International Journal of Mathematics And its Applications
Volume 5, Issue 4–F (2017), 951–957.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

$f\omega\alpha-$Continuous Maps in Fine-Topological Spaces

Research Article

P.V. Jain[1] and Kusumlata Rajak[2]*

1 Department of Mathematics and Computer Science, Rani Durgawati University, Jabalpur, Madhya Pradesh, India.

2 Department of Mathematics, St. Aloysius College (Autonomous), Jabalpur, Madhya Pradesh, India.

Abstract: In the present paper, we have introduced a new class of continuous functions called $f\alpha g-$continuous maps, $fgp-$continuous, strongly $fg-$continuous, $fg-$closed, $f\omega\alpha-$ closed, pre-$f\alpha-$closed etc. Also we have introduced a new class of maps pre-$f\omega\alpha-$continuous, $f\omega\alpha$ -continuous maps, $f\omega\alpha-$irresolute, $f\omega$ -irresolute maps, $f\omega\alpha-$irresolute homeomorphism and studied some properties of these functions.

MSC: 54XX, 54CXX.

Keywords: $\omega\alpha-$closed set, $\omega\alpha-$open set, $\omega\alpha-$continuous, $\omega\alpha-$irresolute and fine-irresolute map.

1. Introduction

A weaker form of continuous functions called generalized continuous (briefly, g-continuous) maps was introduced and studied by Balachandran et al [2]. Then many researchers studied on generalizations of continuous maps. Devi et al. ([5, 6]) investigated new class of continuous maps called generalized $\alpha-$continuous and $\alpha-$generalized continuous maps (briefly $g\alpha-$continuous and $\alpha g-$continuous). Recently, Benchalli et al. [3] introduced and studied the properties of $\omega\alpha$-closed sets. Powar P. L. and Rajak K. [14] have introduced fine-topological space which is a special case of generalized topological space. This new class of fine-open sets contains all $\alpha-$open sets, $\beta-$open sets, semi-open sets, pre-open sets, regular open sets etc. and fine-irresolute mapping which include pre-continuous function, semi-continuous functions, $\alpha-$continuous function, $\beta-$continuous function, $\alpha-$irresolute and $\beta-$irresolute functions.

In the present paper, we have introduced a new class of continuous functions called $f\alpha g$-continuous maps, $fgp-$continuous, strongly $fg-$continuous, $fg-$closed, $f\omega\alpha-$ closed, pre-$f\alpha-$closed etc. Also, we have introduced a new class of maps pre$-f\omega\alpha-$continuous maps, $f\omega\alpha-$continuous maps, $f\omega\alpha-$irresolute, $f\omega-$irresolute maps and $f\omega\alpha-$irresolute homeomorphism. Also, we have studied some properties of these functions.

2. Preliminaries

Throughout the paper (X,τ) and (Y,τ') (or simply X and Y) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(X,\tau), Cl(A), Int(A),_\alpha Cl(A)$ and A^C denote the closure of A, the interior of A, the α-closure of A and the compliment of A in X respectively. We recall the following definitions, which are useful in the sequel.

* *E-mail: kusumrajakrdvv@gmail.com*

Definition 2.1. *A subset A of a space (X, τ) is called*

(1). α-open [13] if $S \subseteq int(cl(int(S)))$.

(2). Semi-open [7] if $S \subseteq cl(int(S))$.

(3). Pre-open [7] if $S \subseteq int(cl(S)))$.

(4). β-open [7] if $S \subseteq cl(int(cl(S)))$.

Definition 2.2. *A subset A of a topological space X is called:*

(1). generalized closed (briefly $g-$closed) [8] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X.

(2). α-generalized closed (briefly $\alpha g-$closed) [10] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is open in X.

(3). generalized pre-closed (briefly $gp-$closed) [11] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in$X$.

(4). strongly generalized closed [15] if $Cl(A) \subset U$ whenever $A \subset U$ and U is $g-$open in X.

(5). ω-closed [16] if $Cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

(6). $\omega\alpha$-closed [3] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is ω-open in X.

Definition 2.3. *A map $f : (X, \tau) \to (Y, \tau')$ is called:*

(1). α-continuous [12] (resp. g-continuous [2], αg-continuous [5], gp-continuous [1], strongly g-continuous [15] and ω-continuous [16]) if $f^{-1}(G)$ is α -closed (resp. $g-$closed, $\alpha g-$closed, $gp-$closed, strongly $g-$closed and ω-closed) set in (X, τ) for every closed set G of (Y, τ').

(2). $g-$closed [9] (resp. $\alpha g-$closed [6], α-closed [12], and ω-closed [16]) if $f(G)$ is $g-$closed (resp. αg-closed, α-closed and ω-closed) in (Y, ?') for every closed set G in (X, τ).

(3). ω^*-closed [16] (resp. ω^*-open [16]) if f(G) is ω-closed (resp. ω-open) in (Y, τ') for every ω-closed (resp. ω-open) set G in (X, τ).

(4). pre-α-closed [6] (resp. pre-α-open [6]) if f(G) is α-closed (resp. α-open) in (Y, τ') for every α-closed (resp. $\alpha-$open) set G in (X, τ).

(5). pre-αg-closed [6] (resp. pre-αg-open [6]) if f(G) is αg-closed (resp. αg-open) in (Y, τ') for everyα-closed (resp. α-open) set G in (X, τ).

(6). ω-irresolute [16] if $f^{-1}(G)$ is ω –closed set in (X, τ) for each ω-closed set G of (Y, τ').

Definition 2.4. *A map $f : (X, \tau) \to (Y, \tau')$ is called $\omega\alpha$-continuous [4] if the inverse image of every closed set in (Y, τ') is $\omega\alpha$-closed in (X, τ).*

Definition 2.5. *A map $f : (X, \tau) \to (Y, \tau')$ is called pre-$\omega\alpha$-continuous [4] if $f^{-1}(G)$ is $\omega\alpha$-closed in (X, τ) for every α-closed set G in (Y, τ').*

Definition 2.6. *A map $f : (X, \tau) \to (Y, \tau')$ is said to be $\omega\alpha$-irresolute [4] if the inverse image of every $\omega\alpha$-open set in (Y, τ') is $\omega\alpha$-open in (X, τ).*

Definition 2.7. *A map $f : (X, \tau) \to (Y, \tau')$ is called $\omega\alpha$-closed [4] (resp. $\omega\alpha$-open [4]) if for each closed (resp. open) set F of (X, τ), $f(F)$ is an $\omega\alpha$-closed (resp. $\omega\alpha$-open) set in (Y, τ').*

Definition 2.8. *A map $f : (X, \tau) \to (Y, \tau')$ is called pre-$\omega\alpha$-closed [4] if for each α -closed set F of (X, τ), $f(F)$ is an $\omega\alpha$ -closed set in (Y, τ').*

Definition 2.9. *Let (X, τ) be a topological space we define $\tau(A_\alpha) = \tau_\alpha(say) = \{G_\alpha(\neq X) : G_\alpha \cap A_\alpha = \varphi$, for $A_\alpha \in \tau$ and $A_\alpha = \varphi, X$, for some $\alpha \in J$, where J is the index set$\}$. Now, we define $\tau_f = \varphi, X, \cup_{\{\alpha\in J\}}\tau_\alpha$. This collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [14]).*

Definition 2.10. *A subset U of a fine space X is said to be a fine-open set of X if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [14]).*

Definition 2.11. *Let A be a subset of a fine space X, we say that a point $x \in X$ is a fine limit point of A if every fine-open set of X containing x must contains at least one point of A other than x (cf. [14]).*

Definition 2.12. *Let A be the subset of a fine space X, the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by f_{Int} (cf. [14]).*

Definition 2.13. *Let A be the subset of a fine space X, the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by f_{cl} (cf. [14]).*

Definition 2.14. *A function $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f^{'})$ is called fine-irresolute(or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [14]).*

3. $f\omega\alpha$-Continuous Maps in Fine-Topological Spaces

In this section we have defined $f\omega\alpha-$continuous maps in fine topological space.

Definition 3.1. *A subset A of a topological space X is called fine-generalized closed (briefly $fg-$closed) if $f_{Cl}(A) \subset U$ whenever $A \subset U$ and U is fine-open in X.*

Definition 3.2. *A subset A of a topological space X is called $f\alpha$-generalized closed (briefly $f\alpha g$-closed) if $f\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is fine-open in X.*

Definition 3.3. *A subset A of a topological space X is called fine-generalized pre-closed (briefly fgp-closed) if and only if $pCl(A) \subset U$ whenever $A \subset U$ and U is fine-open in X.*

Definition 3.4. *A subset A of a topological space X is called strongly fine-generalized closed if $f_{Cl}(A) \subset U$ whenever $A \subset U$ and U is fg-open in X.*

Definition 3.5. *A subset A of a topological space X is called $f\omega$-closed if $f_{Cl}(A) \subset U$ whenever $A \subset U$ and U is fine-semi-open in X.*

Definition 3.6. *A subset A of a topological space X is called $f\omega\alpha$-closed if $f\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is $f\omega$-open in X.*

Definition 3.7. *A map $f : (X, \tau, \tau_f) \to (Y, \tau^{'}, \tau_f^{'})$ is called $f\alpha$-continuous if $f^{-1}(G)$ is $f\alpha$-closed set in X for every fine-closed set G of Y.*

Example 3.8. *Let*$X = \{a, b, c\}$, $\tau = \{X, \varphi, \{a, b\}\}$, $\tau_f = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ *and* $Y = \{1, 2, 3\}$, $\tau' = \{X, \varphi, \{1\}\}$, $\tau'_f = \{X, \varphi, \{1\}, \{1, 2\}, \{1, 3\}\}$. *We define a map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f^{'})$ *by* $f(a) = 1, f(b) = 2, f(c) = 3$. *It may be easily checked that, the only fine-closed sets of* Y *are* $\varphi, Y, \{1, 2\}, \{3\}, \{2\}$ *and their pre-images are* $X, \varphi, \{a, b\}, \{c\}, c$ *which are* $f\alpha$*-closed in* X*. Hence,* f *is* $f\alpha$*-continuous.*

Definition 3.9. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau^{'}, \tau'_f)$ *is called fg-continuous if* $f^{-1}(G)$ *is fg-closed set in* X *for every fine-closed set* G *of* Y*.*

Example 3.10. *Let*$X = \{a, b, c\}$, $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$, $\tau_f = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ *and* $Y = \{1, 2, 3\}$, $\tau' = \{X, \varphi, \{2\}\}$, $\tau'_f = \{X, \varphi, \{2\}, \{1, 2\}, \{2, 3\}\}$. *We define a map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f^{'})$ *by* $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. *It may be easily checked that, the only fine-closed sets of* Y *are* $\varphi, Y, \{1, 3\}, \{3\}, \{1\}$ *and their pre-images are* $X, \varphi, \{1, c\}, \{c\}, \{1\}$ *which are* fg*-clopen in* X*. Hence,* f *is fg-continuous.*

Definition 3.11. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ *is called* $f\alpha g$*-continuous if* $f^{-1}(G)$ *is* $f\alpha g$*-closed set in* X *for every fine-closed set* G *of* Y*.*

Example 3.12. *The above Example 3.10.*

Definition 3.13. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau^{'}, \tau'_f)$ *is called* fgp*-continuous if* $f^{-1}(G)$ *is* fgp*-closed in* X *for every fine-closed set* G *of* Y*.*

Example 3.14. *Let*$X = a, b, c$, $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$, $\tau_f = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ *and* $Y = \{1, 2, 3\}$, $\tau' = \{X, \varphi, \{a\}\}$, $\tau'_f = \{X, \varphi, \{1\}, \{1, 2\}, \{1, 3\}\}$. *We define a map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f^{'})$ *by* $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. *It may be easily checked that, the only fine-closed sets of* Y *are* $\varphi, Y, \{2, 3\}, \{3\}, \{2\}$ *and their pre-images are* $X, \varphi, \{b, c\}, \{c\}, \{b\}$ *which are* $f\alpha g$*-closed in* X*. Hence,* f *is* $f\alpha g$*-continuous.*

Definition 3.15. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau^{'}, \tau'_f)$ *is called strongly* fg*-continuous if* $f^{-1}(G)$ *is strongly* fg*-closed in* X *for every fine-closed set* G *of* Y*.*

Example 3.16. *Let* $X = a, b, c$, $\tau = \{X, \varphi, \{a, b\}\}$, $\tau_f = \{X, \varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ *and* $Y = \{1, 2, 3\}$, $\tau' = \{X, \varphi, \{3\}\}$, $\tau'_f = \{X, \varphi, \{3\}, \{1, 3\}, \{2, 3\}\}$. *We define a map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f^{'})$ *by* $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. *It may be easily checked that, the only fine-closed sets of* Y *are* $\varphi, Y, \{1, 2\}, \{2\}, \{1\}$ *and their pre-images are* $X, \varphi, \{a, b\}, \{b\}, \{a\}$ *which are strongly* fg*-closed in* X*. Hence,* f *is strongly* fg*-continuous.*

Definition 3.17. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau^{'}, \tau'_f)$ *is called* $f\omega$*-continuous if* $f^{-1}(G)$ *is* $f\omega$*-closed in* X *for every fine-closed set* G *of* Y*.*

Definition 3.18. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f^{'})$ *is called fg-closed if f(G) is fg-closed in* Y' *for every fine-closed set* G *in* X*.*

Definition 3.19. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ *is called* $f\alpha g$*-closed if f(G) is* $f\alpha g$*-closed in* Y *for every fine-closed set* G *in* X*.*

Definition 3.20. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ *is called* $f\alpha$*-closed if f(G) is* $f\alpha$*-closed in* Y *for every fine-closed set* G *in* X*.*

Definition 3.21. *A map* $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ *is called*$f\omega$*-closed if f(G) is* $f\omega$ *-closed in* Y *for every fine-closed set* G *in* X*.*

Definition 3.22. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is calledfω*-closed if f(G) is fω-closed for every fω-closedset G in X.*

Definition 3.23. *A map $f : (X, \tau, \tau_f) \to (Y, \tau^{'}, \tau'_f)$ is called fω*-open if f(G) is fω -open for every fω-openset G in X.*

Definition 3.24. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called pre-$f\alpha$-closed (resp. pre-$f\alpha$-open) if $f(G)$ is $f\alpha$-closed (resp. $f\alpha$ -open) in Y for every $f\alpha$-closed (resp. $f\alpha-$open) set G in X.*

Definition 3.25. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called pre-$f\alpha g$-closed (resp. pre-$f\alpha g$-open) if f(G) is $f\alpha g$-closed (resp. $f\alpha$ g-open) in Y for every $f\alpha$-closed (resp. fα-open) set G in X.*

Definition 3.26. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called fω–irresolute if $f^{-1}(G)$ is fω-closed set in X for each fine-closed set G of Y.*

Definition 3.27. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called $f\omega\alpha$-continuous if the inverse image of every fine-closed set in Y is fωα -closed in X.*

Definition 3.28. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called pre- fωα-continuous if $f^{-1}(G)$ is $f\omega\alpha$-closed in X for every $f\alpha$ -closed set G in Y.*

Definition 3.29. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is said to be $f\omega\alpha$-irresolute if the inverse image of every fωα-open set in Y is $f\omega\alpha$-open in X.*

Definition 3.30. *A map $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called fωα-closed (resp. fωα-open) if for each fine-closed (resp. fine-open) set F of X, f(F) is an fωα-closed (resp. fωα -open) set in Y.*

Definition 3.31. *A map $f : (X, \tau) \to (Y, \tau')$ is called pre-$f\omega\alpha$-closed if for each$f\alpha$-closed set F of X, f(F) is an $f\omega\alpha$-closed set in Y.*

4. Main Results

Theorem 4.1. *Iff $: (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is $f\omega\alpha$-continuous and $f\omega^*$ -closed and if A is $f\omega\alpha$-open (or $f\omega\alpha$ -closed) subset of (Y, τ', τ'_f) and (Y, τ', τ'_f) is $f\alpha$-space, then $f^{-1}(A)$ is $f\omega\alpha$-open (or $f\omega\alpha$-closed) in (X, τ, τ_f).*

Proof. Let A be a $f\omega\alpha$-open set in (Y, τ', τ'_f) and G be any $f\omega$-closed set in (X, τ, τ_f) such that $G \subseteq f^{-1}(A)$. Then, $f(G) \subseteq A$. By hypothesis $f(G)$ is $f\omega$-closed and A is $f\omega\alpha$-open in (Y, τ', τ'_f). Therefore $f(G) \subseteq f_\alpha Int(A)$ and so $G \subseteq f^{-1}(f_\alpha Int(A))$. Since f is $f\omega\alpha$-continuous, $f_\alpha Int(A)$ is $f\alpha$-open in Y and Y is $f\alpha$-space, so $f_\alpha Int(A)$ is open in Y. Therefore $f^{-1}(f_\alpha Int(A))$ is $f\omega\alpha$-open in X. Thus, $G \subseteq f_\alpha Int(f^{-1}(f_\alpha Int(A))) \subseteq f_\alpha Int(f^{-1}(A))$, that is $G \subseteq f_\alpha Int(f^{-1}(A))$, $f^{-1}(A)$ is $f\omega\alpha$-open in X. By taking the complements we can show that if A is $f\omega\alpha$-closed in Y, $f^{-1}(A)$ is $f\omega\alpha$-open in X. □

Theorem 4.2. *$f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is bijective, $f\omega$-open and pre-$f\omega\alpha$ -continuous, then f is $f\omega\alpha$-irresolute.*

Proof. Let G be $f\omega\alpha$-closed set in Y and let U be a $f\omega$-open set in X such that $G \subseteq f(U)$. Since G is $f\omega\alpha$-closed set and $f(U)$ is $f\omega$-open in Y and since f is $f\omega^*$-open, $f_\alpha Cl(G) \subseteq f(U)$ and $f_\alpha Cl(G)$ is $f\alpha$-closed in Y. Since f is pre-$f\omega\alpha$-continuous, $f^{-1}(f_\alpha Cl(G))$ is $f\omega\alpha$-closed in X. We have $f_\alpha Cl(f^{-1}(f_\alpha Cl(G))) \subseteq U$ and so $f_\alpha Cl(f^{-1}(G)) \subseteq U$. Therefore $f^{-1}(G)$ is $f\omega\alpha$-closed in X. Hence f is $f\omega\alpha$-irresolute. □

Theorem 4.3. *If $f : (X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is bijective, $f\alpha$-closed and $f\omega$ -irresolute, then the inverse map f^{-1} : $(Y, \tau', \tau'_f) \to (X, \tau, \tau_f)$ is $f\omega\alpha$-irresolute.*

Proof. Let G be $f\omega\alpha$-closed set in (X,τ,τ_f). Let $(f^{-1})^{-1}(G) = f(G)?U$ where U is $f\omega$-open set in $(Y,\tau',\tau_f^{'})$. Then $G \subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is $f\omega$-open in (X,τ,τ_f) and G is $f\omega\alpha$-closed set in (X,τ,τ_f), $f_\alpha Cl(G) \subseteq f^{-1}(U)$ and hence $f(f_\alpha Cl(G)) \subseteq U$. Since f is $f\alpha$-closed and $f_\alpha Cl(G)$ is $f\alpha$-closed in (X,τ,τ_f), $f(f_\alpha Cl(G))$ is $f\alpha$-closed in $(Y,\tau',\tau_f^{'})$. So $f(f_\alpha Cl(G))$ is $\omega\alpha$ -closed set in $(Y,\tau',\tau_f^{'})$. Therefore, $f_\alpha Cl(f(f_\alpha Cl(G))) \subseteq U$, so that $f_\alpha Cl(f(G)) \subseteq U$. Thus $f(G)$ is $\omega\alpha$-closed set in $(Y,\tau',\tau_f^{'})$. Hence, f^{-1} is $f\omega\alpha$-irresolute. □

Theorem 4.4. *A map $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is $f\omega\alpha$-open if and only if for any subset S of Y and for any closed set F containing $f^{-1}(S)$ there exists an $f\omega$-closed set K of Y containing S such that $f^{-1}(K) \subseteq F$.*

Proof. Suppose $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is $f\omega\alpha$-open. Let S be a subset of Y and F be closed set of X containing $f^{-1}(S)$. Then $K = Y - f(X-F)$ is an $f\omega\alpha$-closed set containing S such that $f^{-1}(K) \subseteq F$. Conversely suppose that U is an open set of X. Then, $f^{-1}(Y - f(U)) \subseteq X - f^{-1}[f(U)] \subseteq X - U$ and $X - U$ is closed. By hypothesis, there is an $f\omega\alpha$-closed set K of Y such that $Y - f(U) \subseteq K$ and $f^{-1}(K) \subseteq X - U$. Therefore, $U \subseteq X - f^{-1}(K)$. Hence $Y - K \subseteq f(U) \subseteq f[X - f^{-1}(K)] \subseteq Y - K$ which implies $f(U) \subseteq Y - K$. Since $Y - K$ is $f\omega\alpha$-open, $f(U)$ is $f\omega\alpha$-open and thus f is $f\omega\alpha$-open map. □

Theorem 4.5. *If a map $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is $f\omega\alpha$-irresolute and pre-$f\omega\alpha$-closed and A is $f\omega\alpha$-closed set of X, then f(A) is $f\omega\alpha$-closed set in Y.*

Proof. Let A be a $f\omega\alpha$-closed set in X. Let G be a $f\omega$-open set in Y such that $f(A) \subseteq G$. Then $f^{-1}(G)$ is $f\omega$-open set in X such that $A \subseteq f^{-1}(G)$. Hence, $f_\alpha Cl(A) \subseteq f^{-1}(G)$, since A is $f\omega\alpha$ -closed and $f^{-1}(G)$ is $f\omega$-open. Since f is pre-$f\omega\alpha$-closed map, $f(f_\alpha Cl(A)$ is $f\omega\alpha$-closed set contained in the $f\omega$-open set G. Therefore,$f_\alpha Cl(f(f_\alpha Cl(A))) = f(f_\alpha Cl(A)) \subseteq G$: This implies $f_\alpha Cl(f(A)) \subseteq G$. Hence, f(A) is $f\omega\alpha$ -closed in Y. □

Theorem 4.6. *If A is $f\omega\alpha$-closed in X and if $f : (X,\tau,\tau_f) \to (Y,\tau',\tau_f')$ is surjection, $f\omega$ irresolute and $f\alpha$-closed, then f(A) is $f\omega\alpha$-closed in Y.*

Proof. Let G be any $f\omega$-open set in Y such that $f(A) \subseteq G$. Then, $A \subseteq f^{-1}(G)$ and by hypothesis A is $f\omega\alpha$-closed in X, $f_\alpha Cl(A) \subseteq f^{-1}(G)$. Thus, $f(f_\alpha Cl(A)) \subseteq G$ and $f(f_\alpha Cl(A))$ is $f\alpha$-closed set. Also, $f_\alpha Cl(f(A)) \subseteq f_\alpha Cl(f(f_\alpha Cl(A))) = f(f_\alpha Cl(A)) \subseteq G$. Therefore, $f_\alpha Cl(f(A)) \subseteq G$ and hence $f(A)$ is $f\omega\alpha$-closed set in Y. □

5. Conclusion

We have introduced a new class of generalized continuous functions called $f\alpha g-$continuous maps, $fgp-$ continuous, $f\omega\alpha$-continuous maps, $f\omega\alpha-$irresolute, $f\omega-$irresolute maps and $f\omega\alpha-$ irresolute homeomorphism. The generalized continuities may be useful in defining generalized homeomorphism to find the wider homeomorphic images and it may applicable in image processing. There are various applications of topology in various field such as Image Processing, Biology, Chemistry, Stock Market, Data Mining, Quantum Physics, Statistics, Clinical research etc. We shall try to study some applications of our newly developed concept.

References

[1] Arockiarani, K.Balachandran and J.Dontchev, *Some characterizations of gp-irresolute and gp continuous maps between topological spaces*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 20(1999), 93-104.

[2] K.Balachandran, P.Sundaram and H.Maki, *On generalized continuous maps in topological spaces*, Mem. Fac. Kochi Univ. Ser. A. Math., 12(1991), 5-13.

[3] S.S.Benchalli, P.G.Patil and T.D.Rayanagoudar, *$\omega\alpha$-closed sets in topological spaces*, The Global J. of Appl. Math.and Math. Sci., 2(1-2)(2009), 53-63.

[4] S.S.Benchalli and P.G.Patil, *Some New Continuous Maps in Topological Spaces*, Journal of Advanced Studies in Topology, 1(2)(2010), 16-21.

[5] R.Devi, K.Balachandran and H.Maki, *On generalized α -continuous maps and α-generalized continuous maps*, Far East J.Math. Sci., Special Volume, I(1997), 1-15.

[6] R.Devi, K.Balachandran and H.Maki, *Generalized α -closed maps and α-generalized closed maps*, Indian. J. Pure appl.Math., 29(1)(1998), 37-49.

[7] N.Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70(1963), 36-41.

[8] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.

[9] S.R.Malghan, *Generalized closed maps*, J. Karnataka Univ. Sci., 27(1982), 82-88.

[10] H.Maki, R.Devi and K.Balachandran, *Associated topologies of generalized α-closed sets and α generalized closed sets*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 15(1994), 51-63.

[11] H.Maki, J.Umehara and T.Noiri, *Every topological space is pre-$T_{\frac{1}{2}}$*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 17(1996), 33-42.

[12] A.S.Mashhour, I.A.Hasanein and S.N.EL-Deeb, *α -continuous and α-open mappings*, Acta Math. Hung., 41(3-4)(1983), 213-218.

[13] O.Njastad, *On some classes of nearly open sets*, Pacic. J. Math., 15(1965), 961-970.

[14] P.L.Powar and K.Rajak, *Fine irresolute mappings*, Journal of Advance Studies in Topology, 3(4)(2012), 125-139.

[15] A.Pushpalatha, *Studies on generalizations of mappings in topological spaces*, Ph.D. Thesis, Bharathiar University, Coimbatore, (2000).

[16] P.Sundaram and Sheik M.John, *Weakly closed sets and weak continuous maps in topological spaces*, In Proc. 82nd, volume 49, Indian Sci. Cong Calcutta, (1995).