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Strong Form of Some Nano Open Sets

Research Article

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Abstract: The purpose of this paper, nano $\mathcal{I}_{\mathcal{R}}$-set, nano $\mathcal{R}_{\mathcal{S}}$-set, nano pre-regular, nano β-regular and nano weak $\mathcal{O}_{\mathcal{N}}$-set are introduced and investigated on the line of research.

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1. Introduction and Preliminaries

Lellis Thivagar et al [2] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological spaces and Rajasekaran et.al [7] introduced the notion of nano $t^{\#}$-sets and nano t_α-sets and nano topological spaces. In this paper, nano $\mathcal{I}_{\mathcal{R}}$-set, nano $\mathcal{R}_{\mathcal{S}}$-set, nano pre-regular, nano β-regular and nano weak $\mathcal{O}_{\mathcal{N}}$-set are introduced and investigated on the line of research.

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 1.1 ([4]). *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

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(2). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. *That is,* $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. *That is,* $B_R(X) = U_R(X) - L_R(X)$.

Property 1.2 ([2]). *If* (U, R) *is an approximation space and* $X, Y \subseteq U$*; then*

(1). $L_R(X) \subseteq X \subseteq U_R(X)$;

(2). $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;

(3). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6). $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7). $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

(8). $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;

(9). $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

(10). $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.3 ([2]). *Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$. *Then by the Property 1.2, R(X) satisfies the following axioms:*

(1). *U and* $\phi \in \tau_R(X)$,

(2). *The union of the elements of any sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$,

(3). *The intersection of the elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ *is a topology on U called the nano topology on U with respect to X. We call* $(U, \tau_R(X))$ *as the nano topological space. The elements of* $\tau_R(X)$ *are called as nano open sets and* $[\tau_R(X)]^c$ *is called as the dual nano topology of* $[\tau_R(X)]$.

Remark 1.4 ([2]). *If* $[\tau_R(X)]$ *is the nano topology on U with respect to X, then the set* $B = \{U, \phi, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 1.5 ([2]). *If* $(U, \tau_R(X))$ *is a nano topological space with respect to X and if* $H \subseteq U$, *then the nano interior of H is defined as the union of all nano open subsets of A and it is denoted by* $Nint(H)$. *That is,* $Nint(H)$ *is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by* $Ncl(H)$. *That is,* $Ncl(H)$ *is the smallest nano closed set containing H.*

Definition 1.6 ([2]). *A subset H of a nano topological space* $(U, \tau_R(X))$ *is called;*

(1). *nano pre open set if* $H \subseteq Nint(Ncl(H))$.

(2). *nano semi open set if* $H \subseteq Ncl(Nint(H))$.

(3). nano α-open set if $H \subseteq Nint(Ncl(Nint(H)))$.

(4). nano β-open if $H \subseteq Ncl(Nint(Ncl(H)))$.

(5). nano b-open set if $H \subseteq Ncl(Nint(H)) \cup Nint(Ncl(H))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 1.7. *A subset H of a space $(U, \tau_R(X))$ is called;*

(1). a nano t-set [1] if $Nint(H) = Nint(Ncl(H))$,

(2). a nano $t^{\#}$-set [6] if $Nint(H) = Ncl(Nint(H))$,

(3). a nano t_α-set [6] if $Nint(H) = Ncl(Nint(Ncl(H)))$,

(4). a nano $\mathcal{B}$-set [1] if $H = U \cap V$, where $U \in \tau$ and V is a nano t-set.

Definition 1.8 ([7])**.** *A subset H of a space $(U, \tau_R(X))$ is said to be nano semi-regular if H is nano semi-open and a nano t-set.*

Remark 1.9 ([6])**.** *The diagram holds for any subset of a space $(U, \tau_R(X))$:*

		nano α-open	$\longrightarrow$	***nano pre-open***
nano open	$\nearrow$	$\downarrow$		$\downarrow$
	$\searrow$	***nano semi-open***	$\longrightarrow$	***nano b-open***
				$\downarrow$
				nano β-open

In this diagram, none of the implications is reversible.

2. Nano $\mathcal{I}_{\mathcal{R}}$-set

Definition 2.1. *A subset H of a space $(U, \tau_R(X))$ is called a nano $\mathcal{I}_{\mathcal{R}}$-set if $H = P \cap Q$ where P is nano open and Q is a nano semi-regular.*

Example 2.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. Then $\{a\}$ is nano $\mathcal{I}_{\mathcal{R}}$-set.*

Proposition 2.3. *In a space $(U, \tau_R(X))$,*

(1). Every nano open set is a nano $\mathcal{I}_{\mathcal{R}}$-set.

(2). Every nano semi-regular set is a nano $\mathcal{I}_{\mathcal{R}}$-set.

Proof. This is obvious from the definition of a nano $\mathcal{I}_{\mathcal{R}}$-set. □

Remark 2.4. *The converses of (1) and (2) in Proposition 2.3 are not true as seen from the following Example.*

Example 2.5. *In Example 2.2,*

(1). then $\{a, c\}$ is nano $\mathcal{I}_{\mathcal{R}}$-set but not nano open set.

(2). then $\{a, b, d\}$ is nano $\mathcal{I}_{\mathcal{R}}$-set but not nano semi-regular.

Theorem 2.6. *For a subset H of a space $(U, \tau_R(X))$, the following are equivalent:*

(1). H is nano open;

(2). H is nano pre-open and a nano $\mathcal{I}_\mathcal{R}$-set.

Proof. (1)⇒(2): This is obvious by Remark 1.9 and (1) of Proposition 2.3.
(2)⇒(1): Given H is nano pre-open and a nano $\mathcal{B}$-set. Since H is a nano $\mathcal{I}_\mathcal{R}$-set, $H = P \cap Q$ where P is nano open and Q is nano semi-regular. Then we have $H \subset P = Nint(P)$. Also H is nano pre-open implies $H \subset Nint(Ncl(H)) \subset Nint(Ncl(Q)) = Nint(Q)$ for Q is a nano t-set being nano semi-regular. Thus $H \subset Nint(P) \cap Nint(Q) = Nint(P \cap Q) = Nint(H)$ and hence H is nano open. □

Remark 2.7. *The notions of nano pre-open and being a nano $\mathcal{I}_\mathcal{R}$-set are independent.*

Example 2.8. *In Example 2.2,*

(1). then $\{a, c\}$ is nano $\mathcal{I}_\mathcal{R}$-set but not nano pre-open.

(2). then $\{b\}$ is nano pre-open set but not nano $\mathcal{I}_\mathcal{R}$-set.

Definition 2.9. *A subset H of a space $(U, \tau_R(X))$ is called nano pre-regular if H is nano pre-open and an nano $t^\#$-set.*

Example 2.10. *In Example 2.2, then $\{d\}$ is nano pre-regular.*

Proposition 2.11. *In a space $(U, \tau_R(X))$,*

(1). Every nano pre-regular set is a nano pre-open.

(2). Every nano pre-regular set is a nano $t^\#$-set.

Remark 2.12. *The converses of (1) and (2) in Proposition 2.11 are not true as seen from the following Example.*

Example 2.13. *In Example 2.2,*

(1). then $\{a, b\}$ is nano pre-open set but not nano pre-regular.

(2). then $\{b, c\}$ is nano $t^\#$-set but not nano pre-regular.

3. Nano $\mathcal{R}_\mathcal{S}$-set

Definition 3.1. *A subset H of a space $(U, \tau_R(X))$ is called a nano $\mathcal{R}_\mathcal{S}$-set if $H = P \cap Q$ where P is nano open and Q is nano pre-regular.*

Example 3.2. *In Example 2.2, then $\{a, b, d\}$ is nano $\mathcal{R}_\mathcal{S}$-set.*

Proposition 3.3. *In a space $(U, \tau_R(X))$,*

(1). Every nano open set is a nano $\mathcal{R}_\mathcal{S}$-set.

(2). Every nano pre-regular set is a nano $\mathcal{R}_\mathcal{S}$-set.

Proof. Proof follows directly from the definition of a nano $\mathcal{R}_\mathcal{S}$-set. □

Remark 3.4. *The converses of (1) and (2) in Proposition 3.3 are not true as seen from the following Example.*

Example 3.5. *In Example 2.2,*

(1). then $\{d\}$ is nano $\mathcal{R}_{\mathcal{S}}$-set but not nano open set.

(2). then $\{b, d\}$ is nano $\mathcal{R}_{\mathcal{S}}$-set but not nano pre-regular.

Theorem 3.6. *For a subset H of a space $(U, \tau_R(X))$, the following are equivalent:*

(1). H is nano open;

(2). H is nano semi-open and a nano $\mathcal{R}_{\mathcal{S}}$-set.

Proof. (1)⇒(2): (2) follows from Remark 1.9 and (1) of Proposition 3.3.
(2)⇒(1): Given H is nano semi-open and a nano $\mathcal{R}_{\mathcal{S}}$-set. Since H is a nano $\mathcal{R}_{\mathcal{S}}$-set, $H = P \cap Q$ where P is nano open and Q is nano pre-regular. Then $H \subset P = Nint(P)$ and $H \subset Q$. Also H is nano semi-open implies $H \subset Ncl(Nint(H)) \subset Ncl(Nint(Q)) = Nint(Q)$ since Q is a nano $t^{\#}$-set being nano pre-regular. Thus $H \subset Nint(P) \cap Nint(Q) = Nint(P \cap Q) = Nint(H)$ and hence H is nano open. □

Remark 3.7. *The notions of nano semi-open and being a nano $\mathcal{R}_{\mathcal{S}}$-set are independent.*

Example 3.8. *In Example 2.2,*

(1). then $\{a, c\}$ is nano semi-open but not nano $\mathcal{R}_{\mathcal{S}}$-set.

(2). then $\{b\}$ is nano $\mathcal{R}_{\mathcal{S}}$-set but not nano semi-open.

Definition 3.9. *A subset H of a space $(U, \tau_R(X))$ is called nano β-regular if H is nano β-open and a nano t_α-set.*

Example 3.10. *In Example 2.2, then $\{\phi, U\}$ is nano β-regular.*

Proposition 3.11. *In a space $(U, \tau_R(X))$,*

(1). Every nano β-regular set is a nano β-open.

(2). Every nano β-regular set is a nano t_α-set.

Remark 3.12. *The converses of (1) and (2) in Proposition 3.11 are not true as seen from the following Example.*

Example 3.13.

(1). Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, U\}$. Then $\{b, c, d\}$ is nano β-open but not nano β-regular.

(2). Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{b, c\}$. Then the nano topology $\tau_R(X) = \{\phi, \{b, c\}, U\}$. Then $\{a\}$ is a nano t_α-set but not nano β-regular.

4. Nano Weak $\mathcal{O}_{\mathcal{N}}$-set

Definition 4.1. *A subset H of a space $(U, \tau_R(X))$ is called a nano weak $\mathcal{O}_{\mathcal{N}}$-set if $H = P \cap Q$, where P is nano open and Q is nano β-regular.*

Example 4.2. *In Example 2.2, then $\{a, b, d\}$ is nano weak $\mathcal{O}_{\mathcal{N}}$-set.*

Proposition 4.3. *In a space $(U, \tau_R(X))$, every nano β-regular set is a nano weak $\mathcal{O}_{\mathcal{N}}$-set.*

Remark 4.4. *The converse of (1) in Proposition 4.3 are not true as seen from the following Example.*

Example 4.5. *In Example 2.2, then $\{b, d\}$ is nano weak $\mathcal{O}_{\mathcal{N}}$-set but not nano β-regular.*

Proposition 4.6. *In a space $(U, \tau_R(X))$, every nano weak $\mathcal{O}_{\mathcal{N}}$-set is nano β-open.*

Proof. Let H be a nano weak $\mathcal{O}_{\mathcal{N}}$-set. Then $H = G \cap V$, where G is nano open and V is nano β-regular. Hence V is nano β-open. So $H = G \cap V \subseteq G \cap Ncl(Nint(Ncl(V))) \subseteq Ncl(G \cap Nint(Ncl(V))) = Ncl(Nint(G) \cap Nint(Ncl(V))) = Ncl(Nint(G \cap Ncl(V))) \subseteq Ncl(Nint(Ncl(G \cap V))) = Ncl(Nint(Ncl(H)))$. Hence H is nano β-open. □

Theorem 4.7. *For a subset H of a space $(U, \tau_R(X))$, the following are equivalent:*

(1). H is nano β-regular.

(2). H is a nano t_α-set and a nano weak $\mathcal{O}_{\mathcal{N}}$-set.

Proof. (1)⇒(2): Proof follows directly since every nano β-regular set is a nano t_α-set by definition and a nano weak $\mathcal{O}_{\mathcal{N}}$-set by (2) of Proposition 4.3.

(2)⇒(1): Let H be a nano t_α-set and a nano weak $\mathcal{O}_{\mathcal{N}}$-set. Since H is a nano weak $\mathcal{O}_{\mathcal{N}}$-set, by Proposition 4.6 H is nano β-open. Thus H is a nano t_α-set as well as nano β-open. Hence H is nano β-regular. □

Remark 4.8. *The concepts of being a nano t_α-set and being a nano weak $\mathcal{O}_{\mathcal{N}}$-set are independent.*

Example 4.9. *In Example 3.13(2),*

(1). then $\{b, c\}$ is nano weak $\mathcal{O}_{\mathcal{N}}$-set but not nano t_α-set.

(2). then $\{a\}$ is nano t_α-set but not nano weak $\mathcal{O}_{\mathcal{N}}$-set.

Theorem 4.10. *For a subset H of a space $(U, \tau_R(X))$, the following are equivalent:*

(1). H is nano open;

(2). H is nano α-open and a nano weak $\mathcal{O}_{\mathcal{N}}$-set.

Proof. (1)⇒(2): Proof follows directly by Remark 1.9 and by (1) of Proposition 4.3.

(2)⇒(1): Given H is nano α-open and a nano weak $\mathcal{O}_{\mathcal{N}}$-set. Since H is a nano weak $\mathcal{O}_{\mathcal{N}}$-set, $H = P \cap Q$ where P is nano open and Q is nano β-regular. Then $H \subset P = Nint(P)$ and $H \subset Q$. Also H is nano α-open implies $H \subset Nint(Ncl(Nint(H))) \subset Nint(Ncl(Nint(Q))) = Nint(Q)$ for Q is a nano t_α-set being nano β-regular. Thus $H \subset Nint(P) \cap Nint(Q) = Nint(P \cap Q) = Nint(H)$ and hence H is nano open. □

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