Int. J. Math. And Appl., 6(1–A)(2018), 23–28
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

On γ-regular Semi-open Sets

A. Vadivel[1,*] and C. Sivashanmugaraja[2]

1 Department of Mathematics, Government Arts College (Autonomous), Karur, Tamilnadu, India.

2 Department of Mathematics, Periyar Arts College, Cuddalore, Tamilnadu, India.

Abstract: We introduce the concepts of γ-regular semi-open set, γ_{rs}-set, γ^{rs}-set, generalized γ_{rs}-set, generalized γ^{rs}-set, regular semi-T_1^{γ} space and regular semi-R_0^{γ} space by using γ-regular open sets.

MSC: 54A05, 54D10.

Keywords: Generalized topology, generalized open sets, γ-open sets, γ-regular semi-open sets, regular semi-T_1^{γ} space, regular semi-R_0^{γ} space.

1. Introduction

The idea of examining generalized open sets in generalized topological spaces was given by À. Csàszàr [4–7]. Generalized $\bigwedge_s$-sets and generalized $\bigvee_s$-sets were introduced by Miguel Caldas and Julian Dontchev in general topology [1, 2]. Maheswari and Prasad in [8, 9] introduced two new classes called semi-T_1 spaces and semi-R_0-spaces. Cameron [3] introduced regular semi-open set which is weaker then regular open set and regular closed set. The complement of regular semi-open is a regular semi-open. In this paper we give the definitions of γ-regular semi-open set, γ_{rs}-set, γ^{rs}-set by using γ-regular open sets. Also we aimed to show that the concepts of $g.\bigwedge_{rs}$-set, $g.\bigvee_{rs}$-set, regular semi-T_1 space and regular semi-R_0 space can be generalized by replacing regular semi-open sets with γ-regular semi-open sets for an arbitrary $\gamma \in \Gamma(X)$. Concepts of this paper should be considered in generalized topological spaces instead of general topology.

1.1. Preliminaries

Let X be an underlying set and $\gamma : expX \to expX$ be a monotonic mapping from the power set exp X of the set X into itself (i.e. such that $A \subset B$ implies $\gamma(A) \subset \gamma(B)$). We denote the collection of all mappings having the property of monotony by $\Gamma(X)$. Let us say that $A \subset X$ is γ-open iff $A \subset \gamma(A)$. We say that $A \subset X$ is γ-closed iff $X - A$ is γ-open. We can construct a set which is equal to the intersection of all γ-closed sets including the set O for $O \subset X$. This constructed set is called the γ-closure of O, denoted by $c_\gamma O$. Similarly, we can construct a set which is equal to the union of all γ-open subsets of O. This set is called the γ-interior of O, denoted by $i_\gamma O$. The concept of regular semi-open set in a topological space was introduced by Cameron [3]. A subset A of a topological space (X, τ) is said to be regular semi-open if $O \subset A \subset cl(O)$ for some regular open set O, where $cl(O)$ denotes the closure of O in (X, τ). A topological space (X, τ) is called a regular

* *E-mail: avmaths@gmail.com*

semi-T_1-space if to each pair of distinct points x, y of (X, τ) there corresponds a regular semi-open set A containing x but not y and a regular semi-open set B containing y but not x. Again a topological space (X, τ) is called a regular semi-R_0 space if every regular semi-open set contains the regular semi-closure of each of its singletons.

2. γ-regular Semi-open Sets

Definition 2.1. *Let $A \subset X$ and $\gamma \in \Gamma(X)$. Then A is γ-regular semi-open iff there exists a γ-regular open set O such that $O \subset A \subset c_\gamma O$.*

Proposition 2.2. *Let X be a set and $\gamma \in \Gamma(X)$. Then X is γ-regular semi-open.*

Proof. If A is the union of all γ-regular open subsets of X and $A \subset F \subset X$, where F is γ-regular closed, then clearly $F = X$ so that $c_\gamma A = X$ and $A \subset X \subset c_\gamma(A)$.

Proposition 2.3. Every γ-regular open set is γ-regular semi-open.

Proof. Let $A \subset X$ be a γ-regular open set. We have $A \subset c_\gamma(A)$. Then it is easily seen that A is γ-regular semi-open. □

The converse of Proposition 2.3 is not true; this may be seen from the following example.

Example 2.4. *Let $X = \{a, b, c, d\}$ and $\gamma : exp\ X \to exp\ X$ be defined as*

$$\gamma A = \begin{cases} A - \{a\}, & a \in A, \\ A, & a \notin A. \end{cases}$$

Then the set $B = \{a, c\}$ is γ-regular semi-open, but it is not a γ-regular open set.

Proposition 2.5. *Any union of γ-regular semi-open set is γ-regular semi-open.*

Proof. Let X be a set, $\gamma \in \Gamma(X)$ and $\{A_i\}_{i \in I}$ a family of γ-regular semi-open subsets of X. Then, there is a γ-regular open set O_i such that $O_i \subset A_i \subset c_\gamma O_i$. Then,

$$\bigcup_{i \in I} O_i \subset \bigcup_{i \in I} A_i \subset \bigcup_{i \in I} c_\gamma O_i.$$

Here $\bigcup_{i \in I} O_i$ is a γ-regular open set. On the other hand $O_i \subset \bigcup_{i \in I} O_i$. Then by using the monotonicity of c_γ

$$c_\gamma O_i \subset c_\gamma(\bigcup_{i \in I} O_i) \Rightarrow \bigcup_{i \in I} c_\gamma O_i \subset c_\gamma(\bigcup_{i \in I} O_i).$$

If we take $\bigcup_{i \in I} O_i = O$ then, $O \subset \bigcup_{i \in I} A_i \subset c_\gamma O$. □

However, the intersection of two γ-regular semi-open sets is not γ-regular semi-open as the following example shows:

Example 2.6. *Let $X = R$ with the usual (Euclidean) topology, i be the interior and c be the closure with respect to the Euclidean topology. Suppose that $\gamma = ci$. The sets $A = [-1, 0]$ and $B = [0, 1]$ are both γ-regular open and hence both of them are γ-regular semi-open. $A \cap B = \{0\}$. On the other hand the only γ-regular open subset of $\{0\}$ is ϕ. As R is a γ-regular open set, ϕ is also γ-regular closed and $c_\gamma \phi = \phi$.*

The subset A of X is γ-regular semi-closed iff $X - A$ is γ-regular semi-open. By using Proposition 2.2 and the Definition of γ-regular semi-closed set we can easily see that ϕ is a γ-regular semi-closed set. Other points that could be mentioned are γ-regular semi closure and γ-regular semi interior. Let $A \subset X$ and $\gamma \in \Gamma(X)$. The intersection of all γ-regular semi-closed sets containing A is the γ-regular semi closure of A, denoted by $rscl_\gamma A$. Similarly, the union of all γ-regular semi-open sets contained in A is called the γ-regular semi interior of A, denoted by $rsi_\gamma(A)$.

3. γ_{rs} and γ^{rs} Sets

Definition 3.1. *Let X be a non-empty set, $\gamma \in \Gamma(X)$ and $B \in X$. Then the set $\gamma_{rs}B$ is the intersection of all γ-regular semi-open sets containing B, that is;*

$$\gamma_{rs}(B) = \bigcap\{O : O \supset B \text{ and } O \text{ is } \gamma\text{-regular semi-open}\}.$$

The set $\gamma^{rs}B$ is the union of all γ-regular semi-closed subsets of B, that is;

$$\gamma^{rs}(B) = \bigcup\{F : F \subset B \text{ and } F \text{ is } \gamma\text{-regular semi-closed}\}.$$

Definition 3.2. *Let X be a nonempty set, $\gamma \in \Gamma(X)$ and $B \in X$. Then*

(a). B is called a γ_{rs}-set iff $B = \gamma_{rs}B$,

(b). B is called a γ^{rs}-set iff $B = \gamma^{rs}B$.

Definition 3.3. *Let X, γ and B be the same as in Definition 3.2 B is called a generalized γ_{rs}-set [g.γ_{rs}-set] if and only if $\gamma_{rs}B \subset F$ whenever $B \subset F$ and F is γ-regular semi-closed. B is called a generalized γ^{rs}-set [g.γ^{rs}-set] iff $B^c = X\backslash B$ is a g.γ_{rs}-set.*

Proposition 3.4. *If A, B and $\{C_\lambda : \lambda \in \Omega\}$ are subsets of X and $\gamma \in \Gamma(X)$, then the following properties are valid:*

(a). $A \subset \gamma_{rs}A$.

(b). If $A \subset B$ then $\gamma_{rs}A \subset \gamma_{rs}B$; $\gamma_{rs} \in \Gamma$.

(c). $\gamma_{rs}\gamma_{rs}A = \gamma_{rs}A$.

(d). $\gamma_{rs}(\bigcup_{\lambda\in\Omega} C_\lambda) = \bigcup_{\lambda\in\Omega} \gamma_{rs}C_\lambda$.

(e). If A is γ-regular semi-open then $\gamma_{rs}A = A$.

(f). $\gamma_{rs}(B)^c = (\gamma^{rs}B)^c$.

(g). $\gamma^{rs}B \subset B$.

(h). If B is γ-regular semi-closed then $\gamma^{rs}B = B$.

(i). $\gamma_{rs}[\bigcap_{\lambda\in\Omega} C_\lambda] \subset \bigcap_{\lambda\in\Omega} \gamma_{rs}C_\lambda$.

(j). $\gamma^{rs}[\bigcup_{\lambda\in\Omega} C_\lambda] \supset \bigcup_{\lambda\in\Omega} \gamma^{rs}C_\lambda$.

Proof.

(a). Clear by definition of $\gamma_{rs}A$.

(b). Suppose that $x \notin \gamma_{rs}B$. Then there exists a γ-regular semi-open set O' such that $O' \supset B$ with $x \notin O'$. Since $B \supset A$, then $x \notin \gamma_{rs}A$ and thus $\gamma_{rs}A \subset \gamma_{rs}B$.

(c). We can write $\gamma_{rs}A \subset \gamma_{rs}\gamma_{rs}A$. Suppose that $\gamma_{rs}\gamma_{rs}A \not\subset \gamma_{rs}A$; so there exists $x \in X$ such that $x \notin \gamma_{rs}A$ and $x \in \gamma_{rs}\gamma_{rs}A$. As $x \notin \gamma_{rs}A$, then there exists a γ-regular semi-open set O such that $x \notin O$ and $O \supset A$. $x \in \gamma_{rs}\gamma_{rs}A$, then $x \in O'$ for every γ-regular semi-open set $O' \supset \gamma_{rs}A$. For every such O', $O' \supset \gamma_{rs}A \supset A$ and $O' \supset A$. This is a contradiction. Thus $\gamma_{rs}\gamma_{rs}A \subset \gamma_{rs}A$.

(d). Suppose that there exists a point x such that $x \notin \gamma_{rs}(\bigcup_{\lambda\in\Omega} C_\lambda)$. Then there exists a γ-regular semi-open set O such that $\bigcup_{\lambda\in\Omega} C_\lambda \subset O$ and $x \notin O$. Thus for each $\lambda \in \Omega$ we have $x \notin \gamma_{rs}C_\lambda$. This implies that $x \notin \bigcup_{\lambda\in\Omega} \gamma_{rs}C_\lambda$. Conversely, suppose that there exists a point $x \in X$ such that $x \notin \bigcup_{\lambda\in\Omega} \gamma_{rs}C_\lambda$. By Definition 3.1, there exist γ-regular semi-open sets O_λ such that $x \notin O_\lambda$, $C_\lambda \subset O_\lambda$ for all $\lambda \in \Omega$. Let $O = \bigcup_{\lambda\in\Omega} O_\lambda$. Then $x \notin \bigcup_{\lambda\in\Omega} O_\lambda$, $\bigcup_{\lambda\in\Omega} C_\lambda \subset O$ and O is γ-regular semi-open. This implies that $x \notin \gamma_{rs}[\bigcup_{\lambda\in\Omega} C_\lambda]$.

(e). It is clear by Definition 3.1

(f). $(\gamma^{rs}B)^c = \bigcap\{F^c | F^c \supset B^c,\ F^c \text{ is } \gamma\text{-regular semi-open }\} = \gamma_{rs}B^c$.

(g). Clear by Definition 3.1

(h). If B is γ-regular semi-closed, then B^c is γ-regular semi-open. We have $B^c = \gamma_{rs}(B^c) = (\gamma^{rs}B)^c$. Hence $B = \gamma^{rs}B$.

(i). $\bigcap_{\lambda\in\Omega} C_\lambda \subset C_\lambda$ for every $\lambda \in \Omega$. By using (a), $\gamma_{rs}[\bigcap_{\lambda\in\Omega} C_\lambda] \subset \bigcap_{\lambda\in\Omega} \gamma_{rs}C_\lambda$.

(j). $\gamma^{rs}[\bigcup_{\lambda\in\Omega} C_\lambda] = [\gamma_{rs}(\bigcup_{\lambda\in\Omega} C_\lambda)^c]^c = [\gamma_{rs}(\bigcap_{\lambda\in\Omega} C_\lambda^c)]^c \supset [\bigcap_{\lambda\in\Omega} \gamma_{rs}C_\lambda^c]^c = [\bigcap_{\lambda\in\Omega}(\gamma^{rs}C_\lambda)^c]^c = \bigcup_{\lambda\in\Omega} \gamma^{rs}C_\lambda$. □

Proposition 3.5. *Let $D^{\gamma_{rs}}$ and $D^{\gamma^{rs}}$ be the family of all $g.\gamma_{rs}$-sets and $g.\gamma^{rs}$-sets, respectively. Then*

(a). Every γ_{rs}-set is a $g.\gamma_{rs}$-set.

(b). Every γ^{rs}-set is a $g.\gamma^{rs}$-set.

(c). If $B_\lambda \in D^{\gamma_s}$, then $\bigcup_{\lambda\in\Omega} B_\lambda \in D^{\gamma_{rs}}$.

(d). If $B_\lambda \in D^{\gamma^{rs}}$, then $\bigcap_{\lambda\in\Omega} B_\lambda \in D^{\gamma^{rs}}$.

Proof.

(a). Assume that A is a γ_{rs}-set and $A \subset F$, whenever F is γ-regular semi-closed. Since A is a γ_{rs}-set, then $\gamma_{rs}A = A$. Thus $A = \gamma_{rs}A \subset F$.

(b). Assume that A is a γ^{rs}-set. By Proposition 3.4(f), $\gamma_{rs}A^c = (\gamma^{rs}A)^c = A^c$. Hence A^c is a γ_{rs}-set. By (a) A^c is $g.\gamma_{rs}$.

(c). Let $\bigcup_{\lambda\in\Omega} B_\lambda \subset F$ and F be γ-regular semi-closed. For every $\lambda \in \Omega$, $B_\lambda \subset F$ and F is γ-regular semi closed. By hypothesis, $\gamma_{rs}B_\lambda \subset F$. Then $\bigcup_{\lambda\in\Omega} \gamma_{rs}B_\lambda \subset F$. By Proposition 3.4(d) $\bigcup_{\lambda\in\Omega} \gamma_{rs}B_\lambda = \gamma_{rs}\bigcup_{\lambda\in\Omega} B_\lambda \subset F$.

(d). $(\bigcap_{\lambda\in\Omega} B_\lambda)^c = \bigcup_{\lambda\in\Omega} B_\lambda^c$. For every $\lambda \in \Omega$, B_λ^c is a $g.\gamma_{rs}$-set. Then by (c) $\bigcup_{\lambda\in\Omega} B_\lambda^c = (\bigcap_{\lambda\in\Omega} B_\lambda)^c$ is a $g.\gamma_{rs}$-set. □

Proposition 3.6.

(a). ϕ is a γ_{rs}-set, and X is a γ^{rs}-set.

(b). The union of $\gamma_{rs}(\gamma^{rs})$-sets is a $\gamma_{rs}(\gamma^{rs})$-set.

(c). The intersection of $\gamma_{rs}(\gamma^{rs})$-sets is a $\gamma_{rs}(\gamma^{rs})$-set.

(d). B is a γ_{rs}-set if and only if B^c is a γ^{rs}-set.

Proof.

(a). It is obvious.

(b). Suppose that $\{A_i\}_{i\in I}$ is a family of γ_{rs}-sets. By Proposition 3.4 (d), $\gamma_{rs}(\bigcup_{i\in I} A_i) = \bigcup_{i\in I} \gamma_{rs} A_i = \bigcup_{i\in I} A_i$. Let $\{B_i\}_{i\in I}$ be a family of γ^{rs}-sets. By Proposition 3.4 (j), $\bigcup_{i\in I} B_i \supset \gamma^{rs}(\bigcup_{i\in I} B_i) \supset \bigcup_{i\in I} \gamma^{rs} B_i = \bigcup_{i\in I} B_i$.

(c). Let $\{A_i\}_{i\in I}$ be a family of γ_{rs}-sets. By Proposition 3.4 (i), $\gamma_{rs} \bigcap_{i\in I} A_i \subset \bigcap_{i\in I} \gamma_{rs} A_i = \bigcap_{i\in I} A_i$. Let $\{B_i\}_{i\in I}$ be a family of γ^{rs}-sets. Then $\gamma^{rs}(\bigcap_{i\in I} B_i) = [\gamma_{rs}(\bigcup_{i\in I} B_i^c)]^c = [\bigcup_{i\in I} \gamma_{rs} B_i^c]^c = \bigcap_{i\in I} \gamma^{rs} B_i = \bigcap_{i\in I} B_i$.

(d). Let A be a γ_{rs}-set. $A = \gamma_{rs} A$. Then $A^c = [\gamma_{rs} A]^c = \gamma^{rs} A^c$. Thus A^c is a γ^{rs}-set. Conversely let A^c be a γ^{rs}-set. Then $A^c = \gamma^{rs} A^c$ and $A = [\gamma^{rs} A^c]^c = \gamma_{rs} A$. Thus A is a γ_{rs}-set. □

Theorem 3.7. *Let X be a set. Then*

(a). For every $x \in X$, x is a γ-regular semi-open set or x^c is a $g.\gamma_{rs}$-set.

(b). For every $x \in X$, x is a γ-regular semi-open set or x is a $g.\gamma^{rs}$-set.

Proof.

(a). Suppose that $\{x\}$ is not γ-regular semi-open. Then the only γ-regular semi-closed set F containing $\{x\}^c$ is X. Thus $\gamma_{rs}(\{x\}^c) \subset F = X$ and $\{x\}^c$ is a $g.\gamma_{rs}$-set.

(b). It is easy from (a) and Definition 3.3. □

Proposition 3.8. *If A is a $g.\gamma_{rs}$-set of X and $A \subset B \subset \gamma_{rs} A$, then B is a $g.\gamma_{rs}$-set of X.*

Proof. Since $A \subset B \subset \gamma_{rs} A$, we have $\gamma_{rs} A = \gamma_{rs} B$. Let F be any γ-regular semi-closed subset of X such that $B \subset F$. Since $A \subset B$ and A is $g.\gamma_{rs}$-set, we have $\gamma_{rs} B = \gamma_{rs} A \subset F$. □

Proposition 3.9. *A subset B of X is a $g.\gamma^{rs}$-set if and only if $U \subset \gamma^{rs} B$ whenever $U \subset B$ and U is γ-regular semi-open.*

Proof. ($\Rightarrow$) Let U be a γ-regular semi-open set such that $U \subset B$. Since U^c is γ-regular semi-closed and $U^c \supset B^c$, we have $U^c \supset \gamma_{rs}(B^c)$. Thus, $U \subset \gamma^{rs} B$ by Definition 3.3.
($\Leftarrow$) Let F be a γ-regular semi-closed set such that $B^c \subset F$. Since F^c is γ-regular semi-open and $F^c \subset B$, by assumption we have $F^c \subset \gamma^{rs} B$. Then, $F \supset (\gamma^{rs} B)^c = \gamma_{rs}(B^c)$ by Proposition 3.4(f), and B^c is a $g.\gamma_{rs}$-set, i.e. B is a $g.\gamma^{rs}$-set. □

Corollary 3.10. *Let B be a $g.\gamma^{rs}$-set. Then, for every γ-regular semi-closed set F such that $\gamma^{rs} B \cup B^c \subset F$, $F = X$ holds.*

Proof. The assumption $\gamma^{rs} B \cup B^c \subset F$ implies $(\gamma^{rs} B)^c \cap B \supset F^c$. Since B is a $g.\gamma^{rs}$-set, then we have $\gamma^{rs} B \supset F^c$ by Proposition 3.9 and hence $\phi = (\gamma^{rs} B)^c \cap \gamma^{rs} B \supset F^c$. Therefore, we have $X = F$. □

Corollary 3.11. *Let B be a $g.\gamma^{rs}$-set. Then $\gamma^{rs} B \cup B^c$ is γ-regular semi-closed if and only if B is a γ^{rs}-set.*

Proof. ($\Rightarrow$) By Corollary 3.10, $\gamma^{rs} B \cup B^c = X$. Thus $(\gamma^{rs} B)^c \cap B = \phi$. By Proposition 3.4(g) $B = \gamma^{rs} B$.
($\Leftarrow$) It is obvious. □

4. Regular Semi-T_1^γ and Regular Semi-R_0^γ Spaces

Definition 4.1. *Let X be a set and $\gamma \in \Gamma(X)$. Then X is called a regular semi-T_1^γ space if for every $x, y \in X$, $x \neq y$, there is a γ-regular semi-open set A such that $x \in A$, $y \notin A$ and there is a γ-regular semi-open set B such that $y \in B$, $x \notin B$.*

Proposition 4.2. *Let X be a non-empty set and $\gamma \in \Gamma(X)$. Then X is a regular semi-T_1^γ space if and only if every singleton is γ-regular semi-closed.*

Proof. ($\Rightarrow$) For every $x \in X$ and $y \in \{x\}^c$, $x \neq y$ and by assumption there is a γ-regular semi-open set B_y such that $y \in B_y$ and $x \notin B_y$, $B_y \subset \{x\}^c$. Then for every $y \in \{x\}^c$, $y \in B_y \subset \{x\}^c \Rightarrow \{x\}^c = \bigcup_{y \in \{x\}^c} B_y$. By Proposition 2.3 $\{x\}^c$ is γ-regular semi-open. Thus $\{x\}$ is γ-regular semi-closed for every $x \in X$.
($\Leftarrow$) We have $y \in \{x\}^c$ and $x \in \{y\}^c$ for every $x, y \in X$, $x \neq y$. By assumption $\{x\}^c$ and $\{y\}^c$ are γ-regular semi-open sets. □

Corollary 4.3. *X is a regular semi-T_1^γ space if and only if every subset of X is a γ_{rs}-set.*

Proof. ($\Rightarrow$) Let $B \subset X$ and for an $x \in X$, $x \notin B$. By Proposition 4.2, $\{x\}^c$ is a γ-regular semi-open set and contains B. Then $\gamma_{rs}B \subset \{x\}^c$ and $x \notin \gamma_{rs}B$. Thus we have $\gamma_{rs}B \subset B$.
($\Leftarrow$) By assumption $\{x\}$ is a γ_{rs}-set. So $\gamma_{rs}\{x\} = \{x\}$ and there is, for $y \neq x$, a γ-regular semi-open set U such that $x \in U$, $y \notin U$. □

Definition 4.4. *Let X be a set and $\gamma \in \Gamma(X)$. Then X is called a regular semi-R_0^γ space if every γ-regular semi-open subset of X contains the γ-regular semi closure of its singletons.*

Proposition 4.5. *X is a regular semi-R_0^γ space if and only if every γ-regular semi-open subset of X is the union of γ-regular semi-closed sets.*

Proof. ($\Rightarrow$) Suppose that X is a regular semi-R_0^γ space. For any γ-regular semi-open subset A of X, $A = \bigcup_{x \in A} rsc_\gamma\{x\}$.
($\Leftarrow$) Suppose that $A \subset X$ is γ-regular semi-open and $A = \bigcup_{\lambda \in \Omega} B_\lambda$, B_λ is γ-regular semi-closed for every $\lambda \in \Omega$. If $x \in B_\lambda$ then $rsc_\gamma\{x\} \subset B_\lambda$. Thus $rsc_\gamma\{x\} \subset B_\lambda \subset A$. □

Proposition 4.6. *Every regular semi-T_1^γ space is a regular semi-R_0^γ space.*

Proof. Suppose that X is a regular semi-T_1^γ space. By Proposition 4.2, X is a regular semi-R_0^γ space. □

Example 4.7. *In Example 2.4, X is a regular semi-T_1^γ space because every singleton is γ-regular semi-closed. By Proposition 4.6, this set is also a regular semi-R_0^γ-space.*

References

[1] Miguel Caldas Gueva, *On maps and generalized $\bigwedge_s$-sets*, East West J. Math., 2(2000), 181-190.

[2] Miguel Caldas and Julian Dontchev, *G.$\bigwedge_s$-sets and G.$\bigvee_s$-sets*, Mem. Fac. Sci. Kochi Univ., 21(2000), 21-30.

[3] D.E.Cameron, *Properties of s-closed spaces*, Proc. Amer. Math. Soc., 72(1978), 581-586.

[4] À.Csàszàr, *Generalized open sets*, Acta Math. Hungar., 75(1997), 65-87.

[5] À.Csàszàr, *On the γ-interior and γ-closure of a set*, Acta Math. Hungar., 80(1998), 89-93.

[6] À.Csàszàr, *Generalized topology, generalized continuity*, Acta Math. Hungar., 96(2002), 351-357.

[7] À.Csàszàr, *Generalized open sets in generalized topologies*, Acta Math. Hungar., 106(1-2)(2005), 53-66.

[8] S.N.Maheswari and R.Prasad, *Some new separation axioms*, Ann. Soc. Sci. Bruxelles, 89(1975), 395-402.

[9] S.N.Maheswari and R.Prasad, *On R_{0_s}-spaces*, Portugal Math., 34 (1975), 213-217.