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Quasi Supra N-closed Map and Supra N-normal Space

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Abstract: The purpose of this paper is to give a new type of map called Quasi supra N-closed map and we obtain its basic properties and also we introduce the concept of supra N-normal spaces and study some fundamental properties.

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Keywords: Quasi supra N-closed map, Quasi supra N-open map, supra N-normal space, weakly supra N-normal space.

1. Introduction

Supra topological spaces was introduced by A.S.Mashhour et al. (see [4]) who studied, s-continuous functions and s*-continuous functions. A.K.Das (see [1]) studied about the decomposition of normality in general topology. The notion of this paper is to bring out and characterize the concept of Quasi supra N-closed map and Also to introduce the concept of supra N-normal space and weakly supra N-normal space and study some fundamental properties.

2. Preliminaries

Definition 2.1 ([4]). *A subfamily μ of X is said to be supra topology on X if*

(1). $X, \phi \in \mu$

(2). If $A_i \in \mu$, $\forall i \in j$ then $\cup A_i \in \mu$

(X, μ) is called supra topological space. The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

Definition 2.2 ([4]). *The supra closure of a set A is denoted by $cl^\mu(A)$, and is defined as*

$$supra\ cl(A) = \cap\{B : B \text{ is supra closed and } A \subseteq B\}.$$

The supra interier of a set A is denoted by $int^\mu(A)$, and is defined as

$$supra\ int(A) = \cup\{B : B \text{ is supra open and } A \supseteq B\}.$$

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Definition 2.3 ([4]). *Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if $\tau \subseteq \mu$.*

Definition 2.4. *A subset A of a space X is called*

(1). supra semi-open set [3], if $A \subseteq cl^{\mu}(int^{\mu}(A))$.

(2). supra α -open set [2], if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$.

(3). supra Ω closed set [5], if $scl^{\mu}(A) \subseteq int^{\mu}(U)$,whenever $A \subseteq U$, U is supra open set.

(4). supra N-closed set [6] if $\Omega cl^{\mu}(A) \subseteq U$, whenever $A \subseteq U$, U is supra α open set.

The complement of the above mentioned sets are their respective open and closed sets and vice-versa.

Definition 2.5. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1). supra N-continuous [6] if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

(2). supra N-irresolute [6] if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

(3). perfectly supra N-continuous [9] if $f^{-1}(V)$ is supra clopen in (X, τ) for every supra N-closed set V of (Y, σ).

(4). Strongly supra N-continuous [9] if $f^{-1}(V)$ is supra closed in (X, τ) for every supra N-closed set V of (Y, σ).

(5). perfectly contra supra N-irresolute [8] if $f^{-1}(V)$ is supra N-closed and supra N-open in (X, τ) for every supra N-open set V of (Y, σ).

(6). supra N-closed map [7] if f(V) is supra N-closed in (Y, σ) for every supra closed set V of (X, τ).

Definition 2.6 ([7]). *A supra topological space (X, τ) is T^{μ}_{N}-space if every supra N-closed set is supra closed in (X, τ).*

3. Quasi Supra N-closed Map

Definition 3.1. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be Quasi supra N-closed map (resp Quasi supra N-open map) if the image of every supra N-closed set(resp supra N-open set) in (X, τ) is supra closed set(resp supra open set) in (X, τ).*

Theorem 3.2. *Every Quasi supra N-closed map is supra closed map.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a Quasi supra N-closed map. Let V be supra closed set in X. Then V is supra N-closed set in X, since every supra closed set is supra N-closed set. Since f is Quasi supra N-closed map, $f(V)$ is supra closed set in Y. Hence f is supra closed map. □

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.3. *Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. supra N-closed sets in (X, τ) are $\{X, \phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. $f : (X, \tau) \to (Y, \sigma)$ be the function defined by $f(a) = a$, $f(b) = c$, $f(c) = b$. Here f is supra closed map but not Quasi supra N-closed map, since $V = \{b\}$ is supra N-closed set in $\{X, \tau\}$ but $f(V) = \{c\}$ is not supra closed set in (Y, σ).*

Theorem 3.4. *Every Quasi supra N-closed map is supra N-closed map.*

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a Quasi supra N-closed map. Let V be supra closed set in X. Then V is supra N-closed set in X, since every supra closed set is supra N-closed set. Since f is Quasi supra N-closed map, $f(V)$ is supra closed set in Y. Then $f(V)$ is supra N-closed set in Y. Hence f is supra N-closed map. □

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.5. *Let $X=Y=\{a,b,c\}$ and $\tau=\{X,\phi,\{a,b\}\}$ and $\sigma=\{Y,\phi,\{a,b\},\{b,c\}\}$. supra N-closed sets in (X,τ) are $\{X,\phi,\{c\},\{b,c\},\{a,c\}\}$. $f:(X,\tau)\rightarrow(Y,\sigma)$ be the function defined by $f(a)=b$, $f(b)=c$, $f(c)=a$. Here f is supra N-closed map but not Quasi supra N-closed map, since $V=\{b,c\}$ is supra N-closed set in $\{X,\tau\}$ but $f(V)=\{a,c\}$ is not supra closed set in (Y,σ).*

Theorem 3.6. *Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be any two functions and $g\circ f:(X,\tau)\rightarrow(Z,\eta)$ is Quasi supra N-closed map. If g is supra continuous injective, then f is Quasi supra N-closed map.*

Proof. Let V be supra N-closed set in X, then $(g\circ f)(V)$ is supra closed set in Z, since $g\circ f$ is Quasi supra N-closed map. Since g is supra continuous, $g^{-1}(g\circ f)(V)$ is supra closed set in Y. implies $f(V)$ is supra closed set in Y. Hence f is Quasi supra N-closed map. □

Theorem 3.7. *If $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ are Quasi supra N-closed maps, then $g\circ f:(X,\tau)\rightarrow(Z,\eta)$ is Quasi supra N-closed map.*

Proof. Let V be supra N-closed set in X,then $f(V)$ is supra closed set in Y, since f is Quasi supra N-closed map. Since every supra closed set is supra N-closed, $f(V)$ is supra N-closed set in Y. Since g is supra N-closed map, $g(f(V))$ is supra closed set in Z. Implies $(g\circ f)(V)$ is supra closed set in Z. Hence $g\circ f$ is supra N-closed map. □

Theorem 3.8. *If $f:(X,\tau)\rightarrow(Y,\sigma)$ is supra N-closed map and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is Quasi supra N-closed maps, then $g\circ f:(X,\tau)\rightarrow(Z,\eta)$ is supra closed map.*

Proof. Let V be supra closed set in X. Since f is supra N-closed map, then $f(V)$ is supra N-closed set in Y. Since g is Quasi supra N-closed map, $g(f(V))$ is supra closed set in Z. Implies $(g\circ f)(V)$ is supra closed set in Z. Hence $g\circ f$ is supra closed map. □

Theorem 3.9. *Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be any two functions. If f is Quasi supra N-closed map and g is supra closed map then $g\circ f$ is strongly supra N-closed map.*

Proof. Let V be supra N-closed set in X, then $f(V)$ is supra closed set in Y, since f is Quasi supra N-closed map. Since g is supra closed map $g(f(V))$ is supra closed set in Z, implies $g(f(V))$ is supra N-closed set in Z. Hence $g\circ f$ is strongly supra N-closed map. □

Theorem 3.10. *Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be any two functions. If f is strongly supra N-closed map and g is Quasi supra N-closed map then $g\circ f$ is Quasi supra N-closed map.*

Proof. Let V be supra N-closed set in X, then $f(V)$ is supra N-closed set in Y, since f is strongly supra N-closed map. Since g is Quasi supra N-closed map $g(f(V))$ is supra closed set in Z. Hence $g\circ f$ is Quasi supra N-closed map. □

Theorem 3.11. *Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be any two functions. If f is strongly supra N-closed map and g is Quasi supra N-closed map then $g\circ f$ is strongly supra N-closed map.*

Proof. Let V be supra N-closed set in X, then $f(V)$ is supra N-closed set in Y, since f is strongly supra N-closed map. Since g is Quasi supra N-closed map $g(f(V))$ is supra closed set in Z, implies $g(f(V))$ is supra N-closed set in Z. Hence $g \circ f$ is strongly supra N-closed map. □

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be supra N-irresolute, surjective and $g : (Y, \sigma) \to (Z, \eta)$ be any functions and $g \circ f$ is Quasi supra N-closed map, then g is supra closed map.*

Proof. Let V be supra closed set in Y, then V is supra N-closed set in Y. Since f is supra N-irresolute, $f^{-1}(V)$ is supra N-closed set in X. Since $g \circ f$ is Quasi supra N-closed map, $(g \circ f)f^{-1}(V)$ is supra closed set in Z. Implies $g(V)$ is supra closed set in Z. Hence g is supra closed map. □

Theorem 3.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be any function and $g : (Y, \sigma) \to (Z, \eta)$ be supra N-continuous injective and $g \circ f$ is Quasi supra N-closed map, then f is strongly supra N-closed map.*

Proof. Let V be supra N-closed set in X. Since $g \circ f$ is Quasi supra N-closed map, then $(g \circ f)(V)$ is supra closed set in Z. Since g is supra N-continuous, $g^{-1}(g \circ f)(V)$ is supra N-closed set in Y. Implies $f(V)$ is supra N-closed set in Y. Hence f is strongly supra N-closed map. □

Theorem 3.14. *Let $f : (X, \tau) \to (Y, \sigma)$ be perfectly contra supra N-irresolute, surjective function and $g : (Y, \sigma) \to (Z, \eta)$ be any function and $g \circ f$ is Quasi supra N-closed map, then g is Quasi supra N-closed map.*

Proof. Let V be supra N-closed set in Y. Since f is perfectly contra supra N-irresolute, then $f^{-1}(V)$ is supra N-closed set and supra N-open set in X. Since $g \circ f$ is Quasi supra N-closed map, then $(g \circ f)f^{-1}(V)$ is supra closed set in Z. Implies $g(V)$ is supra closed set in Z. Hence g is Quasi supra N-closed map. □

4. Supra N-normal Spaces

Definition 4.1. *A Space (X, τ) is said to be supra normal if for any pair of disjoint closed sets A and B, there exist disjoint supra open sets U and V such that $A \subset U$ and $B \subset V$.*

Definition 4.2. *A Space (X, τ) is said to be supra N-normal if for any pair of disjoint closed sets A and B, there exist disjoint supra N-open sets U and V such that $A \subset U$ and $B \subset V$.*

Remark 4.3. *Every supra normal space is supra N-normal. converse need not be true. It is seen from the following example.*

Example 4.4. *Let X=$\{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ supra N-closed sets in (X, τ) are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{c, d\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$. Here (X, τ) is supra N-normal but not supra normal, since A=$\{a\}$ and B=$\{b\}$ is supra closed in (X, τ) but A and B is not contained in disjoint supra open sets.*

Theorem 4.5. *For a space X the following are equivalent:*

(1). X is supra N-normal

(2). for every pair of supra open sets U and V whose union is X, there exist supra N-closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(3). For every supra closed set H and every supra open set K containing H, there exists a supra N-open set U such that $H \subset U \subset \bar{U} \subset K$.

Proof. (1)⇒(2) Let U and V be a pair of supra open sets in a supra N-normal space X, such that $X = U \cup V$. Then $X - U$ and $X - V$ are disjoint supra closed sets. Since X is supra N-normal, there exist disjoint supra N-open sets U_1 and V_1 such that $X - U \subset U_1$ and $X - V \subset V_1$. Let $A = X - U_1$ and $B = X - V_1$. Then A and B are supra N-closed sets such that $A \subset U$ and $B \subset V$ and $A \cup B = X$.

(2)⇒(3) Let H be supra closed set and K be an supra open set containing H. Then $X - H$ and K are supra open sets whose union is X. Then by (2), there exist supra N-closed sets H_1 and K_1 such that $H_1 \subset X - H$ and $K_1 \subset K$ and $H_1 \cup K_1 = X$. Then $H \subset X - H_1$ and $X - K \subset X - K_1$ and $(X - H_1) \cap (X - K_1) = \phi$. Let $U = X - H_1$ and $V = X - K_1$. Then U and V are disjoint supra N-open sets such that $H \subset U \subset X - V \subset K$. ie., $H \subset U \subset \bar{U} \subset K$.

(3)⇒(1) Let A and B be any two disjoint supra closed sets of X. Put $G = X - B$, then $B \cap G = \phi$. $A \subset G$ where G is a supra open set. Then by (3), there exist supra N-open set U of X such that $A \subset U \subset \bar{U} \subset G$. Then $\bar{U}$ is supra N-open set and $U \cap \bar{U} = \phi$. Hence A and B are separated by supra N-open sets U and $\bar{U}$. Therefore X is supra N-normal. □

Theorem 4.6. *If $f : (X, \tau) \to (Y, \sigma)$ is a strongly supra N-open map, supra continuous function from a supra N-normal space X on to a space Y, then Y is supra N-normal.*

Proof. Let A and B be disjoint supra closed sets in Y, then $f^{-1}(A)$ and $f^{-1}(B)$ are supra closed sets in X, since f is supra continuous. Since X is supra N-normal, there exist a disjoint supra N-open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Implies $A \subset f(U)$ and $B \subset f(V)$. Since f is strongly supra N-open map, $f(U)$ and $f(V)$ are disjoint supra N-open sets in Y. Hence Y is supra N-normal. □

Theorem 4.7. *If $f : (X, \tau) \to (Y, \sigma)$ be supra closed map, supra N-continuous injection. If Y is supra normal, then X is supra N-normal.*

Proof. Let A and B be disjoint supra closed sets in X. Since f is a supra closed map injection, $f(A)$ and $f(B)$ are disjoint supra closed set in Y. Since Y is supra normal, there exist disjoint supra open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Since f is supra N-continuous $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint supra N-open sets in X. Therefore $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Hence X is supra N-normal. □

Theorem 4.8. *If $f : (X, \tau) \to (Y, \sigma)$ be supra closed map, supra N-irresolute injection and if Y is supra N-normal, then X is supra N-normal.*

Proof. Let A and B be disjoint supra closed sets in X. Since f is a supra closed map injection, $f(A)$ and $f(B)$ are disjoint supra closed set in Y. Since Y is supra N-normal, there exist disjoint supra N-open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Since f is supra N-irresolute $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint supra N-open sets in X. Therefore $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Hence X is supra N-normal. □

Theorem 4.9. *If $f : (X, \tau) \to (Y, \sigma)$ be supra open map, supra continuous surjection and If X is supra normal, then Y is supra N-normal.*

Proof. Let A and B be disjoint supra closed sets in Y. Since f is a supra continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint supra closed set in X. Since X is supra normal, there exist disjoint supra open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is supra N-open map $f(U)$ and $f(V)$ are disjoint supra N-open sets in Y. Therefore $A \subset f(U)$ and $B \subset f(V)$. Hence X is supra N-normal. □

Theorem 4.10. *If $f : (X, \tau) \to (Y, \sigma)$ is supra continuous, Quasi supra N-open map from a supra normal space X on to a space Y, then Y is supra normal.*

Proof. Let A and B be disjoint supra closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets in X, since f is supra continuous. Since X is supra normal, there exist a disjoint supra open set U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since every supra open set is supra N-open, then U and V are supra N-open sets in X. Since X is Quasi supra N-open map, then $f(U)$ and $f(V)$ are supra open sets in Y. Therefore $A \subset f(U)$ and $B \subset f(V)$. Hence Y is supra normal. □

Theorem 4.11. *If $f : (X, \tau) \to (Y, \sigma)$ is supra continuous, Quasi supra N-open map from a supra N-normal space X on to a space Y, then Y is supra normal.*

Proof. Let A and B be disjoint supra closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets in X, since f is supra continuous. Since X is supra N-normal, there exist a disjoint supra N-open set U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since X is Quasi supra N-open map $f(U)$ and $f(V)$ are supra open sets in Y. Therefore $A \subset f(U)$ and $B \subset f(V)$. Hence Y is supra normal. □

Definition 4.12. *A supra topological space (X, τ) is said to be weakly supra N-normal if every pair of disjoint supra N-closed sets are contained in disjoint open sets.*

Theorem 4.13. *Every weakly supra N-normal space is supra normal.*

Proof. Let (X, τ) be weakly supra N-normal space. Let A and B be disjoint supra closed sets in (X, τ), then A and B are disjoint supra N-closed sets, since every supra closed set is supra N-closed set. Since X is weakly supra N-normal, there exist disjoint supra open sets U and V such that $A \subset U$ and $B \subset V$. Hence (X, τ) is supra normal. □

Converse of the above theorem need not be true. It is shown by the following example.

Example 4.14. *Let X=$\{a, b, c, d\}$ and $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ supra N-closed sets in (X, τ) are $\{X, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$. Here (X, τ) is supra normal but not weakly supra N-normal, since A=$\{a\}$ and B=$\{b\}$ is supra N-closed in (X, τ) but A and B is not contained in disjoint supra open sets.*

Theorem 4.15. *Every weakly supra N-normal space is supra N-normal.*

Proof. Let (X, τ) be weakly supra N-normal space. Let A and B be disjoint supra closed sets in (X, τ), then A and B are disjoint supra N-closed sets, since every supra closed set is supra N-closed set. Since X is weakly supra N-normal, there exist disjoint supra open sets U and V such that $A \subset U$ and $B \subset V$. Since every supra open set is supra N-open set U and V are supra N-open sets in X. Hence (X, τ) is supra N-normal. □

Converse of the above theorem need not be true. It is shown by the following example.

Example 4.16. *Let X=$\{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ supra N-closed sets in (X, τ) are $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{c, d\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$. Here (X, τ) is supra N-normal but not weakly supra N-normal, since A=$\{a\}$ and B=$\{b\}$ is supra N-closed in (X, τ) but A and B is not contained in disjoint supra open sets.*

Theorem 4.17. *A supra topological space X is weakly supra N-normal iff for every supra N-closed set A and a supra N-open set U containing A, there is an open set V such that $A \subset V \subset \bar{V} \subset U$.*

Proof. Suppose X is a weakly supra N-normal space. Let U be a supra N-open set containing a supra N-closed set A. Then A and X-U are disjoint supra N-closed sets in X. Since X is weakly supra N-normal, there exist disjoint supra open sets V and W containing A and $X-U$ respectively. Then $A \subset V \subset X-W \subset U$. ie., $A \subset V \subset \bar{V} \subset U$.

Conversely, Let A and B be two disjoint supra N-closed sets in X. Let U=X-B be a supra N-open set containing A. Thus by hypothesis, there exist a supra open set V such that $A \subset V \subset \bar{V} \subset U$. Then V and $X-\bar{V}$ are disjoint supra open sets containing A and B respectively. Hence X is weakly supra N-normal. □

Theorem 4.18. *If $f:(X,\tau) \rightarrow (Y,\sigma)$ be strongly supra N-closed map, supra continuous injective and If Y is weakly supra N-normal, then X is weakly supra N-normal.*

Proof. Let A and B be disjoint supra N-closed sets in X. Since f is a strongly supra N-closed map, $f(A)$ and $f(B)$ are disjoint supra N-closed set in Y. Since Y is weakly supra N-normal, there exist disjoint supra open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Implies $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Since f is supra continuous $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint supra open sets in X. Hence X is weakly supra N-normal. □

Theorem 4.19. *If $f:(X,\tau) \rightarrow (Y,\sigma)$ be supra open map, perfectly contra supra N-irresolute surjective and if X is weakly supra N-normal, then Y is weakly supra N-normal.*

Proof. Let A and B be disjoint supra N-closed sets in Y. Since f is a perfectly contra supra N-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint supra N-closed set and supra N-open set in X. Since X is weakly supra N-normal, there exist disjoint supra open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Implies $A \subset f(U)$ and $B \subset f(V)$. Since f is supra open map $f(U)$ and $f(V)$ are disjoint supra open sets in Y. Hence Y is weakly supra N-normal. □

Theorem 4.20. *If $f:(X,\tau) \rightarrow (Y,\sigma)$ be supra open map, supra N-irresolute surjective and if X is weakly supra N-normal, then Y is weakly supra N-normal.*

Proof. Let A and B be disjoint supra N-closed sets in Y. Since f is a supra N-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint supra N-closed set in X. Since X is weakly supra N-normal, there exist disjoint supra open sets U and V such that $f^{-1}(A) \subset$U and $f^{-1}(B) \subset$ V. Since f is supra open map $f(U)$ and $f(V)$ are disjoint supra open sets in Y. Therefore $A \subset f(U)$ and $B \subset f(V)$. Hence Y is weakly supra N-normal. □

Theorem 4.21. *If $f:(X,\tau) \rightarrow (Y,\sigma)$ be Quasi supra N-closed map, supra continuous injective and if Y is weakly supra N-normal, then X is weakly supra N-normal.*

Proof. Let A and B be disjoint supra N-closed sets in X. Since f is a Quasi supra N-closed map, $f(A)$ and $f(B)$ are disjoint supra closed set and hence $f(A)$ and $f(B)$ are disjoint supra N-closed set in Y. Since Y is weakly supra N-normal, there exist disjoint supra open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Implies $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Since f is supra continuous $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint supra open sets in X. Hence X is weakly supra N-normal. □

Theorem 4.22. *If $f:(X,\tau) \rightarrow (Y,\sigma)$ be supra N-closed map, supra continuous injective and if Y is weakly supra N-normal and X is T_N^{μ}-space, then X is weakly supra N-normal.*

Proof. Let A and B be disjoint supra N-closed sets in X. Since X is T_N^{μ}-space, A and B are disjoint supra closed sets in X. Since f is a supra N-closed map, $f(A)$ and $f(B)$ are disjoint supra N-closed set in Y. Since Y is weakly supra N-normal, there exist disjoint supra open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Implies $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Since f is supra continuous $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint supra open sets in X. Hence X is weakly supra N-normal. □

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