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Fuzzy Graph Cellular Spaces

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Abstract: In the present work the initiation of fuzzy cellular graph and fuzzy graph cellular space are introduced. And fuzzy cellular graph and fuzzy $\mathcal{C}$ cellular are compared. Moreover some of the properties of fuzzy cellular graph are studied.

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1. Introduction

In topology, Spaces of continuous functions are amongst the most natural and important objects. And many of the research in function space concerns continuous functions. But in the real world one have to face problems of noncontinuous functions. Stallings tried to fulfill this lack and he initiated the concept of almost continuous functions in [8]. An almost continuous function is one whose graph can be approximated by graphs of continuous functions. During a study of almost continuous functions, S. A. Naimpally developed a new function space topology which he called the graph topology which enable him to make determined efforts to deal with almost continuous function. In [6], Naimpally investigated some further properties in graph topology. K.K Azad [2] introduced the concept of graph of a function in fuzzy topological space. With the aid of these investigation, the idea of fuzzy cellular graph and fuzzy graph cellular space are initiated. In our later works, we introduced fuzzy jointly continuous cellular [1], and fuzzy $\mathcal{C}$ cellular. In the present work the relations between fuzzy cellular graph and fuzzy $\mathcal{C}$ cellular are established. And it is also shown that under which condition the domain will gives the expected results. For instance it is shown that fuzzy cellular graph of $\mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})$ is fuzzy jointly continuous cellular if $\mathfrak{X}$ is fuzzy τ_{cel} regular space. In additionally, several results in this regard were studied.

2. Preliminaries

Definition 2.1 ([10])**.** *Let X be a non-empty set and I be the unit interval [0, 1]. A fuzzy set in X is an element of the set I^X of all functions from X to I.*

Definition 2.2 ([7])**.** *For any $x \in X$ and every $\alpha \in$ (0, 1), the fuzzy set x_α with membership function*

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$$x_\alpha(y) = \begin{cases} \alpha, & y = x; \\ 0, & y \neq x. \end{cases}$$

Definition 2.3 ([3]). *A base for a fuzzy topological space (X, τ) is sub collection $\mathcal{B}$ of τ such that each member A of τ can be written as $A = \vee_{j \in I} A_j$ where each $A_j \in \mathcal{B}$.*

Definition 2.4 ([3]). *A subbase for a fuzzy topological space (X, τ) is a sub collection S of τ such that the collection of infimum of finite subfamilies if S forms a base for (X, τ)*

Definition 2.5 ([7]). *Let $(X_i,\ \delta_i)$ be a fuzzy topological space for each index $i \in I$. The product fuzzy topology $\delta = \Pi_{i \in I}\delta_i$ on the set $X = \Pi_{i \in I}X_i$ is the coarsest fuzzy topology on X making all the projection mappings $\pi_i : X \to X_i$ fuzzy continuous.*

Definition 2.6 ([5]). *A fuzzy topological space (X, τ) is called fuzzy regular if for each pair consisting fuzzy point P_x^α and fuzzy closed set η disjoint from P_x^α there exists fuzzy open sets γ_1 and γ_2 containing P_x^α and η respectively.*

Definition 2.7 ([7]). *A fuzzy topological (X, τ) is called a fuzzy Hausdorff or T_2 space if for any pair of distinct fuzzy points x_t and y_r, there exist fuzzy open sets U and V such that $x_t \in U,\ y_r \in V$ and $U \wedge V$.*

Definition 2.8 ([2]). *Let X and Y be any two fuzzy topological space. For a mapping f: X→Y,the graph g: X→ $X \times Y$ of f is defined by g(x) = (x, f(x)), for each x∈ X.*

Definition 2.9 ([1]). *A fuzzy topological space (X, τ) is called a fuzzy cellular (X, τ_{cel})(in short, $\mathfrak{X}$) if for every family $\Omega = \{\eta_i \in I^X : \eta_i$'s are fuzzy $G_\delta$sets, $i \in I\}$ there exists a countable family $\Gamma = \{\delta_i \in I^X : \delta_i$'s are fuzzy G_δ sets, $i \in I\}$ such that $\Gamma \subseteq \Omega$ and $cl(\vee\delta_i) = \vee\eta_i$, for every $i \in I$.*

Definition 2.10 ([1]). *In a fuzzy cellular space $\mathfrak{X}$, every member of τ_{cel} is called a fuzzy τ_{cel} - open set and its complement is fuzzy τ_{cel} - closed set.*

Definition 2.11 ([1]). *Let $\mathfrak{Y} = (Y, \tau_{1_{cel}})$ and $\mathfrak{Z} = (Z, \tau_{2_{cel}})$ be any two fuzzy cellular spaces. A function F: $\mathfrak{Y} \to \mathfrak{Z}$ is called a fuzzy cellular continuous function iff the inverse image of each fuzzy $\tau_{2_{cel}}$ - open set in $\mathfrak{Z}$ is fuzzy $\tau_{1_{cel}}$ - open in $\mathfrak{Y}$.*

Definition 2.12 ([1]). *Let $\mathfrak{Y}$ and $\mathfrak{Z}$ be any two fuzzy cellular spaces and $\mathfrak{C}(\mathfrak{Y}, \mathfrak{Z})$ be the set of all fuzzy cellular continuous functions from $\mathfrak{Y}$ to $\mathfrak{Z}$, then $(\mathfrak{C}(\mathfrak{Y},\ \mathfrak{Z}),\ \tau_{cel})$ is a fuzzy cellular space and it is denoted by $\mathfrak{C}_{\tau_{cel}}(\mathfrak{Y},\ \mathfrak{Z})$.*

Definition 2.13 ([1]). *Let $\mathcal{S}$ be the family of fuzzy cellular spaces. A fuzzy cellular τ_{cel} in $\mathfrak{C}(\mathfrak{Y},\ \mathfrak{Z})$ is said to be fuzzy $\mathcal{S}$ splitting if and only if for every fuzzy cellular space $\mathfrak{X}$ in $\mathcal{S}$, the fuzzy cellular continuity of the function $F_1 :\ \mathfrak{X}\ \times\ \mathfrak{Y}\ \to\ \mathfrak{Z}$ implies that of the function $F_2 :\ \mathfrak{X}\ \to\ \mathfrak{C}_{\tau_{cel}}(\mathfrak{Y},\ \mathfrak{Z})$.*

Definition 2.14 ([1]). *Let $\mathcal{S}$ be the family of fuzzy cellular spaces. A fuzzy cellular τ_{cel} in $\mathfrak{C}(\mathfrak{Y},\ \mathfrak{Z})$ is said to be fuzzy $\mathcal{S}$ jointly continuous cellular if and only if for every fuzzy cellular space $\mathfrak{X}$ in $\mathcal{S}$, the fuzzy cellular continuity of the function $F_2 :\ \mathfrak{X}\ \to\ \mathfrak{C}_{\tau_{cel}}(\mathfrak{Y},\ \mathfrak{Z})$ implies that of the function $F_1 :\ \mathfrak{X}\ \times\ \mathfrak{Y}\ \to\ \mathfrak{Z}$.*

Proposition 2.15 ([1]). *Let $\mathfrak{Y}$ and $\mathfrak{Z}$ be any two fuzzy cellular spaces. A fuzzy cellular τ_{cel} on $\mathfrak{C}$ $(\mathfrak{Y}, \mathfrak{Z})$ is fuzzy $\mathcal{S}$ jointly continuous if and only if the fuzzy cellular evaluation function e : $\mathfrak{C}_{\tau_{cel}}$ $(\mathfrak{Y}, \mathfrak{Z}) \times \mathfrak{Y} \to \mathfrak{Z}$ defined by e(f, γ) = f(γ) is fuzzy cellular continuous, where γ is a fuzzy set in $\mathfrak{Y}$.*

Definition 2.16 ([9]). *Let $\mathfrak{Y}$ and $\mathfrak{Z}$ be two fuzzy cellular spaces. Let $\mathcal{U} = \{\mu \in I^Y : \mu$ is fuzzy cellular compact in $\mathfrak{Y}\}$ and $\mathcal{V}$= $\{\eta \in I^Z : \eta$ is fuzzy τ_{cel} open in $\mathfrak{Z}\}$. Then fuzzy $\mathcal{C}$ cellular on the class $\mathfrak{C}(\mathfrak{Y}, \mathfrak{Z})$ is the cellular generated by a sub base $\{\mathcal{N}_{\mu,\ \eta} : \mu \in \mathcal{U},\ \eta \in \mathcal{V}\}$ where $\mathcal{N}_{\mu,\ \eta} = \{f \in \mathfrak{C}(\mathfrak{Y}, \mathfrak{Z}) : f(\mu) \leq \eta\}$. The class with this fuzzy cellular is called a fuzzy $\mathcal{C}$ cellular space.*

3. Fuzzy Graph Cellular Spaces

Definition 3.1. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces and $\mathfrak{F}$ denote the set of all functions on $\mathfrak{X}$ to $\mathfrak{Y}$. For $f \in \mathfrak{F}$ the fuzzy cellular graph of f, denoted by $\mathfrak{G}(f)$ is defined by $\{(\mu \times f(\mu)) : \mu \in I^X\} \subset \mathfrak{X} \times \mathfrak{Y}$. And $\mathfrak{F}_\mu = \{ f \in \mathfrak{F} : \mathfrak{G}(f) \leq \mu\}$ for μ in $\mathfrak{X} \times \mathfrak{Y}$.*

Proposition 3.2. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces and let μ and γ in $\mathfrak{X} \times \mathfrak{Y}$, then $\mathfrak{F}_{\mu\wedge\gamma} = \mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$.*

Proof. Assume that $\mathfrak{F}_{\mu\wedge\gamma} = \phi$. If f$\in \mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$, $\mathfrak{G}(f) \leq \mu$ *and* $\mathfrak{G}(f) \leq \gamma$. Then $\mathfrak{G}(f) \leq \mu \wedge \gamma$. This implies that f$\in \mathfrak{F}_{\mu\wedge\gamma}$. This contradicts to the fact that $\mathfrak{F}_{\mu\wedge\gamma} = \phi$. Hence $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma = \phi$ and $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma = \mathfrak{F}_{\mu\wedge\gamma}$.
Suppose $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma = \phi$. Consider f$\in \mathfrak{F}_{\mu\wedge\gamma}$ then $\mathfrak{G}(f) \leq \mu \wedge \gamma$. This implies that $\mathfrak{G}(f) \leq \mu$ and $\mathfrak{G}(f) \leq \gamma$. Therefore f $\in \mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$. This contradicts to the fact that $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma = \phi$. Hence $\mathfrak{F}_{\mu\wedge\gamma} = \phi$ and $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma = \mathfrak{F}_{\mu\wedge\gamma}$.
Assume that neither $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$ *nor* $\mathfrak{F}_{\mu\wedge\gamma}$ is empty. If f$\in \mathfrak{F}_{\mu\wedge\gamma}$, $\mathfrak{G}(f) \leq \mu \wedge \gamma$. This implies that f $\in \mathfrak{F}_\mu$ *and* $f \in \mathfrak{F}_\gamma$. Thus f $\in \mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$. Hence $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma \subset \mathfrak{F}_{\mu\wedge\gamma}$.
Let g $\in \mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$ then $\mathfrak{G}(g) \leq \mu$ and $\mathfrak{G}(g) \leq \gamma$. This implies that $\mathfrak{G}(g) \leq \mu \wedge \gamma$. Thus g $\in \mathfrak{F}_{\mu \wedge \gamma}$. Hence $\mathfrak{F}_{\mu\wedge\gamma} \subset \mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma$.
Therefore, $\mathfrak{F}_\mu \wedge \mathfrak{F}_\gamma = \mathfrak{F}_{\mu\wedge\gamma}$. □

Proposition 3.3. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces and if $\mu = \vee \mu_\alpha \times \gamma_\alpha$ is an fuzzy τ_{cel} - open in $\mathfrak{X} \times \mathfrak{Y}$ and $\mathfrak{F}_\mu \neq \phi$ then $\{ \mu_\alpha : \alpha \in J\}$ is a fuzzy cover of $\mathfrak{X}$.*

Proof. Assume that $\mu = \vee \mu_\alpha \times \gamma_\alpha$ is an fuzzy τ_{cel}- open in $\mathfrak{X} \times \mathfrak{Y}$. If $\{ \mu_\alpha : \alpha \in J\}$ is not a fuzzy cover of $\mathfrak{X}$ then there exist a fuzzy point x in $1_{\mathfrak{X}} - \vee \mu_\alpha$, (x, f(x)) cannot belongs to μ for any f $\in \mathfrak{F}$. This implies that $\mathfrak{F}_\mu$ is empty. But $\mathfrak{F}_\mu$ is not empty. Therefore $\{ \mu_\alpha : \alpha \in J\}$ is a fuzzy cover of $\mathfrak{X}$. □

Proposition 3.4. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces and let μ and γ be any two fuzzy sets in $\mathfrak{X} \times \mathfrak{Y}$. Let $\mathfrak{F}_\mu \neq \phi$, $\mu \leq \gamma$ if and only if $\mathfrak{F}_\mu \leq \mathfrak{F}_\gamma$.*

Proof. Suppose $\mathfrak{F}_\mu \neq \phi$, then there exists a function f $\in \mathfrak{F}$ and $\mathfrak{G}(f) \leq \mu$. This implies that the projection of μ into the coordinate space $\mathfrak{X}$, P(μ) = $1_{\mathfrak{X}}$. Let $\mathfrak{F}_\mu \leq \mathfrak{F}_\gamma$ and (x, y) be a fuzzy point in μ and let f $\in \mathfrak{F}_\mu \leq \mathfrak{F}_\gamma$. This implies that $\mathfrak{G}(f) \leq \mu$ and $\mathfrak{G}(f) \leq \gamma$. Let g $\in \mathfrak{F}$ be defined by

$$g(z) = \begin{cases} f(z), & \text{z} \neq \text{x;} \\ y, & \text{z} = \text{x.} \end{cases}$$

where x, y, and z are fuzzy points. Then for $z \neq x$, (z, g(z)) = (z, f(z)) $\in \mathfrak{G}(f)$. And (x, g(x)) =(x, y) $\in \mu$. Therefore $\mathfrak{G}(g) \leq \mu$ or g $\in \mathfrak{F}_\mu$. Since $\mathfrak{F}_\mu \leq \mathfrak{F}_\gamma$, g $\in \mathfrak{F}_\gamma$. And also $\mathfrak{G}(g) \leq \gamma$ since g $\in \mathfrak{F}_\gamma$. Therefore(x,g(x)) = (x, y) $\in \gamma$. This implies that $\mu \leq \gamma$.
Conversely assume that $\mu \leq \gamma$ and $\mathfrak{F}_\mu \neq \phi$ and f $\in \mathfrak{F}_\mu$. This implies that $\mathfrak{G}(f) \leq \mu$ which implies that $\mathfrak{G}(f) \leq \gamma$. Therefore f $\in \mathfrak{F}_\gamma$. Hence $\mathfrak{F}_\mu \leq \mathfrak{F}_\gamma$. □

Definition 3.5. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces. A fuzzy cellular induced on $\mathfrak{F}$ by a fuzzy base consisting of sets of the form $\{\mathfrak{F}_\mu$ for every fuzzy τ_{cel}- open μ in $\mathfrak{X} \times \mathfrak{Y}\}$ is said to be fuzzy graph cellular Ω for $\mathfrak{F}$. Then the pair $(\mathfrak{F}, \Omega)$ is called fuzzy graph cellular space.*

Definition 3.6. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces. A function f$\in \mathfrak{F}$ is called fuzzy cellular almost continuous function if for each fuzzy τ_{cel}- open μ in $\mathfrak{X} \times \mathfrak{Y}$ containing $\mathfrak{G}(f)$,there exists a function $g \in \mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ such that $\mathfrak{G}(g) \leq \mu$.*

Proposition 3.7. *Let $\mathfrak{X}$ and $\mathfrak{Y}$ be any two fuzzy cellular spaces. Then the family of fuzzy cellular almost continuous functions on $\mathfrak{X}$ to $\mathfrak{Y}$ is in $(\mathfrak{F},\ \Omega)$.*

Proof. By Definitions 3.2, 3.3, the family of fuzzy cellular almost continuous functions on $\mathfrak{X}$ to $\mathfrak{Y}$ is in $(\mathfrak{F},\ \Omega)$. □

Definition 3.8. *A fuzzy cellular space $\mathfrak{X}$ is called fuzzy τ_{cel} regular space if for each fuzzy point x and each fuzzy τ_{cel} closed set γ such that $x \wedge \gamma$=0 there exist fuzzy τ_{cel} open sets μ and η such that $x \in \mu$ and $\gamma \leq \eta$.*

Proposition 3.9. *Let $\mathfrak{X}$ be fuzzy τ_{cel} regular space, for any fuzzy τ_{cel} open set μ with fuzzy point $x \in \mu$, there exists fuzzy τ_{cel} open set ω such that $x \in \omega \leq \tau_{cel}cl(\omega) \leq \mu$.*

Proof. Let $\mathfrak{X}$ be fuzzy τ_{cel} regular space and let μ be fuzzy τ_{cel} open set in $\mathfrak{X}$ with fuzzy point x$\in \mu$. Then μ^c is fuzzy τ_{cel} closed in $\mathfrak{X}$ and x$\notin \mu^c$. Since $\mathfrak{X}$ is fuzzy τ_{cel} regular space, there exist fuzzy τ_{cel} open sets ω *and* η such that x$\in \omega$ and $\mu^c \leq \eta$. Then η^c is a fuzzy τ_{cel} closed in $\mathfrak{X}$ such that $\omega \leq \eta^c \leq \mu$. Thus, x$\in \omega \leq \tau_{cel}cl(\omega)$ and $\tau_{cel}cl(\omega) \leq \eta^c \leq \mu$. This shows that x$\in \omega \leq \tau_{cel}cl(\omega) \leq \mu$. □

Proposition 3.10. *Let $\mathfrak{X}$ be a fuzzy cellular T_2 space and $\mathfrak{Y}$ be a fuzzy cellular space and $\mathfrak{F}$ be the set of all functions from $\mathfrak{X}$ to $\mathfrak{Y}$. Then fuzzy $\mathcal{C}$ cellular contained in the fuzzy graph cellular on $\mathfrak{F}$.*

Proof. Assume that $\mathfrak{X}$ be a fuzzy cellular T_2 space. Let $\mathcal{N}_{\mu,\ \eta} = \{\ f\ \in\ \mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})\ :\ f(\mu) \leq\ \eta,\ \mu\, is\ fuzzy\ cellular\ compact\}$ be subbase for fuzzy $\mathcal{C}$ on $\mathfrak{X}$. The fuzzy set $\gamma = (1_{\mathfrak{X}} \times\ \eta) \vee\ ((1_{\mathfrak{X}} - \mu) \times\ 1_{\mathfrak{Y}})$ is fuzzy τ_{cel} open in $\mathfrak{X}\ \times\ \mathfrak{Y}$. For f $\in\ \mathcal{N}_{\mu,\ \eta}$, then f$(\mu) \leq \eta$. This implies that $\mathfrak{G}(f) \leq\ \gamma$ and f $\in\ \mathfrak{F}_{\gamma}$. Therefore $\in\ \mathcal{N}_{\mu,\ \eta} \subset\ \mathfrak{F}_{\gamma}$. For f $\in\ \mathfrak{F}_{\gamma}$ and $\mathfrak{G}(f) \leq\ \gamma$,this implies that f $(\mu) \subset\ \eta$ and f $\in\ \mathfrak{N}_{\mu,\ \eta}$. Thus $\mathcal{N}_{\mu,\ \eta} = \mathfrak{F}_{\gamma}$. Hence, $\mathcal{N}_{\mu,\ \eta}$ in fuzzy graph cellular. Therefore fuzzy $\mathcal{C}$ cellular contained in the fuzzy graph cellular on $\mathfrak{F}$. □

Proposition 3.11. *Let $\mathfrak{X}$ be a fuzzy cellular T_2, compact space and $\mathfrak{Y}$ be a fuzzy cellular space and $\mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})$ be the set of all fuzzy cellular continuous functions from $\mathfrak{X}$ to $\mathfrak{Y}$. Then fuzzy $\mathcal{C}$ cellular equivalent to the fuzzy graph cellular on $\mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})$.*

Proof. Let $\mathfrak{X}$ be a fuzzy cellular T_2 space and $\mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})$ be the set of all fuzzy cellular continuous functions from $\mathfrak{X}$ to $\mathfrak{Y}$. Assume that $\mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})_{\mu}$ is τ_{cel} open set in $(\mathfrak{C}(\mathfrak{X},\mathfrak{Y}),\ \Omega)$, here $\mu\ =\ \vee\ \mu_{\alpha}\ \times\ \gamma_{\alpha}$ for $\alpha\ \in\ J$. Let f $\in\ \mathfrak{C}(\mathfrak{X},\ \mathfrak{Y})_{\mu}$. Then $\mathfrak{G}(f) \leq\ \mu$. This implies that for each fuzzy point x there exists $\alpha_x \in\ J$ such that (x, f(x)) $\in\ \mu_{\alpha_x} \times \gamma_{\alpha_x}$. By using the fact that f is fuzzy cellular continuous, there exists an fuzzy τ_{cel} open set ω in $\mathfrak{X}$ such that the fuzzy point x $\in\ \omega$ and f$(\omega) \leq\ \gamma_{\alpha_x}$ for each x. The fuzzy cellular space $\mathfrak{X}$ is fuzzy cellular regular space since $\mathfrak{X}$ is fuzzy cellular compact and T_0 space. Therefore there is a fuzzy τ_{cel} open set $\tilde{\mu}_{\alpha_x}$ with x $\in \tilde{\mu}_{\alpha_x} \leq \tau_{cel}cl(\tilde{\mu}_{\alpha_x}) \leq\ \omega_x \wedge \mu_{\alpha_x}$ for each x. Therefore f(x) $\in\ f(\tilde{\mu}_{\alpha_x}) \leq\ f(\tau_{cel}cl(\tilde{\mu}_{\alpha_x})) \leq\ f(\omega_x\ \wedge\ \mu_{\alpha_x}) \leq\ f(\omega_x) \leq\ \gamma_{\alpha_x}$. Since $\mathfrak{X}$ is fuzzy cellular compact and $\{\mu_{\tilde{\alpha_x}}\}$ is a fuzzy τ_{cel} open cover of $\mathfrak{X}$, there is a finite fuzzy τ_{cel} open subcover $\{\mu_{\tilde{\alpha_{x_i}}},\ for\ i = 1,\ 2, ..., n\}$. Then f$(\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}}) \leq\ \gamma_{\alpha_{x_i}}$. Since $\mathfrak{X}$ is fuzzy cellular compact and $\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}})$ is fuzzy τ_{cel} closed in $\mathfrak{X}$, $\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}})$ is fuzzy cellular compact. Therefore f $\in\ \wedge_{i=1}^{n} \mathcal{N}_{\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}}),\ \gamma_{\alpha_{x_i}}}$ is τ_{cel} open set in the fuzzy $\mathcal{C}$ cellular on $\mathfrak{F}$.

Let g $\in\ \wedge_{i=1}^{n} \mathcal{N}_{\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}}),\ \gamma_{\alpha_{x_i}}}$ then g$(\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}})) \leq \gamma_{\alpha_{x_i}}$ and g$(\mu_{\alpha_{x_i}}) \leq \gamma_{\alpha_{x_i}}$. Since $\{\mu_{\tilde{\alpha_{x_i}}} : i = 1,\ 2, ..., n\}$ is a fuzzy τ_{cel} cover of $\mathfrak{X}$, $\mathfrak{G}(g) \leq\ \vee\, \mu_{\tilde{\alpha_{x_i}}} \times \gamma_{\alpha_{x_i}} \leq \mu$ and g $\in\ \mathfrak{C}(\mathfrak{X},\mathfrak{Y})_{\mu}$. This implies that $\wedge_{i=1}^{n}\mathcal{N}_{\tau_{cel}cl(\mu_{\tilde{\alpha_{x_i}}}),\gamma_{\alpha_{x_i}}}\ \subset\ \mathfrak{C}(\mathfrak{X},\mathfrak{Y})_{\mu}$. □

Proposition 3.12. *Let $\mathfrak{X}$ is fuzzy τ_{cel} regular space and $\mathfrak{Y}$ fuzzy cellular space. If $\mathfrak{C}$ is set of all fuzzy cellular continuous functions from $\mathfrak{X}$ to $\mathfrak{Y}$ then the fuzzy graph cellular Ω on $\mathfrak{C}$ is fuzzy cellular jointly continuous.*

Proof. Let f $\in\ \mathfrak{C}$ and x be a fuzzy point in $\mathfrak{X}$.Then (f, x) $\in\ \mathfrak{C} \times \mathfrak{X}$. Since f is fuzzy cellular continuous, there is a fuzzy τ_{cel} open set μ with x $\in \mu$ such that f$(\mu) \leq \gamma$ where γ is fuzzy τ_{cel} open set with f(x) $\in \gamma$.

By using the fact that $\mathfrak{X}$ is fuzzy τ_{cel} regular space, there is a fuzzy τ_{cel} open ω with x$\in \omega \leq \tau_{cel}cl(\omega) \leq \mu$. This implies that f(x)∈ $f(\omega) \leq f(\tau_{cel}cl(\omega)) \leq f(\omega) \leq \gamma$. $1_{\mathfrak{X}} - \tau_{cel}cl(\omega)$ fuzzy τ_{cel} open, since $\tau_{cel}cl(\omega)$ is fuzzy τ_{cel} closed in $\mathfrak{X}$. Let $\lambda = 1_{\mathfrak{X}-\tau_{cel}cl(\omega)} \times 1_{\mathfrak{Y}} \vee 1_{\mathfrak{X}} \times \gamma$ then $\mathfrak{G}(f) \leq \lambda$. This implies that f $\in \mathfrak{C}_\lambda$ and $\mathfrak{C}_\lambda$ is fuzzy τ_{cel} open in fuzzy graph cellular Ω on $\mathfrak{C}$ and (f, x)∈ $\mathfrak{C}_\lambda \times \omega$ where $\mathfrak{C}_\lambda \times \omega$ is fuzzy τ_{cel} open in $(\mathfrak{C}, \Omega) \times \mathfrak{X}$. Let (g, y) be any point of $\mathfrak{C}_\lambda \times \omega$ then $\mathfrak{G}(g) \leq \lambda$ and $y \in \lambda$. And (y, g(y))∈ $1_{\mathfrak{X}} \times \gamma$, Since $\mathfrak{G}(g) \leq \lambda$ and y∈ $\omega \leq \tau_{cel}cl(\omega)$. This implies that e(g,y)=g(y)∈ γ. Thus for every point (g, y)∈ $\mathfrak{C}_\lambda \times \omega$, e(g, y)=g(y) $\in \gamma$. This implies that e is fuzzy cellular continuous at (f, x) with respect to the fuzzy graph cellular on $\mathfrak{C}$. Hence the fuzzy graph cellular Ω on $\mathfrak{C}$ is fuzzy cellular jointly continuous. □

Proposition 3.13. *Let $\mathfrak{X}$, $\mathfrak{Y}$ and $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$ be fuzzy cellular spaces. If fuzzy cellular τ_{cel} in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ is fuzzy cellular jointly continuous then fuzzy $\mathcal{C}$ cellular contained in fuzzy cellular in fuzzy cellular space $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$.*

Proof. Let $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$ be fuzzy cellular space and $\mathcal{N}_{\mu,\ \eta}$ be a subbase for the fuzzy $\mathcal{C}$ cellular in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$. Let f $\in \mathcal{N}_{\mu,\ \eta}$ then f(μ) $\leq \eta$. It shows that e(f, x) = f(x) $\in \eta$ for each $x \in \mu$. This implies that (f, x) $\in e^{-1}(\eta)$ for each $x \in \mu$. It is given that τ_{cel} in $\mathfrak{C}(\mathfrak{X},\mathfrak{Y})$ is fuzzy cellular jointly continuous. By the proposition 2.1 the evalution map is fuzzy cellular continuous. Therefore $e^{-1}(\eta)$ is fuzzy τ_{cel} open in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y}) \times \mathfrak{X}$. Thus (f, x) $\in \lambda \times \gamma \leq e^{-1}(\eta)$ where λ is τ_{cel} open in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ contains the fuzzy point x and γ *is fuzzy* τ_{cel} *open in* $\mathfrak{X}$. Since μ is fuzzy cellular compact in $\mathfrak{X}$, the fuzzy τ_{cel} open cover $\{\gamma_i : where\ \gamma_i\ is\ fuzzy\ \tau_{cel} open\ in\ \mathfrak{X},\ i \in\ I\}$ has sub cover $\{\gamma_{i_j} : where\ \gamma_{i_j}\ is\ fuzzy\ \tau_{cel} open\ in\ \mathfrak{X},\ j = 1, 2, ..., n\}$. Assume that $\omega = \wedge_{i=1}^{n} \lambda_i$ where λ_i is fuzzy τ_{cel} open in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$. That is ω is finite intersection of fuzzy τ_{cel} open sets in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$. Therefore ω is fuzzy τ_{cel} open in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ and f $\in \omega$, since f $\in \lambda_i$ for each i. Let g $\in \omega$ and x∈ μ,then x∈ γ_{i_j} for some j. Since g $\in \lambda_i$, (g,x)∈ $\lambda_i \times \gamma_{i_j} \leq e^{-1}(\eta)$. This implies that e(g, x) = g(x) $\in \eta$ for fuzzy point x in μ. Therefore g(μ) $\leq \eta$. This implies that, g∈ $\mathcal{N}_{\mu,\eta}$. Since g is arbitrary, f $\in \omega \leq \mathcal{N}_{\mu,\eta}$. This shows that $\mathcal{N}_{\mu,\eta}$ is fuzzy τ_{cel} open in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$. Hence fuzzy $\mathcal{C}$ cellular contained in fuzzy cellular in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$. □

Proposition 3.14. *Let $\mathfrak{X}$ be fuzzy cellular regular space, $\mathfrak{Y}$ be fuzzy cellular space and $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ be set of all continuous functions from $\mathfrak{X}$ to $\mathfrak{Y}$. Then fuzzy $\mathcal{C}$ cellular on $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ contained in fuzzy graph cellular Ω in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$.*

Proof. Let $\mathfrak{X}$ be fuzzy cellular regular space, $\mathfrak{Y}$ be fuzzy cellular space and $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ be set of all continuous functions from $\mathfrak{X}$ to $\mathfrak{Y}$. By Proposition 3.8 fuzzy graph cellular Ω in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$ is fuzzy jointly continuous. And by Proposition 3.9 fuzzy $\mathcal{C}$ cellular on $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ contained in fuzzy cellular in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$. This implies that fuzzy $\mathcal{C}$ cellular on $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ contained in fuzzy graph cellular Ω in $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$. □

4. Conclusion

From the above discussion, one can say that if $\mathfrak{F}_\mu$ defined by $\mathfrak{F}_\mu = \{ f \in \mathfrak{F} : \mathfrak{G}(f) \leq \mu\}$ for μ in $\mathfrak{X} \times \mathfrak{Y}$, then $\{\mu_\alpha\}$ covers $\mathfrak{X}$ where $\mu = \vee \mu_\alpha \times \gamma$. Fuzzy graph cellular Ω on $\mathfrak{C}_{\tau_{cel}}(\mathfrak{X}, \mathfrak{Y})$ is not a fuzzy jointly continuous cellular. This true only if $\mathfrak{X}$ be fuzzy cellular regular space. Fuzzy $\mathcal{C}$ cellular equivalent to fuzzy graph cellular in $\mathfrak{C}(\mathfrak{X}, \mathfrak{Y})$ only if $\mathfrak{X}$ is both fuzzy cellular T_2 and fuzzy cellular compact space.

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