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# Best Proximity Points for Multiplicative Modified Rational Proximal Contraction Mapping on Multiplicative Metric Spaces

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**Abstract:** In this paper, we prove best proximity point theorems in multiplicative metric spaces satisfying multiplicative modified rational proximal contraction condition of the first kind. The presented results extend, generalize and improve some known results from best proximity point theory. We also give some examples to illustrate and validate our definitions and results.

**MSC:** 47H10, 54H25.

**Keywords:** Best proximity points, fixed points, multiplicative metric spaces, multiplicative modified rational proximal contraction mappings of the first kind.



## 1. Introduction

In 2008, Bashirov et al., [1] studied the usefulness of a new calculus, called multiplicative calculus due to Michael Grossman and Robert Katz in the period from 1967 till 1970. By using the concepts of multiplicative absolute values, Bashirov et al., [1] defined a new distance so called multiplicative distance. Afterward, Ozavsar and Cevike [2] introduced the concept of multiplicative metric spaces by using the idea of multiplicative distance and gave some topological properties in such space. They also introduced the concept of multiplicative Banach's contraction mapping and proved fixed point results for such mapping in multiplicative metric spaces. The notation of best proximity point is introduced in [3]. It turns out that many of the contractive conditions which are investigated for fixed points ensure the existence of best proximity points. Precisely, we introduce the notions of multiplicative modified rational proximal contraction mappings of the first kind, then we establish some corresponding best proximity theorems for such contraction and give some illustrative example of our main results. Our main results generalize, extend and improve the corresponding results on the topics given in the literature.

## 2. Preliminaries

In this section, we give some definitions and basic concept of multiplicative metric space for our consideration. Throughout this paper, we denote, $\mathbb{N}$, $\mathbb{R}^+$ and $\mathbb{R}$ the sets of positive integers, positive real numbers and real numbers, respectively.

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**Definition 2.1** ([1]). *Let $X$ be a nonempty set. A mapping $d : X \times X \to R$ is said to be multiplicative metric if it is satisfying the following conditions:*

*(1). $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$*

*(2). $d(x, y) = d(y, x)$ for all $x, y \in X$*

*(3). $d(x, z) \leq d(x, y) \cdot d(y, z)$ for all $x, y, z \in X$ (multiplicative triangle inequality)*

*Also, the ordered pair $(X, d)$ is called multiplicative metric space.*

**Example 2.2** ([2]). *Let $d : (\mathbb{R}^+)^n \times (\mathbb{R}^+)^n \to R$ be defined as follows.*

$$d(x, y) = \left|\frac{x_1}{y_1}\right| \cdot \left|\frac{x_2}{y_2}\right| \ldots \left|\frac{x_n}{y_n}\right|$$

*where $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in (R^+)^n$ and $|\cdot| : \mathbb{R}^+ \to \mathbb{R}^+$ is defined as follows*

$$|a| = \begin{cases} a & \text{if } a \geq 1 \\ \frac{1}{a} & \text{if } a < 1 \end{cases}$$

*Then $((\mathbb{R}^+)^n, d)$ is a multiplicative metric space.*

**Definition 2.3** ([2]). *Let $(X, d)$ be a multiplicative metric space, $x \in X$ and $\epsilon > 1$. Define the following set $B_\epsilon(x) = \{y \in X : d(x, y) < \epsilon\}$, which is called the multiplicative open ball of radius $\epsilon$ with center $x$. Similarly, one can describe the multiplicative closed ball as follows: $\overline{B}_\epsilon(x) = \{y \in X : d(x, y) \leq \epsilon\}$.*

**Definition 2.4** ([2]). *Let $(x, d)$ be a multiplicative metric space, $\{x_n\}$ be a sequence in $X$ and $x \in X$. If, for any multiplicative open ball $B_\epsilon(x)$, there exists a natural number $N$ such that, for all $n \geq N$, $x_n \in B_\epsilon(x)$, then the sequence $\{x_n\}$ is said to be multiplicative convergent to the point $x$, which is denoted by $x_n \to x$ as $n \to \infty$.*

**Lemma 2.5** ([2]). *Let $(X, d)$ be a multiplicative metric space, $\{x_n\}$ be a sequence in $X$ and $x \in X$. Then $x_n \to x$ as $n \to \infty$ if and only if $d(x_n, x) \to 1$ as $x_n \to x$ as $n \to \infty$.*

**Lemma 2.6** ([2]). *Let $(X, d)$ be a multiplicative metric space and $\{x_n\}$ be a sequence in $X$. If the sequence $\{x_n\}$ is multiplicative convergent, then the multiplicative limit point is unique.*

**Definition 2.7** ([2]). *Let $(X, d)$ be a multiplicative metric space and $\{x_n\}$ be a sequence in $X$. The sequence $\{x_n\}$ is called a multiplicative Cauchy sequence if, for all $\varepsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_m, x_n) < \varepsilon$ for all $m, n \geq N$.*

**Lemma 2.8** ([2]). *Let $(X, d)$ be a multiplicative metric space and $\{x_n\}$ be a sequence in $X$. Then $\{x_n\}$ is a multiplicative Cauchy sequence if and only if $d(x_m, x_n) \to 1$ as $m, n \to \infty$.*

**Theorem 2.9** ([2]). *Let $(X, d)$ be a multiplicative metric space. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in $X$ such that $x_n \to x$ and $y_n \to y \in X$ as $n \to \infty$. Then $d(x_n, y_n) \to (x, y)$ as $n \to \infty$.*

**Definition 2.10** ([2]). *Let $(X, d)$ be a multiplicative metric space and $A \subseteq X$. Then, we call $x \in A$, a multiplicative interior point of $A$ if there exists an $\varepsilon > 1$ such that $B_\epsilon(x) \subseteq A$. The collection of all interior points of $A$ is called multiplicative interior of $A$ and denoted by int$(A)$.*

**Definition 2.11** ([2]). *Let $(X, d)$ be a multiplicative metric space and $A \subseteq X$. If every point $A$ is a multiplicative interior point of $A$, i.e, $A = int(A)$, then $A$ is called a multiplicative open set.*

**Definition 2.12** ([2]). *Let $(X, d)$ be a multiplicative metric space A subset $S \subseteq X$ is called multiplicative closed in $(X, d)$ if $S$ contains all of its multiplicative limit points.*

**Theorem 2.13** ([2]). *Let $(X, d)$ be a multiplicative metric space. A subset $S \subseteq X$ is multiplicative closed if and only if $XnS$, the complement of $S$, is multiplicative open.*

**Theorem 2.14** ([2]). *Let $(X, d)$ be a multiplicative metric space and $S \subseteq X$. Then the set $S$ is multiplicative closed if and only if every multiplicative convergent sequence in $S$ has a multiplicative limit point that belongs to $S$.*

**Theorem 2.15** ([2]). *Let $(X, d)$ be a multiplicative metric space and $S \subseteq X$. Then $(S, d)$ is complete if and only if $S$ is multiplicative closed.*

**Theorem 2.16** ([2]). *Let $(X, d_x)$ and $(Y, d_y)$ be two multiplicative metric spaces, $f : X \to Y$ be a mapping and $\{x_n\}$ be any sequence in $X$. Then $f$ is multiplicative continuous at the point $x \in X$ if and only if $f(x_n) \to f(x)$ as $n \to \infty$ for every sequence $\{x_n\}$ with $x_n \to x$ as $n \to \infty$.*

Next we give the notations $A_0$, $B_0$ and $d(A, B)$ for nonempty subsets A and B of a multiplicative metric space $(X, d)$ in the same sense in metric spaces. Let A and B be non-empty subsets of a multiplicative metric space $(X, d)$, we recall the following notations and notions that will be used in what follows.

$$d(A, B) = \inf\{d(x, y) : x \in A \text{ and } \ y \in B\}$$

$$A_0 = \{x \in A : d(x, y) = d(A, B) \text{ for some } \ y \in B\},$$

$$B_0 = \{y \in B : d(x, y) = d(A, B) \text{ for some } \ x \in A\}$$

**Definition 2.17** ([7]). *Let $A$ be non-empty subset of a multiplicative metric space $(X, d)$. A mapping $g : A \to A$ is said to be isometry if $d(gx, gy) = d(x, y)$ for all $x, y \in A$.*

**Definition 2.18** ([7]). *Let $A$ and $B$ be non-empty subsets of a multiplicative metric space $(X, d)$. A point $x \in A$ is called a best proximity point of a mapping $T : A \to B$ if it satisfies the condition that $d(x, Tx) = d(A, B)$.*

**Definition 2.19** ([7]). *A subset $A$ of a multiplicative metric space $(X, d)$ is said to be approximately compact with respect to $B$, if every sequence $\{x_n\}$ in $A$ satisfies the condition that $d(y, x_n) \to d(y, A)$ as $n \to \infty$ for some $y \in B$ has a convergent subsequence.*

**Definition 2.20** ([6]). *Let $(X, d)$ be a metric space and $A$ and $B$ be two non-empty subsets of $X$. Then $T : A \to B$ is said to be a rational proximal contraction of the first kind if there exists non-negative real numbers $\alpha$, $\beta$, $\gamma$, $\delta$ with $\alpha+\beta+2\gamma+2\delta < 1$ such that the conditions $d(u_1, Tx_1) = d(A, B)$ and $d(u_2, Tx_2) = d(A, B)$ imply that*

$$d(u_1, u_2) \leq \alpha d(x_1, x_2) + \frac{\beta\,[1 + d(x_1, u_1)]\, d(x_2, u_2)}{1 + d(x_1, x_2)} + \gamma\,[d(x_1, u_1) + d(x_2, u_2)] + \delta\,[d(x_1, u_2) + d(x_2, u_1)]$$

*for all $u_1, u_2, x_1, x_2 \in A$.*

**Definition 2.21** ([7]). *Let $A$ and $B$ be nonempty subsets of a multiplicative metric space $(X, d)$. A mapping $T : A \to B$ is called multiplicative proximal contraction if there exists $\alpha \in [0, 1)$ satisfying the following condition.*

$$\left.\begin{array}{l} d(u, Tx) = d(A, B) \\ d(v, Ty) = d(A, B) \end{array}\right\} \Rightarrow d(u, v) \leq d(x, y)^{\alpha}$$

*for all $u, v, x, y \in A$.*

**Theorem 2.22** ([7])**.** *Let $(X, d)$ be a complete multiplicative metric space and $A$, $B$ be nonempty closed subsets of $X$ such that $A_0$ and $B_0$ are nonempty and $B$ is approximately compact with respect to $A$. Suppose that $T : A \to B$ and $g : A \to A$ satisfy the following conditions.*

*(1). $T$ is a multiplicative proximal contraction*

*(2). $T(A_0) \subseteq B_0$*

*(3). $g$ is an isometry*

*(4). $A_0 \subseteq g(A_0)$*

*Then there exists a unique point $x^* \in A$ such that $d(gx^*, Tx^*) = d(A, B)$. Moreover for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ is defined by $d(gx_n, Tx_{n-1}) = d(A, B)$ converges to the element $x^*$.*

# 3. Main Results

In this section, we introduce the new class of multiplicative modified rational proximal contraction mappings of the first kind.

**Definition 3.1.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B$ be a non-empty subsets of $X$. Then $T : A \to B$ is called a multiplicative modified rational proximal contraction of the first kind if there exists non-negative real numbers $\alpha$, $\beta$, $\gamma$, $\delta$ with $\alpha + \beta + 2\gamma + 2\delta < 1$ such that the conditions $d(u_1, Tx_1) = d(A, B)$ and $d(u_2, Tx_2) = d(A, B)$. This implies*

$$d(u_1, u_2) \leq \frac{d(x_1, x_2)^{\alpha}.[d\left(x_1, u_1\right).d(x_2, u_2)]^{\beta+\gamma}.[d(x_1, u_2).d(x_2, u_1)]^{\delta}}{d(x_1, x_2)}$$

*for all $x_1, x_2, u_1, u_2 \in A$.*

**Theorem 3.2.** *Let $(X, d)$ be a complete multiplicative metric space and $A$, $B$ be nonempty closed subsets of $X$ such that $A_0$ and $B_0$ are nonempty and $B$ is approximately compact with respect to $A$. Suppose that $T : A \to B$ and $g : A \to A$ satisfy the following conditions.*

*(1). $T$ is a multiplicative modified rational proximal contraction of the first kind*

*(2). $T(A_0) \subseteq B_0$*

*(3). $g$ is an isometry*

*(4). $A_0 \subseteq g(A_0)$*

*Then there exists a unique point $x^* \in A$ such that $d(gx^*, Tx^*) = d(A, B)$. Moreover for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$ is defined by $d(gx_n, Tx_{n-1}) = d(A, B)$ converges to the element $x^*$.*

*Proof.* Let $x_0$ be a fixed point in $A_0$. In view of the fact that $T(A_0) \subseteq B_0$ and $A_0 \subseteq g(A_0)$, there exists an element $x_1 \in A_0$ such that $d(gx_1, Tx_0) = d(A, B)$. Since $T(A_0) \subseteq B_0$ and $A_0 \subseteq g(A_0)$, there exists an element $x_2 \in A_0$ such that $d(gx_2, Tx_1) = d(A, B)$. Since T is a multiplicative modified rational proximal contraction and g is isometry, we get

$$\begin{aligned} d(x_1, x_2) &= d(gx_1, gx_2) \\ &\leq \frac{d\left(x_0, x_1\right)^{\alpha}.d(x_0, gx_1)^{\beta+\gamma}.d(x_1, gx_2)^{\beta+\gamma}d(x_0, gx_2)^{\delta}d(x_1, gx_1)^{\delta}}{d(x_1, x_2)} \end{aligned}$$

$$
\begin{aligned}
&\leq \frac{d\left(x_0,x_1\right)^{\alpha}.d(x_0,x_1)^{\beta+\gamma}.d(x_1,x_2)^{\beta+\gamma}d(x_0,x_2)^{\delta}d(x_1,x_1)^{\delta}}{d(x_1,x_2)} \\
&\leq \frac{d\left(x_0,x_1\right)^{\alpha}.d(x_0,x_1)^{\beta+\gamma}.d(x_1,x_2)^{\beta+\gamma}d(x_0,x_2)^{\delta}}{d(x_1,x_2)} \\
&\leq \frac{d\left(x_0,x_1\right)^{\alpha}.d(x_0,x_1)^{\beta+\gamma}.d(x_1,x_2)^{\beta+\gamma}d(x_0,x_1)^{\delta}d(x_1,x_2)^{\delta}}{d(x_1,x_2)} \\
d(x_1,x_2)^{2-\beta-\gamma-\delta} &\leq d(x_0,x_1)^{\alpha+\beta+\gamma+\delta} \\
d(x_1,x_2) &\leq d(x_0,x_1)^{\frac{\alpha+\beta+\gamma+\delta}{2-\beta-\gamma-\delta}} \\
d(x_1,x_2) &\leq d(x_0,x_1)^{k} \qquad (1)
\end{aligned}
$$

where $k=\frac{\alpha+\beta+\gamma+\delta}{2-\beta-\gamma-\delta}<1$. Again, since $T(A_0)\subseteq B_0$ and $A_0\subseteq g(A_0)$, there exists an element $x_3\in A_0$ such that $d(gx_3,Tx_2)=d(A,B)$. It follows from T is a multiplicative proximal contraction, g is an isometry and (1) that

$$
\begin{aligned}
d(x_3,x_2) &= d(gx_3,gx_2) \\
&\leq d(x_1,x_2)^{K} \\
&\leq d(x_1,x_0)^{K^2}
\end{aligned}
$$

By the same method, for each $n\in N$, we can find $x_n,x_{n+1}\in A_0$ such that

$$
\begin{aligned}
& d(gx_n,Tx_{n-1})=d(A,B) \\
\text{and}\quad & d(gx_{n+1},Tx_n)=d(A,B) \qquad (2)
\end{aligned}
$$

This implies that

$$
\begin{aligned}
d(x_{n+1},x_n) &= d(gx_{n+1},gx_n) \\
&\leq d(x_n,x_{n-1})^{K} \\
&\leq d(x_{n-1},x_{n-2})^{K^2} \\
&\vdots \\
&\leq d(x_1,x_0)^{K^n}
\end{aligned}
$$

for all $n\in\mathbb{N}$. Next we will show that $\{x_n\}$ is a Cauchy sequence. Let $m,n\in\mathbb{N}$ with $m>n$, then we get,

$$
\begin{aligned}
d(x_m,x_n) &\leq d(x_m,x_{m-1}).d(x_m,x_{m-2})\ldots d(x_{n+1},x_n) \\
&\leq d(x_1,x_0)^{K^{m-1}}.d(x_1,x_0)^{K^{m-2}}\ldots d(x_1,x_0)^{K^n} \\
&\leq d(x_1,x_0)^{K^{m-1}+K^{m-2}+\cdots+K^n} \\
d(x_m,x_n) &\leq d(x_1,x_0)^{\frac{K^n}{1-K}}
\end{aligned}
$$

Taking $m,n\to\infty$, in the above inequality, we obtain that $d(x_m,x_n)\to 1$. Hence $\{x_n\}$ is a Cauchy sequence. Since A is closed subsets of complete multiplicative metric space X, then the sequence $\{x_n\}$ converges to some element $x\in A$. Notice that,

$$
d(gx,B)\leq d(gx,Tx_n)
$$

$$\leq d(gx, gx_{n+1}).d(gx_{n+1}, Tx_n)$$
$$= d(gx, gx_{n+1}).d(A, B)$$
$$\leq d(gx, gx_{n+1}).d(gx, B)$$

for all $n \in N$. Since g is continuous and the sequence $\{x_n\}$ converges, then the sequence $\{gx_n\}$ converges to $gx$, that is $d(gx, gx_n) \to 1$ as $n \to \infty$. Therefore, $d(gx, Tx_n) \to d(gx, B)$ as $n \to \infty$. Since B is approximately compact with respect to A, then there exists subsequence $\{Tx_{n_k}\}$ of sequence $\{Tx_n\}$ such that converging to some element $u \in B$. Further, for each $k \in \mathbb{N}$, we have

$$d(A, B) \leq d(gx, u)$$
$$\leq d(gx, gx_{n_k+1}).d(gx_{n_k+1}, Tx_{n_k}).d(Tx_{n_k}, u) \quad (3)$$
$$= d(gx, gx_{n_k+1}).d(A, B).d(Tx_{n_k}, u) \quad (4)$$

Letting $K \to \infty$ in (3), we get $d(gx, u) = d(A, B)$ and hence $gx \in A_0$. From the fact that $A_0 \subseteq g(A_0)$, then $gx = gz$ for some $z \in A_0$. By the isometry of g, we get $d(x, z) = d(gx, gz) = 1$ and thus $x = z$, that is x is an element of $A_0$. Since $T(A_0) \subseteq A_0$, then there exists $x^* \in A$ such that

$$d(x^*, Tx) = d(A, B) \quad (5)$$

From (2), (5) and the multiplicative modified rational proximal contractive condition of T, we have $d(gx_{n+1}, x^*) \leq d(x_n, x)^K$ for all $n \in \mathbb{N}$. This yields that

$$\lim_{n\to\infty} d(gx_{n+1}, x^*) \leq \lim_{n\to\infty} d(x_n, x)^K = 1^K = 1$$

This shows that the sequence $\{gx_n\}$ converges to $x^*$. By Lemma 2.6, we get that $gx = x^*$. Hence $d(gx, Tx) = d(x^*, Tx) = d(A, B)$. Next, to prove the uniqueness, suppose that there exists $x^* \in A$ with $x \neq x^*$ and $d(gx^*, Tx^*) = d(A, B)$. Since g is an isometry and T is a multiplicative modified rational proximal contraction, it follows that $d(x, x^*) \subset d(gx, gx^*) \leq d(x, x^*)^K$, which is a contradiction. Therefore, we get $x = x^*$. This completes the proof. □

**Corollary 3.3.** *Let $(X, d)$ be a complete multiplicative modified rational multiplicative space and $A$, $B$ be a nonempty closed subsets of $X$ such that $A_0$ and $B_0$ nonempty and $B$ is approximately compact with respect to $A$. Let $T : A \to B$ satisfies the following conditions*

*(1). $T$ is a multiplicative modified rational proximal contraction of the first kind*

*(2). $T(A_0) \subseteq B_0$.*

*Then there exists a unique point $x^* \in A$ such that $d(x^*, Tx^*) = d(A, B)$. Moreover, for any fixed $x \in A_0$, the sequence $\{x_n\}$ is defined by $d(x_n, Tx_{n-1}) = d(A, B)$ converges to the element $x^*$.*

**Example 3.4.** *Let $x = \mathbb{R}^2$. Define the mapping $d : X \times X \to \mathbb{R}$ by $d((x_1, x_2), (y_1, y_2)) = e^{|x_1-y_1|+|x_2-y_2|}$ for all $(x_1, x_2), (y_1, y_2) \in X$. It is easy to see that $(x, d)$ is a complete multiplicative metric space. Let $A = \{(0, x) : x \in \mathbb{R}\}$ and $B = \{(1, y) : y \in \mathbb{R}\}$. Then $d(A, B) = e$, $A_0 = A$; $B_0 = B$ and $B$ is approximately compact with respect to $A$. Define two mappings $T : A \to B$ and $g : A \to A$ as follows : $T(0, x) = \left(1, \frac{x}{2}\right)$ and $g(0, x) = (0, -x)$ for all $(0, x) \in A$. For all $(0, x), (0, y) \in A$, we get*

$$d(g(0, x), g(0, y)) = d((0, -x), (0, -y))$$

$$= e^{|-x+y|} = e^{|x+y|} = d((0,x),(0,y))$$

*This implies that g is an isometry. Next, we show that T is a multiplicative modified rational proximal contraction with* $\alpha = \frac{1}{5}$, $\beta = \frac{1}{5}$, $\gamma = \delta = 0$. *Let* $(0,u)$, $(0,v)$, $(0,x)$ *and* $(0,y)$ *be elements in A satisfying* $d(g(0,u),T(0,x)) = d(A,B) = e$, $d(g(0,v),T(0,y)) = d(A,B) = e$. *Then we have* $u = \frac{x}{2}$ *and* $v = \frac{-y}{2}$ *and hence*

$$\begin{aligned} d(g(0,u),T(0,v)) &= d((0,-u),(0,-v)) \\ &= d\left(\left(0,\frac{x}{2}\right),\left(0,\frac{y}{2}\right)\right) \\ &= \left(e^{|x-y|}\right)^{\frac{2}{9}} \\ &= (d(0,x),(0,y))^{\frac{2}{9}}. \end{aligned}$$

*This implies that T is a multiplicative rational proximal contraction with* $K = \frac{2}{9}$. *Now all hypothesis in Theorem 3.2 holds and so there exists a unique point* $x^*$ *in A such that* $d(gx^*,Tx^*) = d(A,B)$. *In this case,* $x^* = (0,0) \in A$ *is a unique element such that*

$$\begin{aligned} d(gx^*,Tx^*) &= d(g(0,0),T(0,0)) \\ &= d((0,0),(1,0)) = e = d(A,B). \end{aligned}$$

## 4. Conclusion

Best proximity point results for multiplicative modified rational proximal contraction mapping in multiplicative metric spaces were investigated under some suitable conditions. These results have been reached by best proximity point theorems for multiplicative modified rational proximal contraction mappings on multiplicative metric spaces.

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