_d)\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)t_1-\theta t_d} - e^{bt_d}\right) - \frac{a}{b}\left(e^{bt_1} - e^{bt_d}\right)\right\} + \frac{c_s\alpha}{2}(T-t_1)^2 + (n+1)c_{tl} \end{bmatrix}, nk + R < Q < (n+1)k \quad (22)$$

Case 2: Model without shortage that is $\delta \to \infty$ then $T = t_1$, $SC = 0$, $LC = 0$ and

$$Q = \frac{a}{(b+\theta)}e^{-\theta t_d}\left[e^{(b+\theta)T} - e^{(b+\theta)t_d}\right] + \frac{a}{b}(e^{bt_d} - 1)$$

then the total relevant cost per unit time is represented by

$$C_2(T) = \frac{1}{T}\begin{bmatrix} A + c_h\left\{\frac{a}{b^2}(1-e^{bt_d}) + \frac{a}{b}t_d + I_0 t_d - \frac{a}{b(b+\theta)}\left(e^{bt_1} - e^{bt_d}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{(t_1-t_d)\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)T-\theta t_d} - e^{bt_d}\right) - \frac{a}{b}\left(e^{bT} - e^{bt_d}\right)\right\} + nc_{tl} + (Q-nk)\,c_{tp} \end{bmatrix}, nk < Q \le nk + R \quad (23)$$

$$C_2(T) = \frac{1}{T}\begin{bmatrix} A + c_h\left\{\frac{a}{b^2}(1-e^{bt_d}) + \frac{a}{b}t_d + I_0 t_d - \frac{a}{b(b+\theta)}\left(e^{bt_1} - e^{bt_d}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{(t_1-t_d)\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)T-\theta t_d} - e^{bt_d}\right) - \frac{a}{b}\left(e^{bT} - e^{bt_d}\right)\right\} + (n+1)c_{tl} \end{bmatrix}, nk + R < Q < (n+1)k \quad (24)$$

Case 3: Model with instantaneous deterioration and completely backlogging. In this situation $t_d = 0$ and $\delta = 0$ then

$$Q = \frac{a}{(b+\theta)}\left[e^{(b+\theta)t_1} - 1\right] + \alpha(T - t_1)$$

$$C_3(t_1,T) = \begin{bmatrix} A + c_h\left\{\frac{a}{b(b+\theta)}\left(1 - e^{bt_1}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{t_1\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)t_1} - 1\right) - \frac{a}{b}\left(e^{bt_1} - 1\right)\right\} + \frac{c_s\alpha}{2}(T-t_1)^2 + nc_{tl} + (Q-nk)\,c_{tp} \end{bmatrix}, nk < Q \le nk + R \quad (25)$$

$$C_3(t_1,T) = \begin{bmatrix} A + c_h\left\{\frac{a}{b(b+\theta)}\left(1 - e^{bt_1}\right) - \frac{ae^{bt_1}}{\theta(b+\theta)}\left(1 - e^{t_1\theta}\right)\right\} \\ +c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)t_1} - 1\right) - \frac{a}{b}\left(e^{bt_1} - 1\right)\right\} + \frac{c_s\alpha}{2}(T-t_1)^2 + (n+1)c_{tl} \end{bmatrix}, nk + R < Q < (n+1)k \quad (26)$$

Case 4: Model with instantaneous deterioration and without shortage. In this situation $t_d = 0$, $\delta \to \infty$ then $T = t_1$ and order amount of quantity is given by $Q = \frac{a}{(b+\theta)}\left[e^{(b+\theta)T} - 1\right]$

$$C_4(T) = \begin{bmatrix} A + c_h\left\{\frac{a}{b(b+\theta)}\left(1 - e^{bT}\right) - \frac{ae^{bT}}{\theta(b+\theta)}\left(1 - e^{\theta T}\right)\right\} + c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)T} - 1\right) - \frac{a}{b}\left(e^{bT} - 1\right)\right\} \\ +nc_{tl} + (Q-nk)\,c_{tp} \end{bmatrix}, nk < Q \le nk + R \quad (27)$$

$$C_4(T) = \begin{bmatrix} A + c_h\left\{\frac{a}{b(b+\theta)}\left(1 - e^{bT}\right) - \frac{ae^{bT}}{\theta(b+\theta)}\left(1 - e^{\theta T}\right)\right\} + c_d\left\{\frac{a}{(b+\theta)}\left(e^{(b+\theta)T} - 1\right) - \frac{a}{b}\left(e^{bT} - 1\right)\right\} \\ +(n+1)c_{tl} \end{bmatrix}, nk + R < Q < (n+1)k \quad (28)$$

4. Illustration of the Model Using Numerical Examples

Example 4.1. *For the numerical illustration we consider an inventory system with the following parameters* $A = 10$, $c_h = 1$, $c_d = 2$, $c_s = 5$, $c_l = 1.5$, $a = 4$, $b = 0.4$, $\theta = 0.002$, $\alpha = 6$, $\delta = 3$, $t_d = 0.5$, $k = 5$, $c_{tl} = 5$, $c_{tp} = 1.25$. *Then using equation* (17), *we get,*

The optimal order quantity $Q^* = 8.32851$,

Optimal time of on hand inventory $t_1^* = 1.16877$,

Optimal cycle $T^* = 1.92336$,

Optimal average cost $C^* = 15.4765$.

Example 4.2. $A = 10$, $c_h = 1$, $c_d = 1$, $c_s = 6$, $c_l = 1.5$, $a = 6$, $b = 0.2$, $\theta = 0.002$, $\alpha = 6$, $\delta = 3$, $t_d = 0.5$, $k = 5$, $c_{tl} = 5$, $c_{tp} = 1.25$. *From equation* (18), *we obtained,*

The optimal order quantity $Q^* = 19.4216$,

Optimal time of on hand inventory $t_1^* = 2.04289$,

Optimal cycle $T^* = 4.51707$,

Optimal average cost $C^* = 18.5067$.

4.1. Sensitivity Analysis

Effect of various parameters used in the model can be seen from the table given below. This table shows the sensitiveness of the various parameter on optimal average cost C^*, optimal ordering quantity Q^*, optimal cycle T^* and optimal time t_1^* of on hand inventory.

Parameters	% Chang in	t_1^*	T^*	Q^*	C^*
θ	-50	+1.88	+0.28	+1.02	-0.32
	-25	+0.93	+0.13	+0.50	-0.16
	+25	-0.89	-0.12	-0.49	+0.15
	+50	-1.76	-0.24	-0.96	+0.30
t_d	-50	-0.51	+0.36	+0.08	+0.27
	-25	-0.27	+0.15	+0.01	+0.12
	+25	+0.30	-0.10	+0.05	-0.11
	+50	+0.62	-0.13	+0.16	-0.20
α	-20	-10.53	+69.14	+1.29	-9.83
	-10	-4.25	+21.52	+0.98	-4.05
	+10	+2.90	-11.21	-1.29	+2.84
	+20	+4.93	-17.76	-2.74	+4.87
δ	-50	+4.39	-33.79	-5.06	+4.24
	-25	+2.63	-24.07	-3.01	+2.52
	+25	-3.87	+80.17	+4.52	-3.63
	+50	-8.77	+450.84	+10.77	-8.09
a	-50	+41.34	-6.53	+3.23	-26.73
	-25	+17.81	-8.50	+1.35	-11.20
	+25	-15.24	+19.85	-1.03	+7.50
	+50	-28.78	+51.53	-1.85	+12.20
b	-50	+23.47	+1.82	+3.23	-6.05
	-25	+10.33	-0.10	+1.35	-2.75
	+25	-8.42	+1.52	-1.03	+2.29
	+50	-15.45	+4.09	-1.85	+4.22

Table 1:

It can be observed from the table that average cost C^* of the system is increases if we increase the parameter θ, α, a, b, where as t_d and δ have reverse effect on average cost C^* that is average cost decreases with increase in the parameters t_d and δ. Also table reflects that α, δ, a and b are more sensitive than θ and t_d.

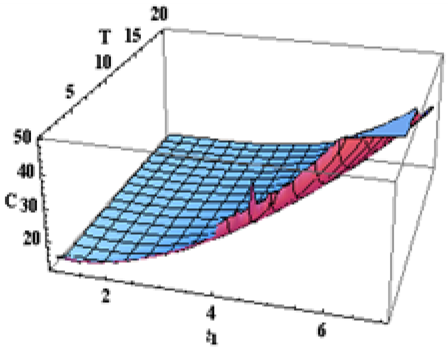

Figure 2: Variation of average cost versus t_1 & T

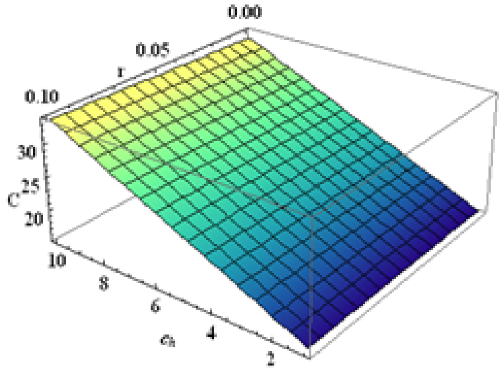

Figure 3: Variation of average cost versus c_h & r

5. Conclusion

In this article, a deterministic inventory system model for non instantaneous deteriorating item with exponentially increasing demand with partial backlogging has been developed. We determine an optimal replacement policy, shortage period, ordering quantity for which the average inventory cost can be minimized. Sensitivity analysis with respect to various parameters has been carried out. From the fig.2 it is observed that as period of on hand inventory that is as the value of t_1 increases the value of average cost is also increases while on the other hand if the value of replenishment time T increases the value of average cost decreases. This indicates that period of on hand inventory t_1 is the major factor to increase the average cost of the system while due to increase in replenishment time T most of the customer leave the retailer or shop because they cannot wait more and as a result less amount of shortage is occur in a big span of time period $[t_1, T]$. Hence average cost is reduces with increase in replenishment time T. It can also be observed from the figure 3 that as the value of holding parameter c_h is increases then the value of average cost of system increases rapidly but as the deterioration parameter θ increases then the average cost of the system increases gradually that is effect of deterioration parameter θ is very much less on average cost of the system. The integrated inventory transportation systems we consider to exhibit more realistic feature on deteriorating inventory control and transportation management.

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