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Absolute Mean Graceful Labeling in Various Graphs

Research Article

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Abstract: In the pesent study of paper, we defined absolute mean graceful labeling for various graphs. We proved all path graphs P_n, cycle C_n, complete bipartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n and double step grid graph DSt_n are absolute mean graceful graphs.

MSC: 05C78.

Keywords: Graphs, bipartite graph, absolute mean graceful labeling.

1. Introduction

Throughout this paper, we will consider $G = (p, q)$, a finite, simple and undirected graph with $V(G)$-vertex set having p vertices and $E(G)$-edge set having q edges. For a graph $G = (V, E)$, a function having domain V or E or $V \cup E$ is known as a graph labeling for G. Graceful labeling for a graph G is well known concept introduced by Rosa [1]. Golomb [2] named this labeling as graceful labeling which was in past known as β-valuation. Kaneria, Makadia and Meghapara [3] proved graceful labeling for grid related graph. Kaneria and Makadia [4] proved graceful labeling for double step grid graph. All path graphs P_n, cycle C_n, complete bipartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n and double step grid graph DSt_n were proved graceful graphs in the early researches in study of graceful lageling. We provided more liberty to domain of vertex labeling function with positive as well as negative integers and defined more flexible labeling as absolute mean graceful labeling. The bijective property for edge labeling function holds same as in graceful labeling. For detail survey of graph labeling, we referred Gallian [5]. Take path P_n, P_n, P_{n-1}, ..., P_3, P_2 and arrange them vertically. A graph obtained by joining horizontal vertices of paths P_n, P_n, P_{n-1}, ..., P_3, P_2 is known as step grid graph of size n and denoted by St_n, where $n \geq 3$. It is obvious that $\mid V(St_n) \mid = \frac{1}{2}(n^2 + 3n - 2)$ and $\mid E(St_n) \mid = n^2 + n - 2$. Similarly take P_2, P_3, ..., P_n, P_n, P_{n-1}, ..., P_3, P_2 and arrange them vertically. By joining horizontal vertices of these paths is known as a double step grid graph of size n and denoted by DSt_n. Here we introduce following definition.

Definition 1.1. *A function f is called an absolute mean graceful labeling of a graph $G = (V, E)$, if $f : V(G) \to \{0, \pm 1, \pm 2, ..., \pm q\}$ is injective and the induced function $f^* : E(G) \to \{1, 2, ..., q\}$ defined as $f^*(e) = \left\lceil \frac{\mid f(u) - f(v) \mid}{2} \right\rceil$ is bijective for every edge $e = (u, v) \in E(G)$. A graph is called absolute mean graceful, if it admits absolute mean graceful labeling.*

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2. Main Results

Theorem 2.1. *Every path graph P_n, $\forall\ n \geq 2$ is absolute mean graceful graph.*

Proof. Let P_n be path graph with vertex set $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and q be number of edges. Obviously $q = n - 1$. Define vertex labeling function $f : V(P_n) \rightarrow \{\ 0,\ \pm1,\ \pm2,\ ...,\ \pm q\}$ defined by $f(v_i) = (-1)^i(i-1)$, $\forall\ \ i = 1, 2, \ldots, q+1$ which is an injective function. Define induced edge labeling function by $f^* : E(P_n) \rightarrow \{1, 2, \ldots, q\}$ defined as $f^*(e) = \left\lceil \frac{|\ f(v_i) - f(v_j)\ |}{2} \right\rceil$ which is a bijective for every edge $e = (v_i, v_j) \in E(P_n)$. Therefore Path graph P_n admits absolute mean graceful labeling. So P_n, $\forall\ n \geq 2$ is absolute mean graceful graph. $\square$

Theorem 2.2. *Every cycle $C_n(n \geq 3)$ is absolute mean graceful graph.*

Proof. Let C_n be cycle graph with vertex set $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and q be number of edges. Obviously $q = n$. Define vertex labeling function $f : V(C_n) \rightarrow \{0, \pm1, \pm2, \ldots, \pm q\}$ defined by

Case 1: When q is an odd number,

$$f(v_i) = \begin{cases} q, & i = 0; \\ (-1)^{i+1}\left[|f(v_{i-1})| - 2\right], & i = 2, 3, \ldots, \frac{q-1}{2}\ (\text{if } q > 3); \\ 0, & i = \frac{q+1}{2}; \\ (-1)^i[|f(v_{i-1})| + 2], & i = \frac{q+3}{2}, \frac{q+5}{2}, \ldots, q. \end{cases}$$

Case 2: When q is an even number,

$$f(v_i) = \begin{cases} q, & i = 1; \\ (-1)^{i+1}\left[|f(v_{i-1})| - 2\right], & i = 2, 3, \ldots, \frac{q+2}{2}; \\ 3 * (-1)^{i+1}, & i = \frac{q+4}{2}; \\ (-1)^{i+1}\left[|f(v_{i-1})| + 2\right], & i = \frac{q+6}{2}, \frac{q+8}{2}, \ldots, q\ (\text{if } q > 4). \end{cases}$$

Define induced edge labeling function by $f^* : E(C_n) \rightarrow \{1, 2, \ldots, q\}$ defined as $f^*(e) = \left\lceil \frac{|f(v_i) - f(v_j)|}{2} \right\rceil$ which is a bijective for every edge $e = (v_i, v_j) \in E(C_n)$. Therefore Cycle graph C_n admits absolute mean graceful labeling. So $C_n, \forall n \geq 3$ is absolute mean graceful graph. $\square$

Theorem 2.3. *Every complete bipartite graph $K_{m,n}$ is absolute mean graceful graph.*

Proof. Let $K_{m,n}$ be complete bipartite graph with vertex set $M = \{u_1, u_2, \ldots, u_m\}$ of m-part and $N = \{v_1, v_2, \ldots, v_n\}$ of n-part such that $V(K_{m,n}) = M \bigcup N$. Clearly $V(K_{m,n}) = m + n$ and $E(K_{m,n}) = mn$. Define vertex labeling function $f : V(K_{m,n}) \rightarrow \{0, \pm1, \pm2, \ldots, \pm q\}$ defined by

$$f(u_i) = \begin{cases} mn, & i = 1 \\ f(u_{i-1}) - 2n, & i = 2, 3, ..., m \end{cases}$$

$$f(v_j) = \begin{cases} -mn, & j = 1 \\ f(v_{j-1}) + 2, & j = 2, 3, ..., n \end{cases}$$

Here vertex labeling function f is injective. Define induced edge labeling function by $f^* : E(K_{m,n}) \rightarrow \{1, 2, ..., q\}$ defined as $f^*(e) = \left\lceil \frac{|\ f(u_i) - f(v_j)\ |}{2} \right\rceil$ which is a bijective for every edge $e = (u_i, v_j) \in E(K_{m,n})$. Therefore $K_{m,n}$ admits absolute mean graceful labeling. Hence, every complete bipartite graph $K_{m,n}$ is absolute mean graceful graph. $\square$

Theorem 2.4. *Every grid graph $P_m \times P_n$ is absolute mean graceful graph.*

Proof. Let $P_m \times P_n$ be grid graph with vertex set

$$V(P_m \times P_n) = \{u_{1,1}, u_{1,2}, ..., u_{1,m}, u_{2,1}, u_{2,2}, ..., u_{2,m}, ..., u_{n,1}, u_{n,2}, ..., u_{n,m}\}.$$

Clearly $V(P_m \times P_n) = mn$ and $E(K_{m,n}) = 2mn - (m+n)$. Define vertex labeling function $f : V(P_m \times P_n) \to \{0, \pm 1, \pm 2, ..., \pm q\}$ defined by

$$f(u_{i,j}) = \begin{cases} q, & i = 1 \text{ and } j = 1 \\ (-1)^{i+j}[|f(u_{i-1,n})| - n], & \forall\, i = 2, 3, ..., m \text{ and } j = 1 \\ (-1)^{i+j}[|f(u_{i,j-1})| - 1], & \forall\, i = 1, 2, ..., m \text{ and } \forall\, j = 2, 3, ..., n \end{cases}$$

Here vertex labeling function f is injective. Define induced edge labeling function by $f^* : E(P_m \times P_n) \to \{1, 2, ..., q\}$ as $f^*(e) = \left\lceil \dfrac{|\, f(u_{i,j}) - f(v_{i,j})\,|}{2} \right\rceil$ which is a bijective for every edge $e = (u_{i,j}, v_{i,j}) \in E(P_m \times P_n)$. Therefore, $P_m \times P_n$ admits absolute mean graceful labeling. Hence, every grid graph $P_m \times P_n$ is absolute mean graceful graph. □

Theorem 2.5. *Every step grid graph St_n is absolute mean graceful graph.*

Proof. Let St_n be grid graph with vertex set

$$V(St_n) = \{u_{1,1}, u_{1,2}, ..., u_{1,n}; u_{2,1}, u_{2,2}, ..., u_{2,n}; u_{3,2}, u_{3,3}, ..., u_{3,n}; u_{4,3}, u_{4,4}, ..., u_{4,n}; ...; u_{n,n-1}, u_{n,n}\}.$$

Clearly $V(St_n) = \dfrac{n^2 + 3n - 2}{2}$ and $E(St_n) = n^2 + n - 2$. Define vertex labeling function $f : V(St_n) \to \{0, \pm 1, \pm 2, ..., \pm q\}$ defined by

$$\begin{aligned} f(u_{1,1}) &= q, \\ f(u_{i+1,i}) &= 1 - [(n-i)(n-i+1)], && \forall\, i = 1, 2, ..., n-1 \\ f(u_{i,j}) &= (-1)^{i+j}\left[|f(u_{i,j-1})| - 1\right], && (\forall\, i \le j),\ \forall\, j = 2, 3, ..., n. \end{aligned}$$

Here vertex labeling function f is injective. Define induced edge labeling function by $f^* : E(St_n) \to \{1, 2, ..., q\}$ as $f^*(e) = \left\lceil \dfrac{|\, f(u_{i,j}) - f(v_{i,j})\,|}{2} \right\rceil$ which is a bijective for every edge $e = (u_{i,j}, v_{i,j}) \in E(St_n)$. Therefore St_n admits absolute mean graceful labeling. Hence, every grid graph St_n is absolute mean graceful graph. □

Theorem 2.6. *Every double step grid graph DSt_n is absolute mean graceful graph.*

Proof. Let DSt_n be any double step grid graph of size n, where $n \equiv 0\ (mod\ 2)$, $n \ne 2$. We mention each vertices of first row like $u_{1,j}$ $(1 \le j \le n)$ and second row like $u_{2,j}$ $(1 \le j \le n)$ and third row like $u_{3,j}$ $(1 \le j \le n-2)$ and fourth row like $u_{4,j}$ $(1 \le j \le n-4)$. Similarly, the last row like $u_{(0.5n)+1,j}$ $(1 \le j \le 2)$. It can be easily observed that $|V(DSt_n)| = \dfrac{n}{4}(n+6)$ and the number of edges $|E(DSt_n)| = \dfrac{n^2 + 3n - 2}{2}$. Define vertex labeling function $f : V(DSt_n) \to \{0, \pm 1, \pm 2, ..., \pm q\}$ defined by

$$f(u_{i,j}) = \begin{cases} q, & i = 1 \text{ and } j = 1 \\ (-1)^{j+1}[|f(u_{1,j-1})| - 1], & \text{for } i = 1 \text{ and } \forall\, j = 2, 3, ..., n \\ -(q - 2n + 1), & \text{for } i = 2 \text{ and } j = 1 \\ -[(q - 4n + 4) + 4(i-3)], & \forall\, i = 3, 4, ..., \dfrac{n}{2} \text{ and } j = 1 \\ 0, & \text{for } i = \dfrac{n}{2} + 1 \text{ and } j = 1 \\ (-1)^{j}[|f(u_{i,j-1})| - 1], & \forall\, i = 2, 3, ..., \frac{n}{2} \text{ and } \forall\, j = 2, 3, ..., n + 4 - 2i. \end{cases}$$

Here vertex labeling function f is injective. Define induced edge labeling function by $f^* : E(DSt_n) \rightarrow \{1, 2, ..., q\}$ as $f^*(e) = \left\lceil \frac{| f(u_{i,j}) - f(v_{i,j}) |}{2} \right\rceil$ which is a bijective for every edge $e = (u_{i,j}, v_{i,j}) \in E(DSt_n)$. Therefore DSt_n admits absolute mean graceful labeling. Hence, every grid graph DSt_n is absolute mean graceful graph. □

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