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Ranking L-R Type Generalized Trapezoidal Fuzzy Numbers

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Abstract: L-R type fuzzy numbers are quite common in practical life situations involving the triangular fuzzy number, the Gaussian fuzzy number, Cauchy fuzzy number, trapezoidal fuzzy numbers, generalized trapezoidal fuzzy numbers etc. Ranking of these fuzzy numbers plays a vital role in dealing with real life situations with imprecise information. This study presents an average approach for ranking L-R type generalized trapezoidal type fuzzy numbers. Compared with other fuzzy ranking techniques, the ranking proposed in this paper is easily interpreted, flexible and invariant with translating real numbers.

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1. Introduction

In research fields such as decision making, optimization, approximate reasoning, control and data-mining etc., many problems are usually described as mathematical relations with vague information. Decision makers are normally faced with lack of alternatives in an uncertain environment. Imprecise evaluations may be characterized to incomplete, unquantifiable, nonobtainable information and partial ignorance. To model the imprecision in these situations, fuzzy set theory introduced by Zadeh [18] can be utilized.

Many fuzzy ranking techniques have been proposed since 1975. In order to rank different types of fuzzy numbers, in 1975, Zadeh [18], presented the extension principle for fuzzy basic operations involving addition, subtraction, multiplication, division and so on. Kaufmann and Gupta [13], initiated an interval method for triangular and trapezoidal fuzzy numbers based on $\alpha-cut$ method. To overcome few pitfalls of above method some researchers proposed few novel approaches [9, 14, 15]. Since the arithmetic based on above approaches is difficult to evaluate some approximation methods were introduced. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [1], Chen and Hwang [2] and still receives much attention by many researchers in recent years. Almost each method has its own short comings in some aspect, such as difficulty of interpretation, inconsistency with human intuition and indiscrimination. Therefore, it so obvious that there is no best method for comparing fuzzy numbers, where different methods may satisfy different desirable criteria.

To tackle this problem in the fuzzy number literature, there are many proposals for ordering L-R (left and right) type fuzzy

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numbers. For instance, Dubois and Prade [5], extended usual algebraic operations on real numbers to fuzzy numbers, and suggested a standard approximation to fuzzy arithmetic with efficient computation. But frequent use of approximation for multiplication may lead to wrong results. Consequently Giachetti and young [8] discussed the error of the standard approximation and developed a new approximation for triangular and trapezoidal fuzzy numbers to reduce the error. As such many researchers [12] proposed a form of using six parameters to define fuzzy numbers, [10] used piecewise monotonic interpolations to approximate and represent a fuzzy number, [4] Chutia et al. developed a generalised method to find the membership function for functions of triangular fuzzy numbers. Eslamipoor et al. [7] and Haji et al. [11] proposed different methods to rank fuzzy numbers using distance method.

The L-R fuzzy number initialized by Dubois and prade [6], dealt with triangular fuzzy numbers. An L-R fuzzy number can be represented by its mean value, left and right spreads and shape functions. Wang and Kuo [17] proposed an alternative operation of fuzzy arithmetic on L-R fuzzy numbers by three parametrs. Sorini and Stefanini [16] suggested a parametrization for L-R fuzzy numbers which is used to model the shapes of the membership fuctions and obtain operators for the fuzzy arithmetic operations. Chou [3] presented an inverse function arithmetic principle on triangular fuzzy numbers.

In this paper, we focus on L-R type generalized trapezoidal fuzzy numbers. The mean value of L-R type generalized trapezoidal fuzzy numbers is defined for ranking of these fuzzy numbers. The L-R type generalized trapezoidal fuzzy numbers are converted into a crisp data by the above ranking technique and utilized for optimization process.

The paper is organized as follows: Section 2 represents a brief review of basic definition of fuzzy theory. Arithmetic operations of generalized L-R fuzzy numbers are discussed in section 3. In section 4, the ranking method is proposed. Section 5, presents an illustration of the proposed method with the comparison table. In section 6, the conclusion and future discussions are revealed.

2. Basic Definition

Definition 2.1 (Fuzzy Set). *A Fuzzy set $\tilde{A}$ characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval* $[0,1]$. *(i.e)* $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$, *here* $\mu_{\tilde{A}} : X \to [0,1]$ *is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ called the membership function value of $x \in X$ in the fuzzy set $\tilde{A}$. These membership grades are often represented by real numbers ranging from* $[0,1]$.

Definition 2.2 (Fuzzy Set). *A fuzzy set $\tilde{A} = (a,b,c)$ defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:*

(1). $\mu_{\tilde{A}} : R \to [0,1]$ *is continuous.*

(2). $\mu_{\tilde{A}}(x) = 0 forall x \in (-\infty, a] \cup [d, \infty)$.

(3). $\mu_{\tilde{A}}(x)$ *strictly increasing on* $[a,b]$ *and strictly decreasing on* $[c,d]$.

(4). $\mu_{\tilde{A}}(x) = 1$ *for all* $x \in [b,c]$, *where* a < b < c < d.

Definition 2.3 (Trapezoidal Fuzzy Number). *A fuzzy number $\tilde{A} = (a,b,c,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{(x-d)}{(c-d)}, & c \le x \le d \end{cases}$$

Definition 2.4 (Generalized Fuzzy Number). *A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be a generalized fuzzy number is said to be generalized fuzzy number if its membership function has the following characteristics:*

(1). $\mu_{\tilde{A}} : R \to [0, \omega]$ *is continuous.*

(2). $\mu_{\tilde{A}}(x) = 0 for all x \in (-\infty, a] \cup [d, \infty)$.

(3). $\mu_{\tilde{A}}(x)$ *strictly increasing on* $[a, b]$ *and strictly decreasing on* $[c, d]$.

(4). $\mu_{\tilde{A}}(x) = \omega$ *for all* $x \in [b, c]$, *where* $\mathrm{a} < \omega \leq 1$.

Definition 2.5 ([5]). *A shape function* $L\,(or R)$ *is a decreasing function from* $R^+ \to [0, 1]$ *such that*

(1). $L(0) = 1$;

(2). $L(x) < 1, \forall x > 0$;

(3). $L(x) > 0, \forall x < 1$;

(4). $L(1) = 0[or L(x) > 0, \forall x and L(+\infty) = 0$.

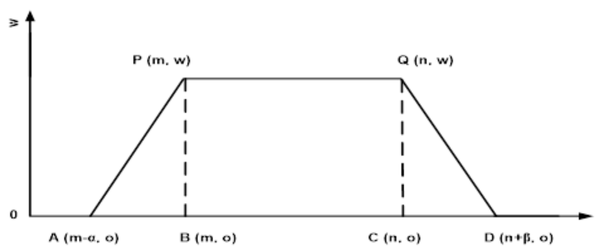
Definition 2.6 (L-R type Generalized Trapezoidal Fuzzy Number). *A fuzzy number $\tilde{A}$ is said to be L-R type if there exists two decreasing functions* $L, R : [0, +\infty] \to [0, 1]$ *with* $L(0) = R(0) = 1, \lim_{\omega \to +\infty} L(\omega) = \lim_{\omega \to +\infty} R(\omega) = 0$ *and positive real numbers* $a_m \geq 0, \alpha > 0, \beta > 0$ *such that*

$$\mu_{\tilde{A}}(\omega) = \begin{cases} L\left(\frac{a_m - \omega}{\alpha}\right), & for \omega \leq a_m, \\ R\left(\frac{\omega - a_m}{\beta}\right), & for \omega \geq a_m, \end{cases}$$

where a_m *is called the centre of* $\tilde{A}$ *and* $\alpha = a_m - a_l$ *nand* $\beta = a_r - a_m$ *are called the left and right propagations, respectively. If* $\alpha = \beta, \widetilde{A}$ *called a symmetric fuzzy number; it is important to stress that for a symmetric membership function, the equality* $L\left(\frac{a_m - \omega}{\alpha}\right) = R\left(\frac{\omega - a_m}{\beta}\right)$ *hlods for* ωR. *If L and R are segments that starts at points* $(a_l, 0)$ *and* $(a_r, 0)$, *respectively, and end at* $(a_m, 1)$, *then we say that $\tilde{A}$ is a triangular fuzzy number (or) A fuzzy number* $\tilde{A} = (m, n, \alpha, \beta; \omega)_{LR}$ *is said to be the L-R type generalized trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega L\left(\frac{m - x}{\alpha}\right), & x \leq m, \alpha > 0, \\ \omega R\left(\frac{x - n}{\beta}\right), & x \geq n, \beta > 0, \\ \omega, & otherwise. \end{cases}$$

where L and R are reference functions.



3. Arithmetic Operations on Generalized L-R Fuzzy Numbers

In this section, the formulas for the elementary operations (addition, subtraction, multiplication) between L-R type generalized trapezoidal fuzzy numbers will be presented. Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; \omega_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; \omega_2)_{LR}$ be any two L-R type generalized trapezoidal fuzzy numbers then

(1). $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2; \min(\omega_1, \omega_2))_{LR}$.

(2). $\tilde{A}_1 \ominus \tilde{A}_2 = (m_1 - \beta_2, n_1 - \alpha_2, \alpha_1 - n_2, \beta_1 - m_2; \min(\omega_1, \omega_2))_{LR}$.

(3). $\lambda\tilde{A}_1 = \begin{cases} (\lambda m_1, \lambda n_1, \lambda\alpha_1, \lambda\beta_1; \omega_1)_{LR} & \lambda > 0 \\ (\lambda\beta_1, \lambda\alpha_1, \lambda n_1, \lambda m_1; \omega_1)_{RL} & \lambda < 0. \end{cases}$

4. Proposed Approach

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function.

(1). $\tilde{A} > \tilde{B}$ iff $R\left(\tilde{A}\right) > R\left(\tilde{B}\right)$.

(2). $\tilde{A} < \tilde{B}$ iff $R\left(\tilde{A}\right) < R\left(\tilde{B}\right)$.

(3). $\tilde{A} \sim \tilde{B}$ iff $R\left(\tilde{A}\right) = R\left(\tilde{B}\right)$ then $\begin{cases} \tilde{A} < \tilde{B}, & if \;\; \omega_1 < \omega_2 \\ \tilde{A} > \tilde{B}, & if \;\; \omega_1 > \omega_2 \\ \tilde{A} = \tilde{B}, & if \;\; \omega_1 = \omega_2 \end{cases}$.

Remark 4.1.

(1). $\tilde{A} > \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} > \tilde{B} \oplus \tilde{C}$.

(2). $\tilde{A} > \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{C} > \tilde{B} \ominus \tilde{C}$.

(3). $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$.

(4). $\tilde{A} > \tilde{B}, \tilde{C} > \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{D}$.

Definition 4.2 (Ranking Function)**.** *Let $\tilde{A} = (m, n, \alpha, \beta; w)_{LR}$ be the L-R type generalized trapezoidal fuzzy numbers then we define the ranking function as follow:*

$$R\left(\tilde{A}\right) = \frac{w}{2}\frac{(m - \alpha + m + n + n + \beta)}{6} = \frac{w\,(2m + 2n - \alpha + \beta)}{12}$$

5. Numerical example

Consider an assignment problem of assigning 3 jobs to 3 machines, whose costs is considered as generalized L-R type generalized trapezoidal fuzzy numbers in 000 dollars. The problem is to find the optimal allocation in an efficient way.

Jobs/Machines	A	B	C
1	(2, 3, 1, 1; 0.7)	(3, 4, 1, 2; 0.5)	(4, 5, 2, 3; 0.8)
2	(2, 6, 0, 4; 0.4)	(5, 7, 3, 4; 0.3)	(2, 3, 1, 1; 0.7)
3	(3, 4, 1, 2; 0.5)	(2, 6, 0, 4; 0.4)	(7, 8, 3, 5; 0.2)

Solution: The given generalized L-R fuzzy cost table is balanced one. Using ranking technique, the rank of L-R generalized trapezoidal fuzzy cost matrix is obtained.

Jobs/Machines	A	B	C
1	0.5833	0.625	1.2667
2	0.6667	0.625	0.5833
3	0.625	0.6667	0.5333

Table 1. **Rank Table**

Proceeding by Hungarian method the optimum allocations are:

Jobs/Machines	A	B	C
1	0	0	0.6834
2	0.0834	0	0
3	0.0917	0.0917	0

The assignment is $1 \to A$, $2 \to B$, $3 \to C$. Hence (2, 3, 1, 1; 0.7), (5, 7, 3, 4; 0.3) and (7, 8, 3, 5; 0.2) are L-R generalized trapezoidal Fuzzy assignments. By L-R generalized trapezoidal fuzzy addition, the minimum cost is: (2, 3, 1, 1; 0.7) + (5, 7, 3, 4; 0.3) + (7, 8, 3, 5; 0.2) = (14, 18, 7, 10; 0.2). Therefore, by ranking method the minimum cost is: 1.1167.

Researchers	Ranking	Optimum Cost
Amit Kumar, Pushpinder Singh,	$\alpha = 0$	0.8
Parmpreet Kaur	$\alpha = 0.5$	1.3
and Amarpreer Kaur	$\alpha = 1$	1.3
Y.L.P.Thorani and	Centroid of Centroids	1.2307
N. Ravi Shankar	using area	
	Mode	1.3
	Spread	2.8
Proposed Method	Average	1.1167

Table 2. **Comparison Table**

6. Conclusion

This paper proposes a new ranking method for ranking L-R type generalized trapezoidal fuzzy numbers. The proposed method is easier in calculation and is efficient for a moderate and optimistic decision maker. It gives a proper ordering of fuzzy numbers overcoming the pitfalls of previous methods. For future research, the proposed method can be applied in real life situations involving decision making and optimization problems.

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