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Tri-b-Continuous Function in Tri Topological Spaces

Research Article

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Abstract: The purpose of this paper is to study the properties of tri-b open sets and tri-b closed sets and introduce tri-b continuous functions in tri topological spaces.

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Keywords: Tri topological spaces, tri-b open sets, tri-b interior, tri-b closure, tri-b continuous function.

1. Introduction

J. C. Kelly [1] introduced bitopological spaces in 1963. The study of tri-topological spaces was first initiated by Martin M. Kovar [2] in 2000, where a non empty set X with three topologies is called tri-topolgical spaces. Tri α Continuous Functions and tri β continuous functions introduced by S. Palaniammal [4] in 2011. In year 2011 Luay Al-Sweedy and A.F.Hassan defined δ^{**}-continuous function in tritopolgical space. In this paper, we study the properties of tri-b open sets and tri-b closed sets and tri-b continuous function in tri topological space.

2. Priliminaries

Definition 2.1 ([3]). *Let X be a nonempty set and T_1, T_2 and T_3 are general topologies on X. Then a subset A of space X is said to be tri-open (123-open) set if $A \subset T_1 \cup T_2 \cup T_3$ and its complement is said to be tri-closed and set X with three topologies called tri topological spaces (X, T_1, T_2, T_3). Tri-open sets satisfy all the axioms of topology.*

Definition 2.2 ([3]). *A subset A of a space X is said to be tri-b open set if $A \subset tri - cl(tri - intA) \cup tri - int(tri - clA)$.*

Definition 2.3 ([3]). *We will denote the tri-b interior (resp. tri-b closure) of any subset, say of A by tri-b $intA$ (tri-b clA), where tri-b $intA$ is the union of all tri-b open sets contained in A, and tri-b clA is the intersection of all tri-b closed sets containing A.*

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3. Tri-b Open and Tri-b Closed Sets

Theorem 3.1. *Arbitrary union of tri-b open sets is tri-b open.*

Proof. Let $\{A\alpha/\alpha \in I\}$ be a family of tri-b open sets in X. For each $\alpha \in I$, $A \subset tri-cl(tri-intA) \cup tri-int(tri-clA)$. Therefore

$$\cup A \subset [\cup\{tricl(triintA)\}] \cup [\cup\{triint(triclA)\}].$$

$$\cup A \subset \{tricl(tri \cup intA)\} \cup \{triint(tri \cup clA)\}.$$

Therefore $\cup A\alpha$is tri-b open. ☐

Theorem 3.2. *Arbitrary intersection of tri-b closed sets is tri-b closed.*

Proof. Let $\{B\alpha\ \alpha \in I\}$ be a family of tri-b closed sets in X. Let $A\alpha = (B\alpha)^c\{A\alpha/\alpha \in I\}$ be a family of tri-b open sets in X. Arbitrary union of tri-b open sets is tri-b open. Hence $\cup A\alpha$ is tri-b open and hence $(\cup A\alpha)^c$ is tri-b closed. That is $\cap A\alpha^c$ is tri-b closed. $\cap B\alpha$ is tri-b closed. Hence arbitrary intersection of tri-b closed sets is tri-b closed. ☐

Note 3.3.

(1). $tri-b\ intA \subset A$.

(2). $trib\ intA$ *is* $tri-b$ *open.*

(3). $tri-b\ intA$ *is the largest* $tri-b$ *open set contained in* A.

Theorem 3.4. *A is tri-b open iff $A = trib\ intA$.*

Proof. A is tri-b open and $A \subset A$. Therefore $A \in \{B/B \subset A, \text{B is tri-b open}\}$. A is in the collection and every other member in the collection is a subset of A and hence the union of this collection is A. Hence $\cup\{B/B \subset A, \text{B is tri-b open}\} = A$ and hence $trib\ intA = A$. Conversely since $tri-b\ intA$ is $tri-b$ open, $A = tri-b\ intA$ implies that A is $tri-b$ open. ☐

Theorem 3.5. *$tri-b\ int(A \cup B) \supset tri-b\ intA \cup tri-b\ intB$.*

Proof. $trib\ intA \subset A$ and $tri-b\ intA$ is tri-b open. $tri-b\ intB \subset B$ and $tri-b\ intB$ is $tri-b$ open. Union of two $tri-b$ open sets is $tri-b$ open and hence $tri-b\ intA \cup tri-b\ intB$ is a $tri-b$ open set. Also $tri-b\ intA \cup trib\ intB \subset A \cup B$. $tri-b\ intA \cup tri-b\ intB$ is one $tri-b$ open subset of $A \cup B$ and $tri-b\ int(A \cup B)$ is the largest $tri-b$ open subset of $A \cup B$. Hence $tri-b\ int(A \cup B) \supset tri-b\ intA \cup tri-b\ intB$. ☐

Note 3.6. *Since intersection of tri-closed sets is $tri-b$ closed, $tri-b\ clA$ is a tri-b closed set.*

Note 3.7. *$tri-b\ clA$ is the smallest $tri-b$ closed set containing A.*

Theorem 3.8. *A is tri-b closed iff $A = tri-b\ clA$.*

Proof. $tri-b\ clA = \cap\{B/B \supset A, \text{B is tri-b closed}\}$. If A is a tri-closed then A is a member of the above collection and each member contains A. Hence their intersection is A. Hence $tri-b\ clA = A$.
Conversely if $A = tri-b\ clA$, then A is tri-closed because $tri-b\ clA$ is a $tri-b$ closed set. ☐

Definition 3.9. *Let $A \subset X$, be a tri topological space. $x \in X$ is called a $tri-b$ limit point of A, if every $tri-b$ open set U containing x, intersects $A - \{x\}$. (ie) every $tri-b$ open set containing x, contains a point of A other than x.*

4. Tri-b Continuous Function

Definition 4.1. *Let (X, T_1, T_2, T_3) and (Y, T_1', T_2', T_3') be two tri topological spaces. A function $f : X \to Y$ is called a $tri-b$ continuous function if $f^{-1}(V)$ is tri-b open in X, for every tri-b open set V in Y.*

Example 4.2. *Let $X = \{1, 2, 3\}$, $T_1 = \{\emptyset, \{1\}, X\}$, $T_2 = \{\emptyset, \{1\}, \{1, 3\}, X\}$, $T_3 = \{\emptyset, \{1\}, \{1, 2\}, X\}$. Let$Y = \{a, b, c\}$, $T_1' = \{\emptyset, \{a\}, Y\}$, $T_2' = \{\emptyset, \{a\}, \{a, c\}, Y\}$, $T_3' = \{\emptyset, \{a\}, \{a, b\}, Y\}$. Let $f : X \to Y$ be a function defined as $f(1) = a$; $f(2) = b$; $f(3) = c$; tri-open sets in (X, T_1, T_2, T_3) are $\emptyset, \{1\}, \{1, 2\}, \{1, 3\}, X$. tri-open sets in (Y, T_1', T_2', T_3') are $\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y$. tri-b open sets in (X, T_1, T_2, T_3) are $X, \emptyset, \{1\}, \{1, 2\}, \{1, 3\}$. tri-b open sets in (Y, T_1', T_2', T_3') are $Y, \emptyset, \{a\}, \{a, b\}, \{a, c\}$. Since $f^{-1}(V)$ is tri-b open in X for every tri-b open set V in Y, f is tri-b continuous.*

Definition 4.3. *Let X and Y be two tri-topological spaces. A function $f : X \to Y$ is said to be $tri-b$ continuous at a point $a \in X$ if for every tri-b open set V containing f(a), $\exists$ a tri-b open set U containing a, such that $f(V) \subset V$.*

Theorem 4.4. *$f : X \to Y$ is tri-b continuous iff f is tri-b continuous at each point of X.*

Proof. Let $f : X \to Y$ be tri-b continuous. Take any $a \in X$. Let V be a tri-b open set containing $f(a)$. $f : X \to Y$ is tri-b continuous, Since $f^1(V)$ is tri-b open set containing a. Let $U = f^1(V)$. Then $f(U) \subset V \Rightarrow \exists$ a tri-b open set U containing a and $f(U) \subset V$. Hence f is tri-b continuous at a.

Conversely, Suppose f is tri-b continuous at each point of X. Let V be a tri-b open set of Y. If $f^1(V) = \emptyset$ then it is tri-b open. Take any $a \in f^1(V)$, f is tri-b continuous at a. Hence $\exists$ Ua, tri-b open set containing a and $f(Ua) \subset V$. Let $U = \cup\{Ua/a \in f^1(V)\}$. Now we have to claim that $U = f^1(V)$. $a \in f^1(V) \Rightarrow Ua \subset U \Rightarrow a \in U$. $x \in U \Rightarrow x \in Ua$ for some $a \Rightarrow () \in V \Rightarrow x \in f^1(V)$. Hence $U = f^1(V)$. Each Ua is tri-b open. Hence U is tri-b open. Therefore $f^1(V)$ is tri-b open in X. Hence f is tri-b continuous. □

Theorem 4.5. *Let (X, T_1, T_2, T_3) and (Y, T_1', T_2', T_3') be two tri-topological spaces. Then $f : X \to Y$ is tri-b continuous function iff $f^1(V)$ is tri-b closed in X whenever V is tri-b closed in Y.*

Proof. Let $f : X \to Y$ be tri-b continuous function. Let V be any $tri-b$ closed in Y $\Rightarrow$ is tri-b open in $Y \Rightarrow f^1(V)$ is tri-b open in X $\Rightarrow [f^1(V)]^c$ is tri-b open in X $\Rightarrow f^1(V)$ is tri-b closed in X. Hence $f^1(V)$ is tri-b closed in X whenever V is tri-b closed in Y.

Conversely, suppose $f^1(V)$ is tri-b closed in X whenever V is tri-b closed in Y. V is a tri-b open set in Y $\Rightarrow V^c$ is tri-b closed in Y $\Rightarrow f^1(V^c)$ is tri-b closed in X $\Rightarrow [f^1(V)]^c$ is tri-b closed in X $\Rightarrow f^1(V)$ is tri-b open in X. Hence f is tri-b continuous. □

Theorem 4.6. *Let (X, T_1, T_2, T_3) and (Y, T_1', T_2', T_3') be two tri-topological spaces. Then $f : X \to Y$ is tri-b continuous iff $f[tri - clA] \subset tri - cl[f(A)]\ \forall\ A \subset X$.*

Proof. Suppose $f : X \to Y$ is tri-b continuous. Since $tri - b\ cl[(A)]$ is tri-b closed in Y. Then by Theorem 4.5, $f^{-1}(tri - cl[f(A)])$ is tri-b closed in X, $tri - b\ cl[f^{-1}(tri - b\ cl(f(A))] = f^{-1}(tri - b\ cl(f(A))$. Now $(A) \subset tri - b\ cl[(A)]$, $A \subset f^{-1}(A(A)) \subset f^{-1}(tri - b\ cl(f(A))$. Then $tri - b(A) \subset tri - b\ cl[f^{-1}(tri - b\ cl(f(A))] = f^{-1}(tri - b\ cl(f(A))$ by (1). Then $(tri - b((A)) \subset tri - b((A))$.

Conversely, let $(tri - b(A)) \subset tri - b((A))\ \forall\ A \subset X$. Let F be tri-b closed set in Y, so that $tri - b(F) = F$. Now $f^{-1}(F) \subset X$, by hypothesis, $f(q - b\ cl(f^{-1}(F)) \subset q - b\ cl(f(f^{-1}(F))) \subset q - b\ cl(F) = F$. Therefore $tri - b(f^{-1}(F)) \subset f^{-1}(F)$. But $f^{-1}(F) \subset tri - b\ cl(f^{-1}(F))$ always. Hence $tri - b(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is $tri - b$ closed in X. Hence by Theorem 4.5, f is tri-b continuous. □

5. Tri-b Homomorphism

Definition 5.1. *Let* (X, T_1, T_2, T_3) *and* (Y, T_1', T_2', T_3') *be two tri topological spaces. A function* $f : X \to Y$ *is called* $tri - b$ *open map if* $f(V)$ *tri-b open in* Y *for every* $tri - b$ *open set* V *in* X.

Example 5.2. *In Example 4.2, f is tri-b open map also.*

Definition 5.3. *Let* (X, T_1, T_2, T_3) *and* (Y, T_1', T_2', T_3') *be two tri-topological spaces. Let* $f : X \to Y$ *be a mapping. f is called tri-b closed map if f(F) is tri-b closed in Y for every tri-b closed set F in X.*

Example 5.4. *The function f defined in the Example 4.2 is tri-b closed map.*

Result 5.5. *Let X and Y be two tri-topological spaces. Let* $f : X \to Y$ *be a mapping. f is tri-b continuous iff* $f^{-1} : Y \to X$ *is tri-b open map.*

Definition 5.6. *Let* (X, T_1, T_2, T_3) *and* (Y, T_1', T_2', T_3') *be two tri-b topological spaces. Let* $f : X \to Y$ *be a mapping. f is called a tri-b homeomorphism. If*

(1). f is a bijection.

(2). f is tri-b continuous.

(3). f^1 *is tri-b continuous.*

Example 5.7. *The function f defined in the Example 4.2 is (1) a bijection. (2) f is tri-b continuous. (3)* f^1 *is tri-b continuous. Therefore f is a tri-b homeomorphism.*

6. Conclusion

In this paper the idea of tri-b continous function in tri topological spaces were introduced and studied. Also properties of tri-b open and tri-b closed sets were studied.

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