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Strong Efficient Open Domination in Graphs

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Abstract: Let $G = (V, E)$ be a simple, undirected, finite graph without isolated vertices. A subset D of $V(G)$ is a *Strong(Weak) efficient open dominating set* of G if $|N_s(v) \cap D| = 1$ ($|N_w(v) \cap D| = 1$), for every $v \in V(G)$ where $N_s(v)$ and $N_w(v)$ are strong and weak neighborhood respectively. The minimum cardinality of D is called as *strong(weak) efficient open domination number* and is denoted by $\gamma_{ste}(G)$ ($\gamma_{wte}(G)$) of G. A graph G is *strong(weak) efficient open dominating graph* if it contains a strong(weak) efficient open dominating set. We write a program to check whether the given graph is strong efficient open dominatable or not in C Language.

MSC: 05C69.

Keywords: Strong efficient open dominating set, Strong efficient open dominating graph, Strong efficient open domination Number, Weak efficient open dominating set, Weak efficient open domination number.

1. Introduction

Let $G = (V, E)$ be a finite, simple, undirected graph without isolated vertices. The open neighborhood of a vertex $v \in V(G)$ is $N(v) = \{u \in V(G) \mid uv \in E(G)\}$ and closed neighborhood of $v \in V$ is $N[v] = N(v) \cup \{v\}$. If v is a vertex of $V(G)$, then the degree of v is defined by the cardinality of $N(v)$ and is denoted by $deg\, v$. The minimum and maximum degree of the vertices of G is denoted by δ and Δ respectively. For graph theoretic terminology, we refer to [2] and [4]. The strong neighborhood and weak neighborhood of a vertex $v \in V(G)$ are defined by $N_s(v) = \{u \in V(G) \mid uv \in E(G) \text{ and } deg\, u \geq deg\, v\}$ and $N_w(v) = \{u \in V(G) \mid uv \in E(G) \text{ and } deg\, u \leq deg\, v\}$ respectively. A subset D of V is a dominating set of G if every vertex in $V - D$ is adjacent to atleast one vertex in D. The domination number, $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. In [9], Prof. E. Sampathkumar and L. Pushpalatha have defined strong(weak) domination in graphs. A subset D of V is called a strong(weak) dominating set of G if for every vertex, v in $V - D$ there exists $u \in D$ such that $uv \in E(G)$ and $deg\, u \geq deg\, v$ ($deg\, v \geq deg\, u$). The strong(weak) domination number, $\gamma_s(G)$ ($\gamma_w(G)$) of G is the minimum cardinality of a strong(weak) dominating set of G. D.W.Bange et al [3] defined an efficient dominating set D as a set of vertices of a graph G such that $|N[v] \cap D| = 1$, for every $v \in V(G)$ and N. Meena et al [8] extend this into strong(weak) efficient dominating set D as a set of vertices of a graph G such that $|N_s[v] \cap D| = 1$ ($|N_w[v] \cap D| = 1$), for every $v \in V(G)$. The strong(weak) efficient domination number, $\gamma_{se}(G)$ ($\gamma_{we}(G)$) of G is the minimum cardinality of a strong(weak) efficient dominating set of G. A subset D of V is a total dominating set of G if every vertex in V is adjacent to atleast one vertex in D. The total domination number, $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set of G. Gavlas and Schultz

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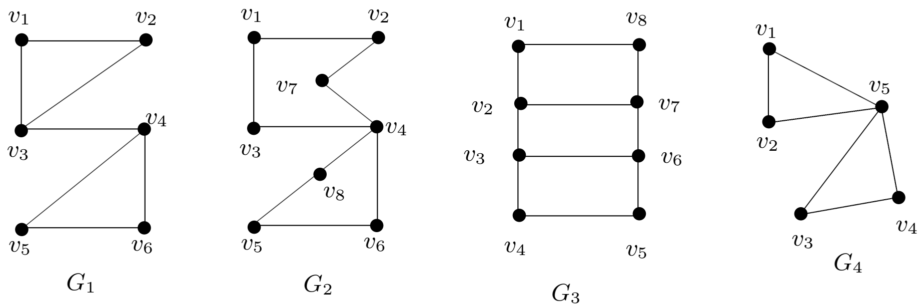
are defined an efficient open dominating set D as a set of vertices of a graph G such that $|N(v) \cap D| = 1$, for every $v \in V(G)$ in [6]. The efficient open domination number, $\gamma_{te}(G)$ of G is the cardinality of a efficient open dominating set of G. In this paper, we introduced a strong(weak) efficient open domination in graphs. A study of domination, total domination and efficient open domination in graphs and its advanced topics are given in [1, 5, 7, 10].

2. Strong(Weak) Efficient Open Domination

Definition 2.1 ([6]). *A subset D of $V(G)$ is called an efficient open dominating set or total perfect dominating set or total efficient dominating set of G if $|N(v) \cap D| = 1$, for every $v \in V(G)$. In other words, A subset D of $V(G)$ is called an efficient open dominating set of G if $N(v), v \in D$, form a partition of $V(G)$. The cardinality of the efficient open dominating set is called an efficient open domination number and is denoted by $\gamma_{te}(G)$. A graph G is called an efficient open dominating graph or EOD graph if it contains an efficient open dominating set. Also says that G is efficient open dominatable.*

Definition 2.2. *A subset D of $V(G)$ is called a strong efficient open dominating set (or SEOD set, for short) of G if $|N_s(v) \cap D| = 1$, for every $v \in V(G)$. A subset D of $V(G)$ is called a weak efficient open dominating set (or WEOD set, for short) of G if $|N_w(v) \cap D| = 1$, for every $v \in V(G)$. The strong (weak) efficient open domination number, denoted by $\gamma_{ste}(G)$ ($\gamma_{wte}(G)$), is the minimum cardinality of a strong (weak) efficient open dominating set of G. We also call the corresponding set that γ_{ste} (γ_{wte}) - set of G. A graph G is called a strong (weak) efficient open dominating graph or SEOD (WEOD) graph if it contains a strong(weak) efficient open dominating set. Also says that G is strong (weak) efficient open dominatable. A strong (weak) efficient open domination is also called as a strong (weak) total efficient domination.*

Example 2.3.



In this example, G_1 is a SEOD graph with a SEOD set $\{v_3, v_4\}$ but not a WEOD graph. G_2 is a WEOD graph with a WEOD set $\{v_1, v_2, v_5, v_6\}$ but not a SEOD graph. G_3 is both SEOD and WEOD graph with SEOD set $\{v_1, v_2, v_3, v_4\}$ and WEOD set $\{v_1, v_4, v_5, v_8\}$. G_4 is neither SEOD nor WEOD graph. The complete bipartite graph, $K_{m,n}$ is both strong and weak efficient open dominatable with $\gamma_{ste}(G) = \gamma_{wte}(G) = 2$ if $m = n$. We oberve that a graph G is SEOD(WEOD) graph if $N_s(v) \neq \phi (N_w(v) \neq \phi)$, for each vertex v in $V(G)$. A strong(weak) neighborhood of a vertex v in SEOD(WEOD) graph is a subset of SEOD(WEOD) set D if $N_s(v)(N_w(v))$ is a singleton set. Note that a SEOD(WEOD) set of a graph need not to be an efficient open domination set of G.

3. C Program to Check Given Graph is Strong Efficient Open Dominatable or not

The following simple **C** program is used to check whether the given graph is Strong Efficient Open Dominatable or not. Also it shows the Strong Efficient Open Dominating set(s) and Strong Efficient Open Dominating number of the given graph. In this program we can check the graphs which have less than 10 vertices.

```
void main()
{
int p,e=1,c,i,j,t,t1,k,l,m;
int a[10][10],d[10],n[10];
int v[10],nk[10],sed[10],N[10][10],s[10][260][10];
clrscr();
printf("*** Strong Efficient Open Domination ***\n");
printf("——————————————————-\n");
printf("Enter the number of vertices \n");
scanf("%d", &p);
for(i=1;i<=p;i++)
{
v[i]=i;
s[1][i][1]=v[i];
}
printf("If there is an edge between the vertices vi and vj then");
printf(" press 1\n otherwise press 0\n");
for(i=1;i<=p;i++)
for(j=i;j<=p;j++)
{
if(i==j)
a[i][j]=0;
else
{
printf("Is there any edge between v%d and v%d\n",v[i],v[j]);
scanf("%d",&a[i][j]);
a[j][i]=a[i][j];
}
}
clrscr();
for(i=1;i<=p;i++)
{
d[i]=0;
for(j=1;j<=p;j++)
if(a[i][j]==1)
d[i]++;
printf("Degree of the vertex v%d is %d\n",v[i],d[i]);
}
for(i=1;i<=p;i++)
{
k=1;
```

```
nk[i]=0;
for(j=1;j<=p;j++)
{
if(a[i][j]==1)
if(d[i]<=d[j])
{
N[i][k]=v[j];
k++;
}
}
nk[i]=k-1;
}
printf("The strong open neighbourhood of a vertex");
for(i=1;i<=p;i++)
{
printf("\nv%d is/are ",v[i]);
for(j=1;j<=nk[i];j++)
printf("v%d\t",N[i][j]);
if(nk[i]==0)
printf("{}");
}
for(i=2;i<=p;i++)
{
j=1;
t1=1;
for(k=1;k<=i-1;k++)
t1=t1*(p-k+1)/k;
for(l=1;l<=t1;l++)
{
t=s[i-1][l][i-1];
if(t!=p)
do
{
for(k=1;k<i;k++)
s[i][j][k]=s[i-1][l][k];
s[i][j][i]=++t;
j++;
}while(t<p);
}
}
printf("\n The strong efficient open dominating set is/are \n");
```

```
for(i=1;i<=p;i++)
{
t1=1;
for(k=1;k<=i;k++)
t1=t1*(p-k+1)/k;
for(j=1;j<=t1;j++)
{
for(k=1;k<=i;k++)
{
if(s[i][j][k]!='\0')
sed[k]=s[i][j][k];
}
c=0;
for(m=1;m<=p;m++)
{
n[m]=0;
for(l=1;l<=nk[m];l++)
if(N[v[m]][l]!='\0')
for(k=1;k<=i;k++)
if(N[v[m]][l]==sed[k])
n[m]++;
if (n[m]==1)
c++;
}
if(c==p)
{
for(k=1;k<=i;k++)
printf("v%d ",sed[k]);
printf("\n");
}
else
e++;
}
}
if(e==pow(2,p))
{
printf("Nil");
printf("\n Given graph is not a strong efficient open dominatable");
}
getch();
}
```

References

[1] M.H.Akhbari and N.Jafari Rad, *Bounds on weak and strong total domination number in graphs*, Electronic Journal of Graph theory and Applictions, 4(1)(2016), 111-118.

[2] R.Balakrishnan and K.Ranganathan, *A textbook of graph theory*, Springer, (2011).

[3] D.W.Bange, A.E.Barkauskas and P.J.Slater, *Efficient dominating sets in graphs*, Application of Discrete Mathematics, 1988(1988), 189-199.

[4] J.A.Bondy and U.S.R.Murthy, *Graph theory with Applications*, North-Holland, (1982).

[5] Dorota Kuziak, Iztok Peterin and Ismael G.Yero, *Efficient open domination in graph products*, DMTCS, 16(1)(2014), 105-120.

[6] H.Gavlas and K.Schultz, *Efficient Open Domination*, Electron. Notes Discrete Math., 11(2002), 681-691.

[7] V.R.Kulli and D.K.Patwari, *Total Efficient domination in graphs*, IRJPA, 6(1)(2016), 227-232.

[8] N.Meena, A.Subramanian and V.Swaminathan, *Strong Efficient Domination in Graphs*, Discrete Mathematics, 154(2006), 1173-1182.

[9] E.Sampathkumar and L.Pushpalatha, *Strong weak domination and domination balance in a graph*, Discrete Math., 161(1996), 235-242.

[10] Teresa W.Haynes, Stephen T.Hedetniemi and Peter J.Slater, *Fundamentals of domination in graphs*, International Journal of Innovative Science, Engineering and Technology, 1(4)(2014).