Int. J. Math. And Appl., 6(1–B)(2018), 325–332
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

An EOQ Model for Quadratic Demand Rate, Parabolic Deterioration, Time Dependent Holding Cost with Partial Backlogging

N. K. Sahoo[1,*] and P. K. Tripathy[2]

1 Department of Mathematics, S.S.S. Mahavidyalaya, Gaudakateni, Dhalpur, Dhenkanal, India.

2 P.G. Department of Statistics, Utkal University, Vani Vihar, Bhubaneswar, India.

Abstract: An economic order quantity model has been developed for deteriorating items with demand rate as a quadratic function of time. In the model, deterioration rate and holding cost are time dependent. The inventory shortage is discussed and partially backlogged. The backlogging rate is dependent on the waiting time for the next replenishment. Results are illustrated with numerical example along with sensitivity analysis for the model with respect to various parameters is carried out.

Keywords: Quadratic Demand Rate, Deterioration Rate, Partial Backlogging.

1. Introduction

Deterioration is defined as decay or damage such that the items, for examples, the frequently used goods like fruits, vegetables, meat, foodstuffs, etc., cannot be used for its original purpose. Most of the physical goods undergo obsolete over time. Highly volatile liquids such as gasoline, alcohol, perfumes, etc., undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic films, etc. deteriorate gradually during their normal storage period. Hariga [1] proposed an optimal inventory models for deteriorating items with time-varying demand. Giri, Goswami and Chaudhuri [2] proposed a model considering deteriorating items with time varying demand. Chang and Dye [3] established an EOQ model for deteriorating items with time varying demand and partial backlogging Khanra and Chaudhuri [4] developed an order-level inventory model for a deteriorating item with time dependent quadratic demand rate. Four inventory models for deteriorating items with time varying demand and partial backlogging are developed by Skouri and Papachristos [5] where cost comparison is analysed. Sana et.al [6] discussed a production inventory model for a deteriorating item with trended demand and shortages. Ghosh and Chaudhuri [7] proposed an inventory model with a quadratic demand rate with time-proportional deterioration and shortages in all cycles. Sahoo et al. [8] studied an EOQ model with ramp type demand rate, linear deterioration rate, unit production cost with shortages and backlogging. Sahoo et al. [9] developed a model for constant deteriorating items with price dependent demand and time-varying holding cost. Sana [10] proposed an inventory model for optimal selling price and lotsize with time varying deterioration and partial backlogging. Tripathy

* *E-mail: narenmaths@yahoo.co.in*

and Pradhan [11] formulated a model using partial backlogging, Weibull demand and variable deterioration rate. Das et al. [12] discussed an economic order quantity model of imperfect quality items with partial backlogging.

In this paper, EOQ model is developed using quadratic demand rate, deterioration rate is dependent of time. Shortage is allowed with partial backlogging. Sensitivity analysis is carried out with numerical example. Graphical representation expresses the variation of total average cost with effects of change of different parameters.

2. Assumptions and Notation

The following assumptions and notations are used in this model

- The demand rate is time dependent and assumed as: $R(t) = \begin{cases} a + bt + ct^2, & t > 0 \\ R_0, & t \leq 0 \end{cases}$, where $a > 0$ is initial demand and $b > 0$, $0 < c < 1$.
- The rate of replenishment is infinite.
- The inventory model involves only one item and planning horizon is infinite.
- The deteriorating rate $\lambda(t) = \alpha + \beta t^2$, $\alpha > 0$, $0 < \beta << 1$ is a function of time.
- The deteriorated units cannot be repaired or replaced during the period under review.
- Holding cost $h(t)$per item per unit-time is time dependent and is assumed as $H(t) = h_1 + h_2 t$, where $h_1 > 0$, $0 < h_2 < 1$.
- A is the ordering cost per order.
- $C_1$is the inventory cost per unit.
- C_2 is the shortage cost per units.
- C_3 is the opportunity cost due to lost sales.
- t_1 is the time at which shortages start.
- T is the length of each ordering cycle.
- Q_A is the maximum inventory level for each ordering cycle.
- Q_B is the maximum amount of demand backlogged for each ordering cycle.
- S is the economic order quantity for each ordering cycle.
- $Q(t)$ is the inventory level at time t.
- When shortage period start, the variable backlogging rate is dependent on the length of the waiting time till the next replenishment. Further longer the waiting time, the smaller the backlogging rate. Hence, those customers who would like to accept backlogging at time t is decreasing with the waiting time $(T - t)$ waiting till the next replenishment. To avoid this situation we have defined $\frac{1}{1+\theta(T-t)}$ as the backlogging rate when inventory is negative with constant positive backlogging parameter θ for time period (t_1, T).

3. Mathematical Model

We consider the deteriorating inventory model with quadratic demand. Replenishment starts at time $t = 0$ when the inventory level attains its maximum, Q_A. The inventory decrease due to demand and deterioration for the time $(0, t_1)$. At time t_1, the inventory level equal to zero, the shortage starts during the time interval $[t_1, T]$ and the demand during this period is partially backlogged. Using assumptions and notations the inventory system depicted in Figure 1 and the inventory system with respect to time t can be depicted by the adopting differential equation.

$$\frac{dQ(t)}{dt} + \lambda(t)Q(t) = -R(t),\ 0 \leq t \leq t_1 \tag{1}$$

With initial condition $Q(t_1) = 0$, and boundary condition $Q(0) = Q_A$.

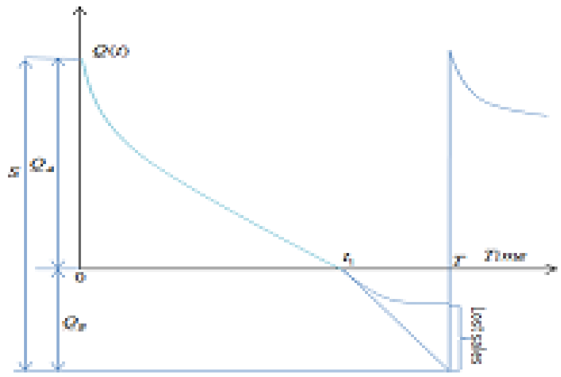

Figure 1. **Graphical Representation of the Inventory System**

Using $\lambda(t) = \alpha + \beta t^2$ and $R(t) = a + bt + ct^2$, we get

$$\frac{dQ(t)}{dt} + (\alpha + \beta t^2)Q(t) = -(a + bt + ct^2),\ \ 0 \leq t \leq t_1 \tag{2}$$

with $Q(t_1) = 0$ and $Q(0) = Q_A$. Solution of equation (2) is

$$Q(t) = \begin{bmatrix} \left(a(t_1 + \frac{\alpha t_1^2}{2} + \frac{\beta\, t_1^4}{12}) + b(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{3} + \frac{\beta\, t_1^5}{15}) + c(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{4} + \frac{\beta\, t_1^6}{18})\right) \\ -\left(a(t + \frac{\alpha t^2}{2} + \frac{\beta\, t^4}{12}) + b(\frac{t^2}{2} + \frac{\alpha t^3}{3} + \frac{\beta\, t^5}{15}) + c(\frac{t^3}{3} + \frac{\alpha t^4}{4} + \frac{\beta\, t^6}{18})\right) \end{bmatrix} e^{-(\alpha\, t + \frac{\beta\, t^3}{3})} \tag{3}$$

Maximum inventory level for each cycle is obtained by putting the boundary condition $Q(0) = Q_A$ in equation (3). Therefore,

$$Q(0) = Q_A = a(t_1 + \frac{\alpha t_1^2}{2} + \frac{\beta\, t_1^4}{12}) + b(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{3} + \frac{\beta\, t_1^5}{15}) + c(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{4} + \frac{\beta\, t_1^6}{18}) \tag{4}$$

During the shortage period $[t_1, T]$, the demand at time t is partially backlogged at the fraction $\frac{1}{1+\theta(T-t)}$. Therefore, the differential equation governing the amount of demand backlogged is

$$\frac{dQ(t)}{dt} = -\frac{R_0}{1+\theta(T-t)},\ \ t_1 < t < T \tag{5}$$

With boundary condition $Q(t_1) = 0$. The solution of equation (5) is

$$Q(t) = -\int \frac{R_0}{1+\theta(T-t)}dt$$

$$= \frac{R_0}{\theta}\left(\log\left(1+\theta(T-t)\right) - \log\left(1+\theta(T-t_1)\right)\right) \tag{6}$$

Maximum amount of demand backlogged per cycle is obtained by putting $t = T$ in equation (6)

$$Q_B = -Q(T) = \frac{R_0}{\theta}\log\left(1+\theta(T-t_1)\right)$$

Hence, the economic order quantity per cycle is

$$S = Q_A + Q_B = a(t_1 + \frac{\alpha t_1^2}{2} + \frac{\beta\, t_1^4}{12}) + b(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{3} + \frac{\beta\, t_1^5}{15}) + c(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{4} + \frac{\beta\, t_1^6}{18}) + \frac{R_0}{\theta}\log\left(1+\theta(T-t_1)\right) \tag{7}$$

The inventory holding cost per cycle is

$$\begin{aligned} I_{HC} &= \int_0^{t_1} H(t)Q(t)dt \\ &= h_1\left(a(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{6} + \frac{\beta t_1^5}{20}) + b(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{8} + \frac{\beta t_1^6}{24}) + c(\frac{t_1^4}{4} + \frac{\alpha t_1^5}{10} + \frac{\beta t_1^7}{28})\right) \\ &+ h_2\left(a(\frac{t_1^3}{6} + \frac{\alpha t_1^4}{24} + \frac{\beta t_1^6}{60}) + b(\frac{t_1^4}{8} + \frac{\alpha t_1^5}{30} + \frac{\beta t_1^7}{70}) + c(\frac{t_1^5}{10} + \frac{\alpha t_1^6}{36} + \frac{\beta t_1^8}{80})\right) \end{aligned} \tag{8}$$

The deterioration cost per cycle is

$$\begin{aligned} I_{DC} &= C_1(Q_A - \int_0^{t_1} R(t)dt) \\ &= C_1\left(a(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^4}{12}) + b(\frac{\alpha t_1^3}{3} + \frac{\beta t_1^5}{15}) + c(\frac{\alpha t_1^4}{4} + \frac{\beta t_1^6}{18})\right) \end{aligned} \tag{9}$$

The shortage cost per cycle is

$$\begin{aligned} I_{CS} &= C_2\left[-\int_{t_1}^{T} Q(t)dt\right] \\ &= \frac{C_2R_0}{\theta}\left(T - t_1 - \frac{1}{\theta}\log\left(1+\theta(T-t_1)\right)\right) \end{aligned} \tag{10}$$

The opportunity cost due to lost sales per cycle is

$$\begin{aligned} I_{OC} &= C_3\left[\int_{t_1}^{T} R_0(1 - \frac{1}{1+\theta(T-t)})dt\right] \\ &= C_3R_0\left(T - t_1 - \frac{1}{\theta}\log(1+\theta(T-t_1))\right) \end{aligned} \tag{11}$$

Therefore, the total average cost per unit time per cycle $C(t_1, T)$ is given by

$$\begin{aligned} C(t_1,T) &= \frac{1}{T}(A + I_{HC} + I_{DC} + I_{CS} + I_{OC}) \\ &= \frac{1}{T}\left[\begin{array}{l} A + h_1\left(a(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{6} + \frac{\beta\, t_1^5}{20}) + b(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{8} + \frac{\beta\, t_1^6}{24}) + c(\frac{t_1^4}{4} + \frac{\alpha t_1^5}{10} + \frac{\beta\, t_1^7}{28})\right) \\ +h_2\left(a(\frac{t_1^3}{6} + \frac{\alpha t_1^4}{24} + \frac{\beta\, t_1^6}{60}) + b(\frac{t_1^4}{8} + \frac{\alpha t_1^5}{30} + \frac{\beta\, t_1^7}{70}) + c(\frac{t_1^5}{10} + \frac{\alpha t_1^6}{36} + \frac{\beta\, t_1^8}{80})\right) \\ +C_1\left(a(\frac{\alpha t_1^2}{2} + \frac{\beta\, t_1^4}{12}) + b(\frac{\alpha t_1^3}{3} + \frac{\beta\, t_1^5}{15}) + c(\frac{\alpha t_1^4}{4} + \frac{\beta\, t_1^6}{18})\right) \\ +\frac{(C_2+\theta C_3)R_0(T-t_1)}{\theta} - \frac{(C_2+\theta C_3)R_0}{\theta^2}\log\left(1+\theta(T-t_1)\right) \end{array}\right] \end{aligned} \tag{12}$$

Our aim is to determine the optimal values of t_1 and T in order to minimize the average total average Cost $C(t_1, T)$ per unit time. Using mathematical software, the optimum values of t_1 and T for the minimum average cost $C(t_1, T)$ is the solution of the equations

$$\frac{\partial C(t_1,T)}{\partial t_1} = 0 \text{ and } \frac{\partial C(t_1,T)}{\partial T} = 0 \tag{13}$$

Provided that they satisfy the sufficient conditions

$$\frac{\partial^2 C(t_1,T)}{\partial t_1^2} > 0, \ \frac{\partial^2 C(t_1,T)}{\partial T^2} > 0 \ \text{ and } \frac{\partial^2 C(t_1,\,T)}{\partial t_1^2}.\frac{\partial^2 C(t_1,\,T)}{\partial T^2} - \left(\frac{\partial^2 C(t_1,\,T)}{\partial t_1 \partial T}\right)^2 > 0$$

Equation (13) can be written as

$$\frac{\partial C(t_1,T)}{\partial t_1} = \frac{1}{T}\left[\begin{array}{l} h_1\left(a(t_1+\frac{\alpha t_1^2}{2}+\frac{\beta t_1^4}{4})+b(t_1^2+\frac{\alpha t_1^3}{2}+\frac{\beta t_1^5}{4})+c(t_1^3+\frac{\alpha t_1^4}{2}+\frac{\beta t_1^6}{4})\right) \\ +h_2\left(a(\frac{t_1^2}{2}+\frac{\alpha t_1^3}{6}+\frac{\beta t_1^5}{10})+b(\frac{t_1^3}{2}+\frac{\alpha t_1^4}{6}+\frac{\beta t_1^6}{10})+c(\frac{t_1^4}{2}+\frac{\alpha t_1^5}{6}+\frac{\beta t_1^7}{10})\right) \\ +C_1\left(a(\alpha t_1+\frac{\beta t_1^3}{3})+b(\alpha t_1^2+\frac{\beta t_1^4}{3})+c(\alpha t_1^3+\frac{\beta t_1^5}{3})\right)-\frac{R_0(C_2+\theta C_3)(T-t_1)}{(1+\theta(T-t_1))} \end{array}\right] = 0 \tag{14}$$

And,

$$\frac{\partial C(t_1,T)}{\partial T} = \frac{1}{T}\left(\frac{R_0(C_2+\theta C_3)(T-t_1)}{(1+\theta(T-t_1))} - C(t_1,T)\right) = 0 \tag{15}$$

4. Numerical Example

Example 4.1. *Let*$A = 16$, $a = 40$, $b = 12$, $c = 0.4$, $h_1 = 4$, $h_2 = 0.2$, $\alpha = 2$, $\beta = 0.1$, $\theta = 4$, $c_1 = 1.5$, $c_2 = 2.5$, $c_3 = 2$ *and* $R_0 = 30$ *in appropriate units. By applying Mathematica-10, we obtain the optimum solution for* t_1 *and* T *of equations* (7) *as* $t_1 = 0.174233$ *and* $T = 0.813875$. *Substituting* t_1 *and* T *in equation, we obtain the optimum average cost as* $C(t_1,T) = 56.6203$, $Q_A = 8.40927$ *and* $S = 17.9295$.

5. Table and Figures

Parameters	% change in parameters	t_1^*	T^*	$C^*(t_1,T)$	% change in $C^*(t_1,T)$
A	+50	0.194803	1.31536	64.3854	+13.71434
	+25	0.185822	1.04285	60.9659	+7.674986
	-25	0.158881	0.6178	50.9789	-9.96356
	-50	0.137547	0.443773	43.3552	-23.4282
a	+50	0.126607	0.853004	58.5866	+3.472783
	+25	0.146754	0.835357	57.7747	+2.038845
	-25	0.213642	0.787986	54.8674	-3.09589
	-50	0.273938	0.788701	51.9759	-8.20271
b	+50	0.17085	0.814329	56.7153	+0.167784
	+25	0.172509	0.814094	56.6685	+0.085128
	-25	0.176029	0.813674	56.5705	-0.08795
	-50	0.177901	0.813494	56.5191	-0.17873
c	+50	0.174212	0.813871	56.6207	+0.000706
	+25	0.174223	0.813873	56.6205	+0.000353
	-25	0.174244	0.813876	56.6201	-0.00035
	-50	0.174254	0.813878	56.6199	-0.00071
h_1	+50	0.140969	0.838665	57.9759	+2.394194
	+25	0.15574	0.82697	57.3791	+1.340155
	-25	0.198201	0.79942	55.6214	-1.76421
	-50	0.23083	0.784173	54.243	-4.19867
h_2	+50	0.174055	0.813893	56.6252	+0.008654
	+25	0.174144	0.813884	56.6227	+0.004239
	-25	0.174323	0.813866	56.6178	-0.00442
	-50	0.174412	0.813857	56.6154	-0.00865
α	+50	0.146995	0.833203	57.721	+1.944002
	+25	0.159343	0.823942	57.2242	+1.066579
	-25	0.192647	0.803069	55.8688	-1.32726
	-50	0.216217	0.791834	54.9042	-3.03089

Parameters	% change in parameters	t_1^*	T^*	$C^*(t_1,T)$	% change in $C^*(t_1,T)$
β	+50	0.174214	0.813871	56.6207	+0.000706
	+25	0.174224	0.813873	56.6205	+0.000353
	-25	0.174243	0.813877	56.6201	-0.00035
	-50	0.174252	0.813879	56.6199	-0.00071
θ	+50	0.180415	0.904131	58.9291	+4.077689
	+25	0.177711	0.85573	57.9165	+2.289285
	-25	0.16965	0.778331	54.9226	-2.99839
	-50	0.163375	0.749526	52.6168	-7.07079
c_1	+50	0.151005	0.832363	57.6115	+1.750609
	+25	0.161851	0.823505	57.1547	+0.943831
	-25	0.188466	0.803419	55.9886	-1.11568
	-50	0.20494	0.792106	55.2332	-2.44983
c_2	+50	0.18352	0.719555	60.0967	+6.139847
	+25	0.179046	0.762703	58.4159	+3.171301
	-25	0.169047	0.875308	54.700	-3.39154
	-50	0.163451	0.950104	52.6445	-7.02186
c_3	+50	0.199883	0.590943	66.3398	+17.1661
	+25	0.188483	0.67608	61.9745	+9.456326
	-25	0.156135	1.06383	49.9834	-11.7218
	-50	0.132799	1.60965	41.6923	-26.3651
R_0	+50	0.205886	0.552996	68.6681	+21.27823
	+25	0.192314	0.64535	63.4331	+12.03243
	-25	0.149483	1.18649	47.5897	-15.9494
	-50	0.114739	2.3911	35.4786	-37.3394

Table 1.

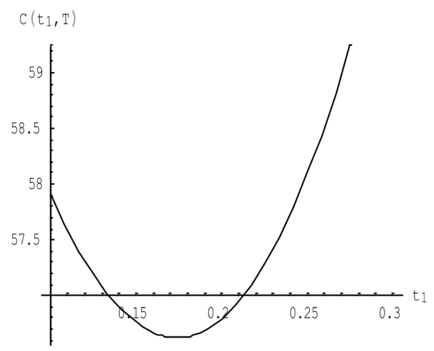

Figure 2. **Total average** $\cos t$ **vs.** t_1 **at** $T = 0.813875$

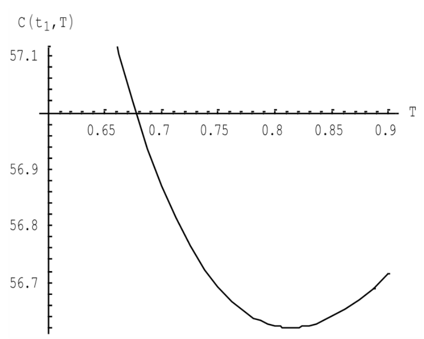

Figure 3. **Total average** $\cos t$ **vs. T at** $t_1 = 0.174233$

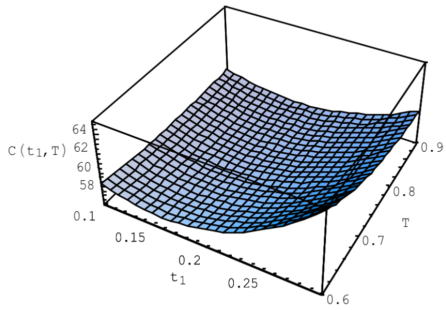

Figure 4. **Total average** $\cos t$ **vs.** t_1 **and T**

6. Sensitivity Analysis

We now study the effects of changes in the system of parameters A, a, b, c, h_1, h_2, α, β, θ, c_1, c_2, c_3 and R_0 on the optimal cost derived by the proposed method. The sensitivity analysis is performed by changing (increasing or decreasing) of parameters by 25% and 50% and taking one parameter at a time and keeping the remaining parameter at their fixed value. The analysis is based on the example-1 and the results are shown in Table 1. The following points are observed:

(1). t_1^* and T^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter A and θ. The model is highly sensitive to the parameter A and θ.

(2). t_1^* decrease (increase) and T^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter a. Here t_1^* insensitive and both T^* and $C^*(t_1,T)$ are highly sensitive with parameter a.

(3). t_1^* decrease (increase) and T^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter b. Here t_1^* and T^* is insensitive and $C^*(t_1,T)$ is moderately sensitive with parameter b.

(4). t_1^* and T^* decrease (increase) while the optimum cost $C^*(t_1,T)$ increase (decrease) slowly with increase (decrease) with parameter of c. Here t_1^* and T^* is insensitive and $C^*(t_1,T)$ is low sensitive with parameter c.

(5). t_1^* decrease (increase) and T^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter h_1. Here t_1^* is insensitive and both T^* and $C^*(t_1,T)$ are highly sensitive with parameternh_1.

(6). t_1^* decrease (increase) and T^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter h_2. Here t_1^* is insensitive and both T^* and $C^*(t_1,T)$ is moderately sensitive with parameter h_2.

(7). t_1^* and T^* decrease (increase) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) with parameter of α. Here t_1^* and $T^*$is insensitive and $C^*(t_1,T)$ is highly sensitive with parameter α.

(8). t_1^* and T^* decrease (increase) while the optimum cost $C^*(t_1,T)$ increase (decrease) slowly with increase (decrease) with parameter of β. Here t_1^* and T^* is insensitive and $C^*(t_1,T)$ is low sensitive with parameter β.

(9). t_1^* decrease (increase) and T^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter c_1. Here t_1^* is insensitive and both T^* and $C^*(t_1,T)$ is highly sensitive with parameter c_1.

(10). T^* decrease (increase) and t_1^* increase (decrease) while the optimum cost $C^*(t_1,T)$ increase (decrease) with increase (decrease) of parameter c_2 and c_3. Here T^* is insensitive and both t_1^* and $C^*(t_1,T)$ is highly sensitive with parameter c_2 and c_3.

(11). T^* decrease (increase) and t_1^* increase (decrease) while the optimum cost $C^*(t_1, T)$ increase (decrease) with increase (decrease) of parameter R_0. Here T^* is insensitive and both t_1^* and $C^*(t_1, T)$ is highly sensitive with parameter R_0.

7. Conclusion

This model has been developed for deteriorating items with quadratic demand rate. And both, deterioration rate and holding cost are time dependent. Here shortage is allowed and partially backlogged. This given model is supported by a numerical example along with sensitivity analysis is carried out to measure the effect of parameters on total average inventory cost. There is scope for extension of this existing model with permissible delay in payment, stochastic demand rate, etc.

References

[1] M.Hariga, *Optimal EOQ models for deteriorating items with time-varying demand*, Journal of the Operational Research Society, 47(1996), 1228-1246.

[2] B.C.Giri, A.Goswami and K.S.Chaudhuri, *An EOQ model for deteriorating items with time varying demand and costs*, Journal of Operational Research Society, 47(1996), 1398-1405.

[3] H.J.Chang and C.Y.Dye, *An EOQ model for deteriorating items with time varying demand and partial backlogging*, J. Oper. Res. Soc., 50(1999), 1176-1182.

[4] S.Khanra and K.S.Chaudhuri, *A note on an order-level inventory model for a deteriorating item with time dependent quadratic demand*, Computers and Operations Research, 30(2003), 1901-1916.

[5] K.Skouri and S.Papachristos, *Four inventory models for deteriorating items with time varying demand and partial backlogging: a cost comparison*, Optimal Control Appl. Methods, 24(2003), 315-330.

[6] S.Sana, S.K.Goyal and K.S.Chaudhuri, *A production inventory model for a deteriorating item with trended demand and shortages*, European Journal of Operational Research, 157(2004), 357-371.

[7] S.K.Ghosh and K.S.Chaudhuri, *An EOQ model with a quadratic demand, time-proportional deterioration and shortages in all cycles*, International Journal of Systems Science, 37(10)(2006), 663-672.

[8] N.K.Sahoo, C.K.Sahoo and S.K.Sahoo, *An EOQ model with ramp type demand rate, linear deterioration rate, unit production cost with shortages and backlogging*, Journal of Rajasthan Academy of Physical Sciences, 8(3)(2009), 319-332.

[9] N.K.Sahoo, C.K.Sahoo and S.K.Sahoo, *An Inventory Model for Constant Deteriorating Items with Price Dependent Demand and Time-varying Holding Cost*, International Journal of Computer Science & Communication, 1(1)(2010), 265-269.

[10] S.S.Sana, *Optimal selling price and lot size with time varying deterioration and partial backlogging*, Appl. Math. Comput., 217(2010), 185-194.

[11] P.K.Tripathy and S.Pradhan, *An integrated partial backlogging inventory model having Weibull demand and variable deterioration rate with the effect of trade credit*, Int. J. Sci. Eng. Res., 2(2011), 1-5.

[12] Roy Das, Sana Monami, Sankar Shib and K.S.Chaudhuri, *An economic order quantity model of imperfect quality items with partial backlogging.International*, Journal of Systems Science, 42(8)(2011), 1409-1419.