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Some Application of Aboodh Transform to First Order Constant Coefficients Complex Equations

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Abstract: In this work, we present a reliable Aboodh transform method to solve first order constant coefficients complex equation. This method provides an effective and efficient way of solving a wide range of linear operator equations.

Keywords: Aboodh Transform, first order constant coefficients complex equation, linear operator equations.

1. Introduction

In real, general solutions of some equation, especially type of elliptic, are not found. For example,

$$u_{xx} + u_{yy} = 0$$

Laplace equation hasn't got general solution in R^2, but it can be written

$$u_{z\overline{z}} = 0$$

and the solution of this equation is

$$u = f(z) + g(\overline{z})$$

Where f is analytic , g is anti analytic arbitrary functions [6]. That is, an equation which has not general solution in real can has general solution in complex space. A partial differential equation system which has two real dependant and two real independent variables can be transformed to a complex equation. For example,

$$u_x - v_y = 0$$
$$u_y + v_x = 0$$

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Couchy-Riemann system transforms to complex equation.

$$w_{\overline{z}} = 0$$

Where $w = u + iv$, $z = x + iy$. All solutions of this complex equation are analytic functions [6]. Moreover any order complex differential equation can be transformed to real partial differential equation system which has two unknowns, two independent variables by separating the real and imaginer parts. The solution of complex equation can be put forward helping solution of this real system [6]. Aboodh transform method which is used several areas of mathematics is a integral transform. We can solve linear differential equations, integral equations, integro-differential equations with Aboodh transform [1–3]. This method can not suitable for solution of nonlinear differential equations because of nonlinear terms .But nonlinear differential equations can solved by using Aboodh transform aid with differential transform method and homotopy perturbation method [4, 5] in this study, we investigate solutions of first order constant coefficients complex equations. These equations were solved by Laplace transform in [6]. The above mentioned equations are solved by Aboodh transform method in this paper. We obtain a formulazition for general first order constant coeffients complex equations. This paper is organized as follows: In section 2, basic definitions and theorems are given. In section 3, we get a formulazition for solve the first order constant coefficients complex partial differential equations and some examples has been given.

2. Basic Definition and Theorems

Definition 2.1. *Let F(t) be a function for $t > 0$. Aboodh transform of F(t)*

$$A\left(F\left(t\right)\right) = \frac{1}{s}\int_0^\infty e^{-st}\cdot f\left(t\right)dt \tag{1}$$

is defined.

Theorem 2.2. *Aboodh transforms of some functions are given in following.*

$F(t)$	– $A(F(t)) = K(s)$
1	– $\frac{1}{s^2}$
t	– $\frac{1}{s^3}$
t^n	– $\frac{n!}{s^{n+2}}$
$\cos\ at$	– $\frac{1}{s^2+a^2}$
$\sin\ at$	– $\frac{a}{s(s^2+a^2)}$

Theorem 2.3. *Aboodh transforms of partial derivative of $f\left(x.t\right)$ are the following.*

(1). $A\left[\frac{\partial f}{\partial t}\right] = sK\left(x.s\right) - \frac{1}{s}f(x.0)$.

(2). $L\left[\frac{\partial f}{\partial x}\right] = \frac{\partial K(x.s)}{\partial x}$.

where $K\left(x.s\right) = A\left[f\left(x.t\right)\right]$.

2.1. Complex Derivatives

Let $w = w(z.\overline{z})$ be complex function. Here

$$z = x + iy.w\left(z.\overline{z}\right) = u\left(x.y\right) + i\cdot v\left(x.y\right).$$

First order derivative according to z and $\overline{z}$ of $w\left(z.\overline{z}\right)$ are defined as following:

$$\frac{\partial w}{\partial z}=\frac{1}{2}\left(\frac{\partial w}{\partial x}-i\frac{\partial w}{\partial y}\right), \tag{2}$$

$$\frac{\partial w}{\partial \overline{z}}=\frac{1}{2}\left(\frac{\partial w}{\partial x}-i\frac{\partial w}{\partial y}\right). \tag{3}$$

3. Solution of Complex Differential From First Order Which is Constant Coefficients

Theorem 3.1. *Let A, B, C are real constants, $F(z.\overline{z})$ is a polynomial of $z.\overline{z}$ and $w=u+iv$ is a complex function. Then the real and imaginal parts of solution of*

$$A\frac{\partial w}{\partial x}+B\frac{\partial w}{\partial x}+Cw=F(z.\overline{z})$$
$$w\left(x.0\right)=f(x)$$

are

$$u=Re\ w=A^{-1}\left[\frac{(A+B)\frac{\partial}{\partial x}(2T_3+(A-B)\cdot s\cdot v(x.0)}{\left[(A+B)\,D+2C\right]^2+\left(\frac{A-B}{s}\right)^2}\right.$$
$$\left.+\frac{2C(2T_3+(A-B)\cdot s\cdot v(x.0)\left(\frac{A-B}{s}\right)(2T_4+(B-A)\cdot s\cdot u(x.0))}{\left[(A+B)\,D+2C\right]^2+\left(\frac{A-B}{s}\right)^2}\right]$$
$$v=Im\ w=A^{-1}\left[\frac{(A+B)\,\frac{\partial}{\partial x}\left(2F_2^*+(B-A)\,u\left(x.0\right)\right)+2C}{\left[(A+B)\,D\right]^2+s^2(A-B)^2}\right.$$
$$\left.+\frac{2C\left(2F_2^*+(B-A)\,u\left(x.0\right)\right)-s\left(B-A\right)\left(2F_1^*+(A-B)\,v\left(x.0\right)\right)}{\left[(A+B)\,D+2C\right]^2+s^2(A-B)^2}\right]$$

Proof.

$$A\frac{\partial w}{\partial x}+B\frac{\partial w}{\partial x}+Cw=F(z.\overline{z}). \tag{4}$$

If it is used equalities (2), (3) in equality (4), following equality is obtained.

$$A\cdot\frac{1}{2}\left(\frac{\partial w}{\partial x}-i\frac{\partial w}{\partial y}\right)+B\cdot\frac{1}{2}\left(\frac{\partial w}{\partial x}+i\frac{\partial w}{\partial y}\right)+Cw=\ F_1\left(x.y\right)+i\ F_2\left(x.y\right). \tag{5}$$

If $w=u+iv$ is written in (5), then following equality is obtained.

$$A\left(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}-i\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}\right)+B\left(\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}+i\frac{\partial u}{\partial y}-\frac{\partial v}{\partial y}\right)+2C=2F_1\left(x.y\right)+2iF_2\left(x.y\right). \tag{6}$$

If (6) is separated to real and imaginer parts, then following equation system is obtained

$$(A+B)\,\frac{\partial u}{\partial x}+(A-B)\,\frac{\partial v}{\partial y}+2Cu=2\ F_1\left(x.y\right) \tag{7}$$

$$(A+B)\,\frac{\partial v}{\partial x}+(B-A)\,\frac{\partial u}{\partial y}+2Cv=2\ F_2\left(x.y\right) \tag{8}$$

If we use Aboodh transform for above equalities (7), (8), then we get following equalities :

$$(A+B)\,\frac{\partial K_1}{\partial x}+(A-B)\left(sK_2-\frac{1}{S}v(x.0)\right)+2CK_1=2K_3 \tag{9}$$

$$(A+B)\frac{\partial K_2}{\partial x}+(B-A)\left(sK_1-\frac{1}{S}u(x.0)\right)+2CK_2=2K_4 \tag{10}$$

Where K_1, K_2, K_3, K_4 are Aboodh transforms of u, v, F_1, F_2 respectively. If (9), (10) is reregulate and is used Crammer rule, then equalities (11), (12) are obtained

$$(A+B)\frac{\partial K_1}{\partial x}+2CK_1+s(A-B)K_2=2K_3+\left(\frac{A-B}{S}\right)v(x.0)$$
$$s(B-A)K_1+(A+B)\frac{\partial K_2}{\partial x}+2CK_2=2K_4+\left(\frac{B-A}{S}\right)u(x.0)$$
$$\begin{vmatrix}(A+B)D+2C & s(A-B)\\ s(B-A) & (A+B)D+2C\end{vmatrix}=[(A+B)D+2C]^2+(s(A-B))^2$$

$$K_1=\frac{\begin{vmatrix}2K_3+\left(\frac{A-B}{s}\right)v(x.0) & s(A-B)\\ 2K_4+\left(\frac{B-A}{s}\right)u(x.0) & (A+B)D+2C\end{vmatrix}}{[(A+B)D+2C]^2+(s(A-B))^2}$$
$$K_1=\frac{(A+B)\frac{\partial}{\partial x}\left(2K_3+\left(\frac{A-B}{s}\right)v(x.0)\right)+2C\left(2K_3+\left(\frac{A-B}{s}\right)v(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2}-\frac{s(A-B)\left(2K_4+\left(\frac{B-A}{s}\right)u(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2} \tag{11}$$
$$K_2=\frac{\begin{vmatrix}(A+B)D+2C & 2K_3+\left(\frac{A-B}{S}\right)v(x.0)\\ s(B-A) & 2K_4+\left(\frac{B-A}{S}\right)u(x.0)\end{vmatrix}}{[(A+B)D+2C]^2+(s(A-B))^2}$$
$$K_2=\frac{(A+B)\frac{\partial}{\partial x}\left(2K_4+\left(\frac{B-A}{s}\right)u(x.0)\right)+2C\left(2K_4+\left(\frac{B-A}{s}\right)u(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2}-\frac{s(A-B)\left(2K_4+\left(\frac{B-A}{s}\right)v(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2} \tag{12}$$

Then

$$u(x.y)=A^{-1}\left[\frac{(A+B)\frac{\partial}{\partial x}\left(2K_3+\left(\frac{A-B}{s}\right)v(x.0)\right)+2C\left(2K_3+\left(\frac{A-B}{s}\right)v(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2}\right.$$
$$\left.-\frac{s(A-B)\left(2K_4+\left(\frac{B-A}{s}\right)u(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2}\right] \tag{13}$$
$$v(x.y)=A^{-1}\left[\frac{(A+B)\frac{\partial}{\partial x}\left(2K_4+\left(\frac{B-A}{s}\right)u(x.0)\right)+2C\left(2K_4+\left(\frac{B-A}{s}\right)u(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2}\right.$$
$$\left.-\frac{s(A-B)\left(2K_4+\left(\frac{B-A}{s}\right)v(x.0)\right)}{[(A+B)D+2C]^2+(s(A-B))^2}\right] \tag{14}$$

□

Example 3.2. *Solve the following equation*

$$4w_z+w_{\overline{z}}=0$$

With the condition

$$w(x,0)=-\frac{1}{3x}.$$

Solution. Coefficients of equation are $A=4$, $B=1$, $C=0$ and $F(z,\overline{z})=0$. From Theorem 3.1 we have

$$u(x,y)=A^{-1}\left[\frac{-\frac{3}{x}}{25D^2+9s^2}\right]$$
$$=A^{-1}\left[\frac{-\frac{3}{x}}{9s^2\left(1+\frac{25D^2}{9s^2}\right)}\right]$$

$$
\begin{aligned}
&= A^{-1}\left[-\frac{1}{3S^2}\left(1-\frac{25D^2}{9s^2}+\left(\frac{5}{3S}\right)^4 D^4-\left(\frac{5}{3S}\right)^6 D^6+\dots\right)\frac{1}{x}\right]\\
&= A^{-1}\left[-\frac{1}{3S^2}\left(\frac{1}{x}-\left(\frac{5}{3S}\right)^2 \cdot\frac{2}{x^3}+\left(\frac{5}{3S}\right)^4 \cdot\frac{4!}{x^5}-\left(\frac{5}{3S}\right)^6 \cdot\frac{6!}{x^7}+\dots\right)\right]\\
&= A^{-1}\left[-\frac{1}{3S^2x}\right]+A^{-1}\left[\left(\frac{5}{3}\right)^2\frac{2}{3s^4x^3}\right]-A^{-1}\left[\left(\frac{5}{3}\right)^4\frac{4!}{3s^6x^5}\right]+A^{-1}\left[\left(\frac{5}{3}\right)^6\frac{6!}{3s^8x^7}\right]-\dots\\
&= -\frac{1}{3x}+\frac{5^2}{3^3}\frac{y^2}{x^3}-\frac{5^4}{3^5}\frac{y^4}{x^5}+\frac{5^6}{3^7}\frac{y^6}{x^7}-\dots\\
&= -\frac{1}{3x}\left(1-\frac{5^2}{3^2}\frac{y^2}{x^2}+\left(\frac{5^2}{3^2}\frac{y^2}{x^2}\right)^2-\left(\frac{5^2}{3^2}\frac{y^2}{x^2}\right)^3+\dots\right)\\
&= -\frac{\frac{1}{3x}}{1+\frac{25y^2}{9x^2}}=-\frac{3x}{9x^2+25y^2}.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
v(x,y) &= A^{-1}\left[\frac{5\frac{\partial}{\partial x}\left(\frac{-3}{-3sx}\right)}{25D^2+9s^2}\right]\\
&= A^{-1}\left[\frac{-\frac{5}{sx^2}}{9s^2\left(1+\frac{25D^2}{9s^2}\right)}\right]\\
&= A^{-1}\left[\frac{-5}{9S^3}\left(1-\frac{25D^2}{9s^2}+\left(\frac{5}{3S}\right)^4 D^4-\left(\frac{5}{3S}\right)^6 D^6+\dots\right)\frac{1}{x^2}\right]\\
&= A^{-1}\left[\frac{-5}{9S^3}\left(\frac{1}{x^2}-\left(\frac{5}{3}\right)^2 \cdot\frac{3!}{s^2x^4}+\left(\frac{5}{3}\right)^4 \cdot\frac{5!}{s^4x^6}-\left(\frac{5}{3}\right)^6 \cdot\frac{7!}{s^6x^8}+\dots\right)\right]\\
&= A^{-1}\left[\frac{-5}{9S^3x^2}+\frac{5^3}{3^4s^5}\frac{3!}{x^4}-\frac{5^5}{3^6s^7}\frac{5!}{x^6}+\dots\right]\\
&= \frac{-5y}{9x^2}+\frac{5^3y^3}{3^4x^4}-\frac{5^5y^5}{3^6x^6}+\dots\\
&= \frac{-5y}{9x^2}\left(1-\frac{5^2y^2}{3^2x^2}+\frac{5^4y^4}{3^4x^4}-\frac{5^6y^6}{3^6x^6}+\dots\right)\\
&= \frac{-5y}{9x^2}\left(1-\frac{5^2y^2}{3^2x^2}+\left(\frac{5^2y^2}{3^2x^2}\right)^2-\left(\frac{5^2y^2}{3^2x^2}\right)^3+\dots\right)\\
&= \frac{-5y}{9x^2}\left(\frac{1}{1+\frac{25y^2}{9x^2}}\right)=-\frac{5y}{9x^2+25y^2}.
\end{aligned}
$$

Hence

$$
\begin{aligned}
w = u+iv &= -\frac{3x}{9x^2+25y^2}-\frac{5iy}{9x^2+25y^2}\\
&= -\frac{1}{3x-5iy}=\frac{1}{z-4\overline{z}}
\end{aligned}
$$

□

Example 3.3. *Solve the following problem*

$$\frac{\partial w}{\partial z}-\frac{\partial w}{\partial \overline{z}}-w=0$$

With the condition

$$w(x,0)=e^{3x}$$

Solution. Coefficients of equation are $A=1$, $B=-1$, $C=-1$ and $F(z,\overline{z})=0$. From Theorem 3.1 we have obtained that

$$u(x,y)=A^{-1}\left[\frac{(-2s)\left(\frac{-2}{s}\right)e^{3x}}{4+4s^2}\right]=A^{-1}\left[\frac{e^{3x}}{1+s^2}\right]=e^{3x}A^{-1}\left[\frac{1}{1+s^2}\right]=e^{3x}\cos y$$

Similarly,

$$v(x,y) = A^{-1}\left[\frac{(-2)\left(\frac{-2}{s}\right)e^{3x}}{4+4s^2}\right] = A^{-1}\left[\frac{e^{3x}}{s(1+s^2)}\right] = e^{3x}A^{-1}\left[\frac{1}{s(1+s^2)}\right] = e^{3x}\sin y$$

Hence

$$\begin{aligned} w = u + iv &= e^{3x}\cos y + ie^{3x}\sin y \\ &= e^{3x+iy} = e^{3\left(\frac{z+\overline{z}}{2}\right)+i\left(\frac{z-\overline{z}}{2i}\right)} = e^{2z+\overline{z}} \end{aligned}$$

□

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References

[1] T.M.Elzaki, S.M.Elzaki and E.M.A.Hilal, *Elzaki and sumudu Transforms for solving some Differential Equations*, Global J. Pure Appl. Math., 8(2)(2012), 167-173.

[2] Tarig M.Elzaki and S.M.Elzaki, *Application of New Transform "Elzaki Transform" to partial Differential Equations*, Global J. Pure Appl. Math., 7(1)(2011), 65-70.

[3] T.M.Elzaki and S.M.Elzaki, *Solution of Integro-Fifferential Equations by using Elzaki Transform*, Global J. Math. Sci., : Theory and Practical, 3(1)(2011), 1-11.

[4] T.M.Elzaki and E.M.A.Hilal, *Homotopy perturbation and Elzaki Transform for Solving Nonlinear partial Differential Equations*, Math. Theory Mod, 2(3)(2012), 33-42.

[5] T.M.Elzaki, *Solution of Nonlinear Differential Equations Using Mixture of Elzaki Transform and Differential Transform Method*, Int. Math. Forum, 7(13)(2012), 631-638.

[6] M.Duz, *On Application of Laplace Transforms*, New Trends in Mathematical Sciences, Accepted.

[7] Murat Duz, *Application of Elzaki Transform to first order Costant Coefficients Complex Equations*, Bullentin of the International Mathematical Virtual Institute, 7(2017), 387-393.

[8] Mohand M.Abdelrahim Mahgoub and Abdelbagy A.Alshikh, *Application of the Differential Transform Method for the Nonlinear Differential Equations*, American Journal of Applied Mathematics, 5(1)(2017), 14-18.

[9] K.S.Aboodh, R.A.Farah, I.A.Almardy and F.A.ALmostafa, *Solution of Partial Integro-Differential Equations by using Aboodh and Double Aboodh Transform Methods*, Global J. Pure Appl. Math., 13(8)(2017), 4347-4360.

[10] K.S.Aboodh, *The New Integral Transform Aboodh Transform*, Global J. Pure Appl. Math., 9(1)(2013), 35-43.

[11] Khalid Suliman Aboodh, *Application of New Transform "Aboodh Transform" to Partial Differential Equations*, Global J. Pure Appl. Math., 10(2)(2014), 249-254.

[12] M.Mohand, K.S.Aboodh and A.Abdelbagy, *On the Solution of Ordinary Differential Equation with Variable Coefficients using Aboodh Transform*, Advances in Theoretical and Applied Mathematics, 11(4)(2016), 383-389.