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A New Generalized Continuity by Using Generalized-Closure Operator

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Abstract: We introduce a new class of generalized closed sets namely, ${}_N D_\beta$–closed sets, which is weaker than g–closed(generalized–closed) [11], *semi**–closed sets [23], *pre**–closed sets [26] and D_α–closed sets [25] in topological spaces. Moreover, we introduce ${}_N D_\beta$–continuous and ${}_N D_\beta$–irresolute functions and study its fundamental properties.

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Keywords: ${}_N D_\beta$–closed, ${}_N D_\beta$–neighborhood, ${}_N D_\beta$–limit point, ${}_N D_\beta$–continuous, ${}_N D_\beta$–irresolute.

1. Introduction

Monsef et al.[1] introduced a new class of generalized open sets in topological spaces called β–open sets. Andrijevic (See also [2]) also defined the notion of β–open(semi–preopen) sets. The class of β–open sets contains the class of α–open sets [21], preopen sets [16] and semiopen sets [10]. The concepts of generalized–closed (g–closed) sets introduced by Levine [11] plays a significant role in topology. This notion has been studied extensively in recent years by many topologists. Dunham [6], [7] defined the new closure operator C^* with the help of g–closed sets in such a way that for any topological space (X,τ), $C^*(E) = \bigcap \{A : E \subset A \in D\}$, where $D = \{A : A \subseteq X, \text{A is g–closed}\}$. Munshi and Bassan [19] introduced the notion of g–continuous functions. The notion of g–continuity is also studied in [4] and [5]. Maheshwari et al.[12] defined and investigated the α–irresolute (resp. Mahmoud et. al [13] defined and studied β–irresolute and Maki et al. [14] introduced g–irresolute)functions. By using the closure operator C^* due to Dunham [7], Robert et al. [23] propounded and investigated a new notion of generalized closed set namely *semi**–closed sets, Missier [17] originated and studied the notion of α^*–open sets and α^*–closed sets and Selvi et al. [26] defined and investigated *pre**–closed sets. In 2016 Sayed et al. [25] introduced and investigated another generalized closed sets namely D_α–closed sets in topological spaces by using the generalized closure operator C^*. The aim of this paper is to continue the study of generalized closed sets. In particular, in section 2 we define a new notion of generalized closed sets namely, ${}_N D_\beta$–closed sets and discus its various characterizations and basic properties and its relationships with already existing some generalized closed sets. In section 3, we define ${}_N D_\beta$–open sets. In section 4, we define ${}_N D_\beta$–continuous, D_α–irresolute functions and ${}_N D_\beta$–irresolute functions and investigate their fundamental properties.

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2. Preliminaries

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A respectively. We recall some generalized open sets and generalized continuities in a topological space.

Definition 2.1. *Let (X, τ) be a topological space. A subset A of the space X is said to be,*

(1). preopen [21] if $A \subseteq Int(Cl(A))$ and preclosed if $Cl(Int(A)) \subseteq A$.

(2). semi–open[10] if $A \subseteq Cl(Int(A))$ and semi–closed if $Int(Cl(A)) \subseteq A$.

(3). α–open [21] if $A \subseteq Int(Cl(Int(A))$ and α–closed if $Cl(Int(Cl(A)) \subseteq A$.

(4). β–open [1] if $A \subseteq Cl(Int(Cl(A)))$ and β–closed if $Int(Cl(Int(A))) \subseteq A$.

(5). generalized closed (briefly g–closed)[11] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.generalized open(briefly g–open) if $X \setminus A$ is g–closed.

(6). pre*–closed set [26] if $Cl^*(Int(A)) \subseteq A$ and pre*–open set if $A \subseteq Int^*(Cl(A))$.

(7). semi*–closed set [23] if $Int^*(Cl(A)) \subseteq A$ and semi*–open set [17] if $A \subseteq Cl^*(Int(A))$.

(8). D_α–closed [25] if $Cl^*(Int(Cl^*(A))) \subseteq A$ and D_α–open if $X \setminus A$ is D_α–closed.

Definition 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be,*

(1). α–continuous [16](resp.β–continuous [1]) if the inverse image of each open set in Y is α–open(resp.β–open) in X.

(2). g–continuous [4] if the inverse image of each open set in Y is g–open in X.

(3). semi*–continuous [18] if the inverse image of each open set in Y is semi*–open in X.

(4). D_α–continuous [25] if the inverse image of each open set in Y is D_α–open in X.

The intersection of all g–closed sets containing A [7] is called the g–closure of A and denoted by $Cl^*(A)$ and the g–interior of A [23] is the union of all g–open sets contained in A and is denoted by $Int^*(A)$. The family of all $_ND_\beta$–closed (resp. D_α–closed, g–closed) sets of X denoted by $_ND_\beta$C(X) (resp.D_αC(X), GC(X)). The family of all $_ND_\beta$–open (resp. D_α–open, g–open) sets of X denoted by $_ND_\beta$O(X) (resp.D_αO(X), GO(X), βO(X)). $D_\alpha O(X, x) = \{$U: $U \in \alpha O(X, \tau)\}$, $D_\alpha C(X, x) = \{$U: $U \in \alpha C(X, \tau)\}$.

Lemma 2.3 ([8])**.** *Let $A \subseteq X$, then*

(1). $X \setminus Cl(X \setminus A) = Int(A)$.

(2). $X \setminus Int(X \setminus A) = Cl(A)$.

Lemma 2.4 ([7])**.** *Let $A \subset X$, then*

(1). $X \setminus Cl^*(A) = Int^*(X \setminus A)$.

(2). $X \setminus Int^*(A) = Cl^*(X \setminus A)$.

Lemma 2.5 ([25])**.** *Let $A \subseteq X$, then*

(1). $X \setminus Cl^*(X \setminus A) = Int^*(A)$.

(2). $X \setminus Int^*(X \setminus A) = Cl^*(A)$.

3. ${}_ND_\beta$–Closed Set

In this section we introduce ${}_ND_\beta$–closed sets and investigate some of their basic properties.

Definition 3.1. *A subset A of a topological space (X, τ) is called* ${}_ND_\beta$*–closed if* $Int(Cl^*(Int(A))) \subseteq A$.

Example 3.2. *Let* $X = \{a,b,c,d\}$ *be any set and* $\tau = \{X, \phi, \{a,c,d\}, \{a,c\}\}$*, then* (X,τ) *be a topological space.*

$$C(X) = \{\phi, X, \{b\}, \{b,d\}\},$$

$$GC(X) = \{\phi, X, \{b\}, \{b,d\}, \{a,b,d\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\},$$

$$GO(X) = \{X, \phi, \{a,c,d\}, \{a,c\}, \{c\}, \{c,d\}, \{a,d\}, \{d\}, \{a\}\},$$

$$D_\alpha C(X) = \{X, \phi, \{b\}, \{b,d\}, \{a,b,d\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,d\}, \{c,d\}, \{c\}, \{d\}, \{a\}\}$$

$${}_ND_\beta C(X) = \{X, \phi, \{b\}, \{b,d\}, \{a,b,d\}, \{a,b\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,d\}, \{c,d\}, \{c\}, \{d\}, \{a\}, \{a,c\}\}.$$

Theorem 3.3. *Let (X, τ) be a topological space, then*

(1). Every β–closed subset of (X, τ)is ${}_ND_\beta$*–closed.*

(2). Every g–closed subset of (X, τ) is ${}_ND_\beta$*–closed.*

(3). Every semi–closed subset of (X, τ) is* ${}_ND_\beta$*–closed.*

(4). Every pre–closed subset of (X, τ) is* ${}_ND_\beta$*–closed.*

(v) Every D_α*–closed subset of (X, τ)is* ${}_ND_\beta$*–closed.*

Proof.

(1). Let A be any β–closed subset of the space X, then we have $Int(Cl(Int(A))) \subseteq A$. We know that $Cl^*(A) \subseteq Cl(A)$, then we have $Cl^*(Int(A) \subseteq Cl(Int(A))$. Therefore $Int(Cl^*(Int(A))) \subseteq Int(Cl(Int(A))) \subseteq A$.

(2). Let A be any g–closed subset of the space X, then we have $Cl^*(A) = A$. Since $Int(A) \subseteq A$, then we have $Cl^*(Int(A)) \subseteq Cl^*(A) = A \Rightarrow Int(Cl^*(Int(A))) \subseteq Int(A) \subseteq A$. Hence $Int(Cl^*(Int(A))) \subseteq A$ i.e. A is ${}_ND_\beta$–closed.

(3). Let A be any *semi**–closed subset of the space X, then we have $Int^*(Cl(A)) \subseteq A$. Since $Int(A) \subseteq A$, therefore we have $Cl^*(Int(A)) \subseteq Cl^*(A) \subseteq Cl(A)$. Thus we have $Int(Cl^*(Int(A))) \subseteq Int(Cl(A) \subseteq Int^*(Cl(A)) \subseteq A$.

(4). Let A be any *pre**–closed subset of the space X, then we have $Cl^*(Int(A)) \subseteq A$. Let $Cl^*(Int(A)) \subseteq A$, thus we have $Int(Cl^*(Int(A))) \subseteq Int(A) \subseteq A$.

(5). Let A be any D_α–closed subset of $(X, Cl^*(Int(Cl^*(A))) \subseteq A$. Since $A \subseteq Cl^*(A)$, $Int(A) \subseteq Int(Cl^*(A)) \Rightarrow Cl^*(Int(A)) \subseteq Cl^*(Int(Cl^*(A))) \subseteq A$. Thus we get $Int(Cl^*(Int(A))) \subseteq Int(A) \subseteq A$. □

Remark 3.4. *The converse of Theorem 3.3 is not true as shown in the following example.*

(1). ${}_ND_\beta$*–closed set need not be β–closed.*

(2). ${}_ND_\beta$*–closed set need not be g–closed.*

(3). ${}_ND_\beta$*–closed set need not be semi*–closed.*

(4). ${}_ND_\beta$–closed set need not be pre*–closed.

(5). ${}_ND_\beta$–closed set need not be D_α–closed.

Example 3.5. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}\}$. Then (X, τ) be a topological space.*

$$
\begin{aligned}
C(X) &= \{\phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}\},\\
GC(X) &= \{\phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\},\\
GO(X) &= \{X, \phi, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}\ \{c, d\}, \{d\}, \{c\}\},\\
\beta C(X) &= \{\phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}, \{c\}, \{d\}, \{c, d\}, \},\\
\beta O(X) &= \{X, \phi, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, b\}\},\\
semi^*C(X) &= \{\phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}, \{c\}, \{d\}, \{c, d\}\},\\
semi^*O(X) &= \{X, \phi, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}, \{a, b, d\}, \{a, b, c\}, \{a, b\}\},\\
pre^*C(X) &= \{\phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}, \{c, d\}, \{c\}, \{d\}, \{a, b, c\}, \{a, b, d\}\},\\
pre^*O(X) &= \{X, \phi, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}, \{a, b\}, \{a, b, d\}, \{a, b, c\}, \{d\}, \{c\}\}\\
D_\alpha C(X) &= \{phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{c, d\}, \{c\}, \{d\}\}\\
D_\alpha O(X) &= \{X, \phi, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}, \{c, d\}, \{d\}, \{c\}, \{a, b\}, \{a, b, d\}, \{a, b, c\}\},\\
{}_ND_\beta C(X) &= \{phi, X, \{a, d\}, \{a, c\}, \{a, c, d\}, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \{c, d\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{b\}\}.\\
{}_ND_\beta O(X) &= \{phi, X, \{b, c\}, \{b, d\}, \{b\}, \{b, c, d\}, \{c, d\}, \{d\}, \{c\}, \{a, b\}, \{a, b, d\}, \{a, b, c\}, \{a, d\}, \{a, c\}, \{a, c, d\}\}.
\end{aligned}
$$

Let $A = \{b, d\}$ is ${}_ND_\beta$–closed in X but it is not a β–closed, not a g–closed, not a pre*–closed and neither semi*–closed nor a D_α–closed.

Theorem 3.6. *Arbitrary intersection of ${}_ND_\beta$–closed sets is ${}_ND_\beta$–closed.*

Proof. Let $\{G_\alpha : \alpha \in \triangle\}$ be a collection of ${}_ND_\beta$–closed sets in X. Then $Int(Cl^*(Int(G_\alpha))) \subseteq G_\alpha$ for each $\alpha \in \triangle$. Since $\bigcap_\alpha G_\alpha \subseteq G_\alpha$ for each $\alpha \in \triangle$, $Int(\bigcap_\alpha G_\alpha) \subseteq Int(G_\alpha)$ for each α. Therefore $Int(\bigcap_\alpha G_\alpha) \subseteq \bigcap_\alpha(Int(G_\alpha)$, $\alpha \in \triangle$. Hence

$$
\begin{aligned}
Int(Cl^*(Int(\bigcap_\alpha G_\alpha))) &\subseteq Int(Cl^*(\bigcap_\alpha Int(G_\alpha)\\
&\subseteq Int(\bigcap_\alpha(Cl^*(Int(G_\alpha)))\\
&\subseteq \bigcap_\alpha(Int(Cl^*(Int(G_\alpha)))\\
&\subseteq \bigcap_\alpha G_\alpha.
\end{aligned}
$$

Therefore $\bigcap_\alpha G_\alpha$ is ${}_ND_\beta$–closed. □

Remark 3.7. *The union of two ${}_ND_\beta$–closed sets need not be ${}_ND_\beta$–closed.*

Example 3.8. *In the Example 3.2, the sets $\{a, b\}$ and $\{b, d\}$ both are ${}_ND_\beta$–closed but their union $\{a, c\} \cup \{c, d\} = \{a, c, d, \}$ is not ${}_ND_\beta$–closed.*

Remark 3.9. *The collection of ${}_ND_\beta C(X)$ does not form a topology.*

Corollary 3.10. *Let A and B are any two subsets of the space X, where A is ${}_N D_\beta$–closed and B is β–closed (resp. g–closed,D_α–closed, pre*–closed,semi*–closed) then $A \cap B$ is ${}_N D_\beta$–closed.*

Proof. It follows directly from the Theorems 3.3 and 3.6. □

Definition 3.11. *Let A be any subset of a space X. The ${}_N D_\beta$–closure of A is the intersection of all ${}_N D_\beta$–closed sets in X containing A i.e. ${}_N D_\beta$–$Cl(A) = \bigcap \{G : A \subseteq G$ and $G \in D_\beta C(X)\}$. It is denoted by ${}_N D_\beta$–$Cl(A)$.*

Theorem 3.12. *Let A be a subset of X. Then A is ${}_N D_\beta$–closed set in X if and only if ${}_N D_\beta$–$Cl(A) = A$.*

Proof. Suppose A is D_β–closed set in X. Since ${}_N D_\beta$–$Cl(A)$ is equal to the intersection of all ${}_N D_\beta$–closed sets in X containing A. Since $A \subseteq {}_N D_\beta$–$Cl(A)$, therefore ${}_N D_\beta$–$Cl(A) = A$. Let ${}_N D_\beta$–$Cl(A) = A$. Then A is D_β–closed set in X. □

Theorem 3.13. *Let (X, τ) be a topological space and suppose A and B be any two subsets of X. Then the following results hold.*

(1). $A \subseteq {}_N D_\beta$–$Cl(A) \subseteq \beta$–$Cl(A)$, ${}_N D_\beta$–$Cl(A) \subseteq Cl^*(A)$, ${}_N D_\beta$–$Cl(A) \subseteq Cl_{D_\alpha}(A)$.

(2). ${}_N D_\beta$–$Cl(A) = \phi$ and ${}_N D_\beta$–$Cl(A) = X$.

(3). If $A \subseteq B$, then ${}_N D_\beta$–$Cl(A) \subseteq {}_N D_\beta$–$Cl(B)$.

(4). ${}_N D_\beta$–$Cl({}_N D_\beta$–$Cl(A)) = {}_N D_\beta$–$Cl(A)$.

(5). ${}_N D_\beta$–$Cl(A) \cup {}_N D_\beta$–$Cl(B) \subseteq {}_N D_\beta$–$Cl(A \cup B)$.

(6). ${}_N D_\beta$–$Cl(A \cap B) \subseteq {}_N D_\beta$–$Cl(A) \cap {}_N D_\beta$–$Cl(A)$.

Proof.

(1). It follows directly from the Theorem 3.3.

(2). It is trivially true.

(3). It is trivially true.

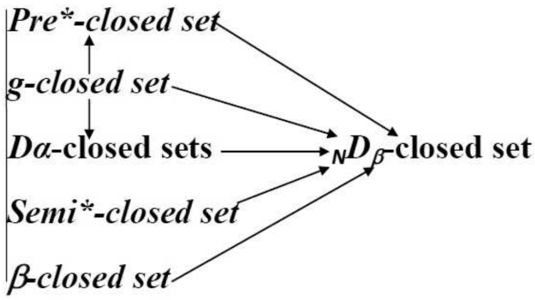
(4). Since ${}_N D_\beta$–$Cl(A)$ is the arbitrary intersection of those ${}_N D_\beta$–closed subsets of X which contain A, therefore by Theorem 3.6 ${}_N D_\beta$–$Cl(A)$ is ${}_N D_\beta$–closed in X. By Theorem 3.12, we have ${}_N D_\beta$–Cl $({}_N D_\beta$–$Cl(A)) = {}_N D_\beta$–$Cl(A)$.

(5). Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By (iii) ${}_N D_\beta$–$Cl(A) \subseteq {}_N D_\beta$–$Cl(A \cup B)$ and ${}_N D_\beta$–$Cl(B) \subseteq {}_N D_\beta$–$Cl(A \cup B)$. Therefore ${}_N D_\beta$–$Cl(A) \cup {}_N D_\beta$–$Cl(B) \subseteq {}_N D_\beta$–$Cl(A \cup B)$.

(6). Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ therefore from (iii) ${}_N D_\beta$–$Cl(A \cap B) \subseteq {}_N D_\beta$–$Cl(A)$ and ${}_N D_\beta$–$Cl(A \cap B) \subseteq {}_N D_\beta$–$Cl(B)$, it follows that ${}_N D_\beta$–$Cl(A \cap B) \subseteq {}_N D_\beta$–$Cl(A) \cap {}_N D_\beta$–$Cl(B)$. □

3.1. Interrelationship

The following diagram will describe the interrelations among D_β–closed set and other existing generalized–closed sets. None of these implications is reversible as shown by examples given below and well known facts.



4. $_ND_\beta$–open Sets

In this section we introduce D_β–open sets and investigate some of their basic properties.

Definition 4.1. *Let (X, τ) be a topological space. A subset A of a space X is called D_β–open if $X \setminus A$ is $_ND_\beta$–closed. Let $_ND_\beta O(X)$ denotes the collection of all an $_ND_\beta$–open sets in X.*

Theorem 4.2. *A subset A of a space X is $_ND_\beta$–open if and only if $A \subseteq Cl(Int^*(Cl(A)))$.*

Proof. Let A be any $_ND_\beta$–open set. Then $X \setminus A$ is $_ND_\beta$–closed and $Int(Cl^*(Int(X \setminus A))) \subseteq X \setminus A$. By Lemma 2.3 and Lemma 2.5, $A \subseteq (X \setminus Int(Cl^*(Int(A)))) = Cl(Int^*(Cl(A)))$. Conversely, suppose $A \subseteq Cl(Int^*(Cl(A)))$. On taking complement of both sides, $X \setminus (Cl(Int^*(Cl(A)))) \subseteq X \setminus A$. It follows that $X \setminus A$ is $_ND_\beta$–closed i.e. A is $_ND_\beta$–open. □

Theorem 4.3. *Let (X, τ) be a topological space. Then*

(1). Every β–open subset of (X, τ) is $_ND_\beta$–open.

(2). Every g–open subset of (X, τ) is $_ND_\beta$–open.

(3). Every semi*–open subset of (X, τ) is $_ND_\beta$–open.

(4). Every pre*–open subset of (X, τ) is $_ND_\beta$–open.

(5). Every D_α–open subset of (X, τ) is D_β–open.

Proof. It is directly follows from the Theorem 3.3. □

Remark 4.4. *The converse of the above theorem is not true.*

Example 4.5. *Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}\}$. Then (X, τ)be a topological space.*

$$C(X) = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}\},$$
$$GC(X) = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}, \{b, c\}, \{a, b, c\}\},$$
$$GO(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, d\}, \{d\}\},$$
$$\beta C(X) = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}, \{a\}, \{b, d\}, \{d\}, \{a, d\}\},$$
$$\beta O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c\}, \{a, b, c\}, \{b, c\}\},$$
$$semi^*C(X) = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}, \{a, d\}, \{d\}, \{a\}, \{b, d\}\},$$
$$semi^*O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{a, c\}\},$$
$$pre^*C(X) = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}, \{b, c\}, \{d\}, \{a, b, c\}\},$$

$$pre^{*}O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, d\}, \{d\}, \{a, b, c\}\}$$

$$D_{\alpha}C(X) = \{phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}, \{b, c\}, \{a, b, c\}, \{d\}\}$$

$$D_{\alpha}O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, d\}, \{d\}, \{a, b, c\}\},$$

$$_{N}D_{\beta}C(X) = \{phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}, \{c\}, \{b, c\}, \{a, b, c\}, \{d\}, \{a\}, \{b\}, \{a, b\}, \{b, d\}\}.$$

$$_{N}D_{\beta}O(X) = \{phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, d\}, \{d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, c\}\}.$$

Let $A = \{a, c, d\}$ is $_{N}D_{\beta}$–open in X but it is not a β–open, not a g–open, not a pre*–open and neither semi*–open nor a D_{α}–open in X.

Theorem 4.6. *Arbitrary union of $_{N}D_{\beta}$–open set is $_{N}D_{\beta}$–open.*

Proof. It follows from the Theorem 3.6. □

Remark 4.7. *The intersection of two $_{N}D_{\beta}$–open sets need not be $_{N}D_{\beta}$–open as seen from Example 4.5, in which two $_{N}D_{\beta}$–open sets are $A = \{a, c\}$ and $B = \{c, d\}$ but their intersection $A \cap B = \{c\}$ is not $_{N}D_{\beta}$–open set.*

Corollary 4.8. *Let A and B be any two subsets of the space (X, τ). If A is $_{N}D_{\beta}$–open and B is β–open(resp. g–open, D_{α}–open, pre*–open, semi*–open) then $A \cup B$ is $_{N}D_{\beta}$–open.*

Proof. It follows from the Theorems 4.3 and 4.6. □

Definition 4.9. *Let A be any subset of a space X. The $_{N}D_{\beta}$– interior of A is the union of all the $_{N}D_{\beta}$–open sets in X, contained in A i.e. $_{N}D_{\beta}$–interior of $A = \bigcup\{U : U \subset A,\ U \in D_{\beta}O(X)\}$. It is denoted by $_{N}D_{\beta}$–$Int(A)$*

Lemma 4.10. *If A be any subset of X, then*

(1). $X \setminus {}_{N}D_{\beta}\text{–}Cl(A) = {}_{N}D_{\beta}\text{–}Int(X \setminus A)$.

(2). $X \setminus {}_{N}D_{\beta}\text{–}Int(A) = {}_{N}D_{\beta}\text{–}Cl(X \setminus A)$.

Theorem 4.11. *Let A be any subset of X. Then A is $_{N}D_{\beta}$–open if and only if $_{N}D_{\beta}$–$Int(A) = A$.*

Proof. It follows from Theorem 3.12 and Lemma 4.10. □

Theorem 4.12. *Let (X, τ) be a topological space and suppose A and B be any two subsets of X. Then the following results hold.*

(1). $\beta\text{–}Int(A) \subseteq {}_{N}D_{\beta}\text{–}Int(A) \subseteq A$, $Int^{*}(A) \subseteq {}_{N}D_{\beta}\text{–}Int(A)$ and $Int_{D_{\alpha}}(A) \subseteq {}_{N}D_{\beta}\text{–}Int(A)$.

(2). $_{N}D_{\beta}\text{–}Int(A) = X$, $_{N}D_{\beta}\text{–}Int(A) = \phi$

(3). If $A \subseteq B$, then $_{N}D_{\beta}\text{–}Int(A) \subseteq {}_{N}D_{\beta}\text{–}Int(B)$.

(4). $_{N}D_{\beta}\text{–}Int(\,{}_{N}D_{\beta}\text{–}Int(A)) = {}_{N}D_{\beta}\text{–}Int(A)$.

(5). $_{N}D_{\beta}\text{–}Int(A) \cup {}_{N}D_{\beta}\text{–}Int(B) \subseteq {}_{N}D_{\beta}\text{–}Int(A \cup B)$.

(6). $_{N}D_{\beta}\text{–}Int(A) \cap {}_{N}D_{\beta}\text{–}Int(B) \subseteq {}_{N}D_{\beta}\text{–}Int(A \cap B)$.

Definition 4.13. *Let (X, τ) be any topological space and let $x \in X$. A subset G_x of X is said to be $_{N}D_{\beta}$–neighborhood of x if there exists a $_{N}D_{\beta}$–open set U in X such that $x \in U \subset G_x$.*

Theorem 4.14. *Let A be any subset of the topological space (X, τ) and let $x \in X$. Then $x \in {}_ND_\beta$–$Cl(A)$ if and only if every ${}_ND_\beta$–open set U containing x intersects A.*

Proof. We prove the result in the manner that, $x \notin {}_ND_\beta$–$Cl(A)$ if and only if there exists a ${}_ND_\beta$–open set U containing x that does not intersect A. For, let $x \notin {}_ND_\beta$–$Cl(A)$ and suppose $U = X \setminus {}_ND_\beta$–$Cl(A)$ is a ${}_ND_\beta$–open set containing x that does not intersect A. Conversely, if there exists a ${}_ND_\beta$–open set U containing x such that it does not intersect A.Then $X \setminus U$ is a ${}_ND_\beta$–closed set containing A. Since ${}_ND_\beta$–$Cl(A)$ is the smallest ${}_ND_\beta$–closed set containing A and hence $X \setminus U$ will contain ${}_ND_\beta$–$Cl(A)$ and therefore $x \notin {}_ND_\beta$–$Cl(A)$. □

Definition 4.15. *Let A be a subset of the space X. A point $x \in X$ is said to be a D_α–limit point of A if for each D_α–open set U containing x, we have $U \cap (A \setminus \{x\}) \neq \phi$. The set of all D_α–limit points of A is called the D_α–derived set of A and it is denoted by D_α–$Der(A)$.*

Definition 4.16. *Let A be a subset of a space X. A point $x \in X$ is said to be a ${}_ND_\beta$–limit point of A if for each ${}_ND_\beta$–open set U containing x, we have $U \cap (A \setminus \{x\}) \neq \phi$. The set of all ${}_ND_\beta$–limit points of A is called the ${}_ND_\beta$–derived set of A and is denoted by ${}_ND_\beta$–$Der(A)$.*

Remark 4.17. *Since every open set is D_α–open, we have D_α–$Der(A) \subseteq D(A)$ and therefore ${}_ND_\beta$–$Der(A) \subseteq D(A)$ for any subset $A \subseteq X$, where $D(A)$ is the derived set of A. Moreover, since every closed set is D_α–closed, we have $A \subseteq {}_ND_\beta$–$Cl(A)$ $\subseteq Cl_{D_\alpha}(A) \subseteq Cl(A)$.*

5. ${}_ND_\beta$–continuous and ${}_ND_\beta$–irresolute Functions

In this section we introduce ${}_ND_\beta$–continuous functions and study some of their basic properties.

Definition 5.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is called ${}_ND_\beta$–continuous if the inverse image of each open set in Y is ${}_ND_\beta$–open in X.*

Theorem 5.2.

(1). Every β–continuous function is ${}_ND_\beta$–continuous.

(2). Every g–continuous function is ${}_ND_\beta$–continuous.

(3). Every semi*–continuous function is ${}_ND_\beta$–continuous.

(4). Every D_α–continuous function is ${}_ND_\beta$–continuous.

Proof. It follows directly from the Theorem 3.12. □

Remark 5.3. *${}_ND_\beta$–continuous function need not be β–continuous(resp. not g–continuous, not semi*–continuous, not D_α–continuous).*

It follows from the following example.

Example 5.4. *Let X=$\{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$, then (X, τ) is a topological space.$C(X) = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}\}$. Let $Y = \{1, 2, 3\}$, $\sigma = \{\phi, Y, \{1\}, \{1, 2\}\}$, then (Y,σ) be another topological space.*

$$GC(X) = \{X, \phi, \{b, c, d\}, \{c, d\}, \{d\}, \{b, d\}, \{a, d\}, \{a, c, d\}, \{a, b, d\}\},$$

$$GO(X) = \{X, \phi, \{a,b\}, \{a\}, \{a,b,c\}, \{a,c\}, \{b,c\}, \{b\}, \{c\}\},$$

$$D_\alpha C(X) = \{X, \phi, \{b,c,d\}, \{c,d\}, \{d\}, \{b,d\}, \{a,d\}, \{a,c,d\}, \{a,b,d\}, \{b,c\}, \{b\}, \{c\}\},$$

$$D_\alpha O(X) = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,c\}, \{b,c\}, \{b\}, \{c\}, \{a,d\}, \{a,c,d\}, \{a,b,d\}\},$$

$$\beta C(X) = \{X, \phi, \{b,c,d\}, \{c,d\}, \{d\}, \{b,c\}, \{b\}, \{c\}, \{b,d\}\},$$

$$\beta O(X) = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,d\}, \{a,c,d\}, \{a,b,d\}, \{a,c\}\},$$

$$semi^* C(X) = \{X, \phi, \{b,c,d\}, \{c,d\}, \{d\}, \{b,c\}, \{c\}\},$$

$$semi^* C(X) = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,d\}, \{a,b,d\}\},$$

$$_N D_\beta C(X) = \{X, \phi, \{b,c,d\}, \{c,d\}, \{d\}, \{b,d\}, \{a,d\}, \{a,c,d\}, \{a,b,d\}, \{b,c\}, \{b\}, \{c\}, \{a\}, \{a,b\}\{a,c\}\}$$

$$_N D_\beta O(X) = \{X, \phi, \{a\}, \{a,b\}, \{a,b,c\}, \{a,c\}, \{b,c\}, \{b\}, \{c\}, \{a,d\}, \{a,c,d\}, \{a,b,d\}, \{b,c,d\}, \{c,d\}, \{b,d\}\}.$$

Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = f(b) = 3,\ f(c) = 1,\ f(d) = 2$ is $_N D_\beta$–continuous, since the inverse image of each open set in Y is $_N D_\beta$–open in X. But it is not β–continuous since the preimage of an open set $A = \{1, 2\}$ in Y is $\{c,d\}$, which is not β–open(not g–open, not D_α–open, not semi*–open) set in Y.

Theorem 5.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(1). f is $_N D_\beta$–continuous.

(2). $f(_N D_\beta\text{–}Cl(A)) \subset Cl(f(A))$ for every subset A of X.

(3). The inverse image of each closed set in Y is $_N D_\beta$–closed in X.

(4). For each $x \in X$ and each open set $U \subset Y$ containing $f(x)$, there exists a $_N D_\beta$–open set $V \subset X$ containing x such that $f(V) \subset U$.

(5). $_N D_\beta\text{–}Cl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$ for every subset B of Y.

(vi) $f^{-1}(Int(A)) \subset {}_N D_\beta\text{–}Int(f^{-1}(A))$ for every subset A of Y.

Proof. (1) ⇒ (2). Suppose f is $_N D_\beta$–continuous and let A be any subset of X. Let $x \in {}_N D_\beta\text{–}Cl(A)$, then $f(x) \in f(_N D_\beta\text{–}Cl(A))$. Suppose U be a neighborhood of $f(x)$ in Y. Then $f^{-1}(U)$ is $_N D_\beta$–open in X containing x and it intersects A in the point y(other than x). Then the set U intersects $f(A)$ in the point $f(y)$, therefore $f(x) \in Cl(f(A))$ and we get $f(_N D_\beta\text{–}Cl(A)) \subset Cl(f(A))$.

(2) ⇒ (3). Let us assume that the function f is $_N D_\beta$–continuous and suppose A be any closed set in Y. Let $B = f^{-1}(A)$. Since B is $_N D_\beta$*–closed* in X. We show that $_N D_\beta\text{–}Cl(B) = B$. For, $f(B) = f(f^{-1}(A)) \subset A$. Suppose $x \in {}_N D_\beta\text{–}Cl(B)$. Then we have $f(x) \in f(_N D_\beta\text{–}Cl(B)) \subset Cl(f(B)) \subset Cl(A) = A$. Thus $x \in f^{-1}(A) = B$. It follows that $_N D_\beta\text{–}Cl(B) \subset B$. Since $B \subset {}_N D_\beta\text{–}Cl(B)$. Hence we have $B = {}_N D_\beta\text{–}Cl(B)$ i.e. $B = f^{-1}(A)$ is a $_N D_\beta$*–closed* set in X.

(3) ⇒ (1) Since function f is $_N D_\beta$–continuous. Suppose U be any open set in Y. Let $A = Y \setminus U$ be any closed set in Y. Then $f^{-1}(A) = f^{-1}(Y) \setminus f^{-1}(U) = X \setminus f^{-1}(U)$ is a $_N D_\beta$–closed set in X. Therefore $f^{-1}(U)$ is $_N D_\beta$–open in X.

(1) ⇒ (4) Suppose f is $_N D_\beta$–continuous function. Suppose for each $x \in X$ and for each open subset U of Y containing $f(x)$, $f^{-1}(U) \in D_\beta O(X)$. We set $V = f^{-1}(U)$ containing x, we get $f(V) \subset U$.

(4) ⇒ (1) Let U be an open set in Y, containing $f(x)$ for each $x \in X$, then there exists a $_N D_\beta$–open set V_x containing x such that $f(V_x) \subset U$ and then $x \in V_x \subset f^{-1}(U)$, which shows that $f^{-1}(U)$ is open in X. Hence f is $_N D_\beta$–continuous.

(2) ⇒ (5) Let B be any subset of Y and $A = f^{-1}(B)$ is the subset of X. By hypothesis $f(_N D_\beta\text{–}Cl(A)) \subset Cl(f(A))$ for every

subset A of X, then we have $f\ ({}_ND_\beta\text{–}Cl(f^{-1}(B))) \subset Cl(f(f^{-1}(B))) \subset Cl(B)$ and therefore we get $({}_ND_\beta\text{–}Cl(f^{-1}(B)) \subset f^{-1}(Cl(B))$.

(5) $\Rightarrow$ (6) Let F be any subset of Y. By hypothesis ${}_ND_\beta\text{–}Cl((f^{-1}(Y \setminus F)) \subseteq f^{-1}(Cl(Y \setminus F))$. This Shows that $({}_ND_\beta\text{–}Cl(X \setminus (f^{-1}(F)) \subseteq f^{-1}(Y \setminus Int(F))$. Therefore $X\setminus\ ({}_ND_\beta\text{–}Int(f^{-1}(F))) \subseteq X \setminus f^{-1}(Int(F)))$. Hence we get $f^{-1}(Int(F)) \subseteq {}_ND_\beta\text{–}Int(f^{-1}(F))$.

(6) $\Rightarrow$ (1) We show that f is ${}_ND_\beta$–continuous. Let V be any open set in Y. Then $Int(V) = V$. By hypothesis $f^{-1}(Int(V)) \subseteq ({}_ND_\beta\text{–}Int(f^{-1}(V))$. Thus we get $f^{-1}(V)) \subseteq ({}_ND_\beta\text{–}Int(f^{-1}(V))$. Since $({}_ND_\beta\text{–}Int(f^{-1}(V)) \subseteq f^{-1}(V))$. Hence we get $({}_ND_\beta\text{–}Int(f^{-1}(V)) = f^{-1}(V)$, which implies that $f^{-1}(V)$ is ${}_ND_\beta$–open in X. □

Remark 5.6. *Composition of two ${}_ND_\beta$–continuous functions need not be ${}_ND_\beta$–continuous.*

Example 5.7. *Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}, \{c\}\}$, $C(X) = \{X, \phi, \{a, b\}, \{b\}\}$, then (X,τ) is a topological space. ${}_ND_\beta C(X) = \{X, \phi, \{a, b\}, \{b\}, \{b, c\}, \{a\}, \{c\}\}$, ${}_ND_\beta O(X) = \{X, \phi, \{a, c\}, \{c\}, \{a\}, \{b, c\}, \{a, b\}\}$. Let $Y = \{x, y, z\}$, $\sigma = \{Y, \phi, \{y, z\}\}$, then (Y, σ) is a topological space. $C(Y)=\{Y, \phi, \{x\}\}$, ${}_ND_\beta C(Y) = \{Y, \phi, \{x, z\}, \{x, y\}, \{x\}, \{y\}, \{z\}\}$, ${}_ND_\beta O(Y) = \{Y, \phi, \{2\}, \{2, 3\}, \{3\}, \{1, 2\}, \{1, 3\}\}$. Let $Z = \{r, s, t\}$, $\eta = \{Z, \phi, \{s\}\}$, then (Z, η) is another topological space. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = y$, $f(b) = z$ and $f(c) = x$ and another function $g : (Y, \sigma) \to (Z, \eta)$ defined as $g(x) = r$, $g(y) = t$, $g(z) = s$. Here both the functions f and g are ${}_ND_\beta$–continuous. Let $A = \{s\}$ be any open set in Z, but $(gof)^{-1}(s) = f^{-1}(g^{-1}(s)) = f^{-1}(z) = \{b\}$, which is not a ${}_ND_\beta$–open set in X.*

Theorem 5.8. *If a function $f : (X, \tau) \to (Y, \sigma)$ is D_α–continuous and $g : (Y, \sigma) \to (Z, \eta)$ is continuous, then $g \circ f : (X, \tau) \to Z, \eta)$ is also D_α–continuous.*

Proof. Let B be any open set in $(Z,\ \eta)$. Since map g is continuous, therefore $g^{-1}(B)$ is open in $(Y,\ \sigma)$. Since f is D_α–continuous, we have $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is D_α–open in X. Thus $g \circ f$ is D_α–continuous. □

Theorem 5.9. *Let $f : (X, \tau) \to (Y, \sigma)$ be ${}_ND_\beta$–continuous and $g : (Y, \sigma) \to (Z, \eta)$ be continuous functions. Then their composition $gof : (X, \tau) \to (Z, \eta)$ is ${}_ND_\beta$–continuous.*

Definition 5.10. *A function $f : (X, \tau) \to (Y, \sigma)$ is called ${}_ND_\beta$–irresolute if the preimage of each ${}_ND_\beta$–closed(${}_ND_\beta$–open) set in Y is ${}_ND_\beta$–closed (${}_ND_\beta$–open) in X.*

Remark 5.11. *Every D_α–irresolute function is ${}_ND_\beta$–irresolute but the converse is not true.*

Example 5.12. *Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{b\}, \{b, c\}\}$, then (X, τ) is a topological space. $C(X) = \{X, \phi, \{a, c, d\}, \{a, d\}\}$. Let $Y = \{r, s, t\}$, $\sigma = \{\phi, Y, \{r, t\}\}$, then (Y, σ) is another topological space. $C(Y) =\{Y, \phi, \{s\}\}$.*

$$D_\alpha C(X) = \{X, \phi, \{a, c, d\}, \{a, d\}, \{a, b\}, \{a, b, c\}, \{d\}, \{a\}, \{a, c\}, \{b, c, d\}, \{a, b, d\}, \{b, d\}, \{c, d\}\},$$

$$D_\alpha O(X) = \{X, \phi, \{b\}, \{b, c\}, \{c, d\}, \{d\}, \{a, b, c\}, \{b, c, d\}, \{b, d\}, \{a\}, \{c\}, \{a, c\}, \{a, b\}\},$$

$${}_ND_\beta C(X) = \{X, \phi, \{a, c, d\}, \{a, d\}, \{a, b\}, \{a, b, c\}, \{d\}, \{a\}, \{a, c\}, \{b, c, d\}, \{a, b, d\}, \{c\}, \{b, d\}, \{b\}, \{c, d\}\},$$

$${}_ND_\beta O(X) = \{X, \phi, \{b\}, \{b, c\}, \{c, d\}, \{d\}, \{a, b, c\}, \{b, c, d\}, \{b, d\}, \{a\}, \{c\}, \{a, c, d\}, \{a, c\}, \{a, b, d\}, \{a, b\}\}.$$

$$D_\alpha C(Y) = \{Y, \phi, \{s\}, \{s, t\}, \{r, s\}\},$$

$$D_\alpha O(Y) = \{Y, \phi, \{r\}, \{r, t\}, \{t\}\},$$

$${}_ND_\beta C(Y) = \{Y, \phi, \{s\}, \{s, t\}, \{r, s\}, \{r\}, \{t\}\},$$

$${}_ND_\beta O(Y) = \{Y, \phi, \{r\}, \{r, t\}, \{t\}, \{s, t\}, \{r, s\}\}.$$

Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = f(d) = r$, $f(c) = t$, $f(b) = s$, is ${}_N D_\beta$–irresolute. Since the preimage of every ${}_N D_\beta$–closed set in X is ${}_N D_\beta$–closed in Y. But it is not D_α–irresolute, if $A = \{s, t\}$ be any D_α–closed set in Y, then $f^{-1}(\{s, t\}) = \{b, c\}$, which is not D_α–closed in X.

Theorem 5.13. *If a function $f : (X, \tau) \to (Y, \sigma)$ is D_α–irresolute and $g : (Y, \sigma) \to (Z, \eta)$ is D_α–irresolute, then $g \circ f : (X, \tau) \to Z, \eta)$ is D_α–irresolute.*

Proof. Let A be any D_α–closed set in the space (Z, η). Since g is D_α–irresolute, therefore $g^{-1}(A)$ is D_α–closed set in Y. Since f is D_α–irresolute, then $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is D_α–closed in X. Hence $(g \circ f)$ is D_α–irresolute. □

Theorem 5.14. *If a function $f : (X, \tau) \to (Y, \sigma)$ is ${}_N D_\beta$–irresolute and $g : (Y, \sigma) \to (Z, \eta)$ is ${}_N D_\beta$–irresolute then $g \circ f : (X, \tau) \to (Z, \eta)$ is also ${}_N D_\beta$–irresolute.*

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