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Decision Making On Triangular Type-2 Fuzzy Soft Set

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Abstract: Molodtsov.D initiated the concept of soft set theory, which has become an effective mathematical tool for dealing with uncertainty. Especially,fuzzy soft set theory is used to solve decision making problems. In this work, the inter relationship between similarity measure, inclusion measure and entropy measure is obtained using triangular type-2 fuzzy soft sets and its complement. A decision making problem is solved by this relationship.

MSC: 05C72.

Keywords: Triangular type-2 fuzzy soft sets, complement of triangular type-2 fuzzy soft sets, similarity measure, inclusion measure, entropy measure.

1. Introduction

The fuzzy set theory and type-2 fuzzy set theory was introduced by L.A.Zadeh in 1965 [7]. A type-2 fuzzy set is characterized by a fuzzy membership function, which can provide us with more degrees of freedom to represent the uncertainty and vagueness of the real world. The type-2 fuzzy set can be used to represent the fuzziness of evaluation of parameters directly. Soft set theory, originally proposed by Molodtsov.D [5], has become an effective mathematical tool to deal with uncertainty. P.K.Maji et al [4] first defined the soft sets and its operation into the decision making problem. The soft set and its existing extensions are used to deal with the parameters involving uncertain words and linguistic terms. Hence, it is necessary to extend soft set theory using type-2 fuzzy sets.

Zhiming Zhang and Shouhua Zhang [9, 10] first introduced the concept of type-2 fuzzy soft sets. Yue Yang et al [6] gives the concept of Interval-valued triangular fuzzy soft sets and its method of dynamic decision making. It can be extended to triangular type-2 fuzzy soft set into decision making. We propose the notion of triangular type-2 fuzzy soft sets and its complement. The inter relationship between similarity measure, inclusion measure and entropy measure are developed by triangular type-2 fuzzy soft sets. We have solved a decision making problem using this relationship.

This paper is organized as follows: In section 2, some basic definitions and few operations of triangular type-2 fuzzy soft sets are given. In section 3, the inter relationship between similarity measure, inclusion measure and entropy measure of triangular type-2 fuzzy soft set is defined. In section 4, a decision making problem is solved through this relationship.

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2. Basic Definitions

Definition 2.1. *A soft set (P, E) is a set of all parameterized family of subsets of the non-empty universe X. For every $\epsilon \in E$ there exists $P(\epsilon)$ such that $P : E \to \rho(X)$, where $\rho(X)$ is a power set of X.*

Definition 2.2. *A fuzzy number $A = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by*

$$\mu_A(X) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leqslant x \leqslant b \\ 1, & b = c \\ \frac{(x-c)}{(b-c)}, & b \leqslant x \leqslant c \end{cases}$$

where $a, b, c, d \in R$. A set with triangular fuzzy numbers is called triangular fuzzy set.

Definition 2.3. *Let X be a non-empty finite set, which is referred as the universal set. A type-2 fuzzy set A,is characterized by a type-2 membership function $\mu_A(x, u) : X \times I \to I$ where $x \in X, I = [0, 1]$ and $u \in J_x \subseteq I$ that is $A = \{((x, u); \mu_A(x, u))/x \in X, u \in J_x\}$ where $0 \leqslant \mu_A(x, u) \leqslant 1$. A can also be expressed as*

$$A = \int_{x\in X} \int_{u\in J_x} \frac{\mu_A(x, u)}{(x, u)} = \frac{p_x(u)/u}{x}; J_x \subseteq I$$

where $p_x(u) = \mu_A(x, u)$. The type-2 triangular number is defined by triangular membership function and is denoted by $A = ([a_1, a_2, a_3], [b_1, b_2, b_3], [c_1, c_2, c_3])$. A set with type-2 triangular fuzzy numbers is called triangular type-2 fuzzy set. The class of all triangular type-2 fuzzy set of the universe X is denoted by $P_{TrT2}(X)$.

Definition 2.4. *A triangular type-2 fuzzy soft set $(\mathscr{P}, A)$ over the universal set X is a set of all parameterized family of subsets of the triangular type-2 fuzzy set A. For every $e \in A, A \subseteq E$ there exists a mapping $\mathscr{P}(\epsilon)$ such that $\mathscr{P} : A \to P_{TrT2}(A)$ where $P_{TrT2}(A)$ denotes the set of all triangular type-2 fuzzy set.*

Definition 2.5. *The union of two triangular type-2 fuzzy soft sets $(\mathscr{P}, A)$ and $(\mathscr{Q}, B)$ over the same universe X is a type-2 fuzzy soft set $(\mathscr{R}, C)$, where $C = A \cup B$ and for all $\epsilon \in C$,*

$$(\mathscr{R}, C) = \begin{cases} \mathscr{P}(\epsilon), & if \epsilon \in A - B \\ \mathscr{Q}(\epsilon), & if \epsilon \in B - A \\ \mathscr{P}(\epsilon) \vee \mathscr{Q}(\epsilon), & if \epsilon \in A \cap B \end{cases}$$

It is denoted by $(\mathscr{P}, A) \cup (\mathscr{Q}, B) = (\mathscr{R}, C)$.

Definition 2.6. *The intersection of two triangular type-2 fuzzy soft sets $(\mathscr{P}, A)$ and $(\mathscr{Q}, B)$ over the same universe X is a type-2 fuzzy soft set $(\mathscr{S}, C)$, where $C = A \cup B$ and for all $\epsilon \in C$,*

$$(\mathscr{S}, C) = \begin{cases} \mathscr{P}(\epsilon), & if \epsilon \in A - B \\ \mathscr{Q}(\epsilon), & if \epsilon \in B - A \\ \mathscr{P}(\epsilon) \wedge \mathscr{Q}(\epsilon), & if \epsilon \in A \cap B \end{cases}$$

It is denoted by $(\mathscr{P}, A) \cap (\mathscr{Q}, B) = (\mathscr{S}, C)$.

Definition 2.7. *The complement of a triangular type-2 fuzzy soft set* $(\mathscr{P}, A)$ *is denoted by* $(\mathscr{P}, A^c)$*, and is defined by* $(\mathscr{P}(\neg\epsilon), A) = (\mathscr{P}, A^c)$, $\mathscr{P}(\neg\epsilon)$*is a mapping given by* $\mathscr{P}(\neg\epsilon) : A^c \to P_{TrT2}(A^c)$ *where* $\mathscr{P}(\epsilon) = \mathscr{P}(\neg\epsilon)$ *for all* $\epsilon \in A^c$*.*

Definition 2.8. *For a triangular type-2 fuzzy number* $A = ([a_1, a_2, a_3], [b_1, b_2, b_3], [c_1, c_2, c_3])$*, the ranking function* R *is given by*

$$\boldsymbol{R}(a) = \frac{1}{16}(a_1 + 2a_2 + a_3 + 2b_1 + 4b_2 + 2b_3 + c_1 + 2c_2 + c_3)$$

3. Similarity Measure, Inclusion Measure, Entropy Measure and Their Relationship

3.1. Similarity Measure for type-2 Fuzzy Soft Set

Let $M_1((\mathscr{P}, A), (\mathscr{Q}, B))$ denotes the similarity measure between the two trapezoidal type-2 fuzzy soft sets $(\mathscr{P}, A)$ and $(\mathscr{Q}, B)$.Then we define

$$M_1((\mathscr{P}, A), (\mathscr{Q}, B)) = max_j \left\{ M_{1_j}((\mathscr{P}, A), (\mathscr{Q}, B)) = \begin{cases} \frac{\sum_{i=1}^{m}(\mathscr{P}(\epsilon_{ij}) \cap \mathscr{Q}(\epsilon_{ij}))}{\sum_{i=1}^{m}(\mathscr{P}(\epsilon_{ij}) \cup \mathscr{Q}(\epsilon_{ij}))} & \text{if } \epsilon \in A \cap B \\ 0 & \text{Otherwise} \end{cases} \right\}$$

where $j = 1$ to n $M_{1_j}((\mathscr{P}, A), (\mathscr{Q}, B))$ denotes the similarity measure between the ϵ_j-th approximations of $\mathscr{P}(\epsilon_{ij})$ and $\mathscr{Q}(\epsilon_{ij})$, $\mathscr{P}(\epsilon_{ij}) = \mathscr{P}(\epsilon_j)(x_i) \in I$ and $\mathscr{Q}(\epsilon_{ij}) = \mathscr{Q}(\epsilon_j)(x_i) \in I$.

3.2. Inclusion Measure for Type-2 Fuzzy Soft Set

Let $M_2((\mathscr{P}, A), (\mathscr{Q}, B))$ denotes the inclusion measure between the two triangular type-2 fuzzy soft sets $(\mathscr{P}, A)$ and $(\mathscr{Q}, B)$.Then we define

$$M_2((\mathscr{P}, A), (\mathscr{Q}, B)) = max_j \left\{ M_{2_j}((\mathscr{P}, A), (\mathscr{Q}, B)) = \begin{cases} \frac{\sum_{i=1}^{m}(\mathscr{P}(\epsilon_{ij}) \cap \mathscr{Q}(\epsilon_{ij}))}{\sum_{i=1}^{m}(\mathscr{P}(\epsilon_{ij})} & \text{if } \epsilon \in A \cap B \\ 0 & \text{Otherwise} \end{cases} \right\}$$

where $j = 1$ to n and $M_{2_j}((\mathscr{P}, A), (\mathscr{Q}, B))$ denotes the inclusion measure between the two ϵ_j-th approximations of $\mathscr{P}(\epsilon_{ij})$ and $\mathscr{Q}(\epsilon_{ij})$, $\mathscr{P}(\epsilon_{ij}) = \mathscr{P}(\epsilon_j)(x_i) \in I$ and $\mathscr{Q}(\epsilon_{ij}) = \mathscr{Q}(\epsilon_j)(x_i) \in I$.

3.3. Entropy Measure for Type-2 Fuzzy Soft Set

Let $M_3((\mathscr{P}, A), (\mathscr{Q}, B))$ denotes the entropy measure between a triangular type-2 fuzzy soft sets (P, A) and $(\mathscr{P}, A^C)$.Then we define

$$M_3((\mathscr{P}, A), (\mathscr{P}, A^C)) = max_j \begin{cases} M_{3_j}((\mathscr{P}, A), (\mathscr{P}, A^c)) \\ = \left\{ \frac{\sum_{i=1}^{m}(\mathscr{P}(\epsilon_{ij}) \cap \mathscr{P}(\epsilon_{ij}^c))}{\sum_{i=1}^{m}(\mathscr{P}(\epsilon_{ij}) \cup \mathscr{P}(\epsilon_{ij}^c))} ; j = 1, 2, 3, ...n \right\} \end{cases}$$

where $M_{3_j}((\mathscr{P}, A), (\mathscr{Q}, B))$ denotes the entropy measure between the two ϵ_j-th approximations of $\mathscr{P}(\epsilon_{ij})$ and $\mathscr{P}(\epsilon_{ij})^c$, $\mathscr{P}(\epsilon_{ij}) = \mathscr{P}(\epsilon_j)(x_i) \in I$ and $\mathscr{P}(\epsilon_{ij}^c) = \mathscr{P}(\epsilon_j^c)(x_i) \in I$.

3.4. Relationship Between Similarity Measure, Inclusion Measure and Entropy Measure

- The relationship between similarity Measure and inclusion measure is given by

$$M_1((\mathscr{P}, A), (\mathscr{Q}, B)) = min\{M_2((\mathscr{P}, A), (\mathscr{Q}, B)), M_2((\mathscr{Q}, A), (\mathscr{P}, B))\}$$

- The relationship between entropy measure and inclusion measure is given by

$$M_3((\mathscr{P},A),(\mathscr{P},A^c)) = M_2((\mathscr{P},A\cup A^c),(\mathscr{P},A\cap A^c))$$

- The relationship between Similarity Measure and entropy measure is given by

$$M_3((\mathscr{P},A),(\mathscr{P},A^c)) = M_1((\mathscr{P},A),(\mathscr{P},A^c))$$

4. Numerical Example

Suppose that a decision maker wants to buy a car. He choose two sets of four cars $\{c_1,c_2,c_3,c_4\}$. He consider three characteristics of the car as parameters, as follows:

- attractive color (h_1)
- new model (h_2) and
- engine capacity (h_3)

From the above infirmations the decision maker wants to choose a car with these three characteristic. Here we are going to choose a best car with these two sets of cars. Now all the available information on cars under consideration can be formulated as triangular type-2 fuzzy soft set. These two sets are tabulated as follows:

$(\mathscr{P},A)$	c_1	c_2	c_3	c_4
h_1	([0.6,0.7,0.8] [0.3,0.4,0.5] [0.4,0.6,0.8])	([0.5,0.6,0.7] [0.3,0.5,0.7] [0.4,0.6,0.8])	([0.2,0.4,0.6] [0.3,0.4,0.5] [0.6,0.7,0.8]	([0.3,0.5,0.7] [0.3,0.5,0.7] [0.6,0.7,0.8])
h_2	([0.2,0.3,0.4] [0.1,0.3,0.5] [0.3,0.5,0.7])	([0.4,0.6,0.8] [0.2,0.3,0.4] [0.1,0.3,0.5])	([0.1,0.4,0.7] [0.2,0.3,0.4] [0.6,0.7,0.8]	([0.1,0.2,0.3] [0.3,0.4,0.5] [0.5,0.6,0.7])
h_3	([0.2,0.4,0.6] [0.4,0.6,0.8] [0.6,0.7,0.8])	([0.1,0.2,0.3] [0.7,0.8,0.9] [0.3,0.4,0.5])	([0.3,0.4,0.5] [0.4,0.6,0.8] [0.1,0.4,0.7]	([0.3,0.4,0.5] [0.5,0.6,0.7] [0.8,0.9,1.0])

Table 1. **Tabular representation of the triangular type-2 fuzzy soft set** $(\mathscr{P},A)$

$(\mathscr{P},A^c)$	c_1	c_2	c_3	c_4
h_1	([0.4,0.3,0.2] [0.7,0.6,0.5] [0.6,0.4,0.2])	([0.5,0.4,0.3] [0.7,0.5,0.3] [0.6,0.4,0.2])	([0.8,0.6,0.4] [0.7,0.5,0.3] [0.4,0.3,0.2]	([0.7,0.5,0.3] [0.7,0.6,0.5] [0.4,0.3,0.2])
h_2	([0.8,0.7,0.6] [0.9,0.7,0.5] [0.7,0.5,0.3])	([0.6,0.4,0.2] [0.8,0.7,0.6] [0.9,0.7,0.5])	([0.9,0.6,0.3] [0.8,0.7,0.6] [0.4,0.3,0.2]	([0.9,0.8,0.7] [0.7,0.6,0.5] [0.5,0.4,0.3])
h_3	([0.8,0.6,0.4] [0.6,0.4,0.2] [0.4,0.3,0.2])	([0.9,0.8,0.7] [0.3,0.2,0.1] [0.7,0.6,0.5])	([0.7,0.6,0.5] [0.6,0.4,0.2] [0.9,0.6,0.3]	([0.7,0.6,0.5] [0.5,0.4,0.3] [0.2,0.1,0.0])

Table 2. **Tabular representation of the complement of triangular type-2 fuzzy soft set** $(\mathscr{P},A^c)$

$(\mathscr{Q}, A)$	c_1	c_2	c_3	c_4
h_1	([0.7,0.8,0.9] [0.1,0.4,0.7] [0.3,0.5,0.7])	([0.3,0.4,0.5] [0.6,0.7,0.8] [0.2,0.5,0.8])	([0.4,0.6,0.8] [0.2,0.5,0.8] [0.3,0.5,0.7]	([0.2,0.5,0.8] [0.3,0.5,0.7] [0.4,0.6,0.8])
h_2	([0.4,0.6,0.8] [0.3,0.5,0.7] [0.2,0.3,0.4])	([0.4,0.5,0.6] [0.5,0.7,0.9] [0.3,0.4,0.5])	([0.3,0.4,0.5] [0.5,0.7,0.9] [0.6,0.7,0.8]	([0.3,0.4,0.5] [0.7,0.8,0.9] [0.4,0.5,0.6])
h_3	([0.2,0.4,0.6] [0.3,0.5,0.7] [0.6,0.7,0.8])	([0.2,0.5,0.8] [0.4,0.5,0.6] [0.7,0.8,0.9])	([0.3,0.4,0.5] [0.2,0.4,0.6] [0.6,0.7,0.8]	([0.3,0.4,0.5] [0.4,0.6,0.8] [0.3,0.5,0.7])

Table 3. **Tabular representation of triangular type-2 fuzzy soft set** $(\mathscr{Q}, B)$

$(\mathscr{Q}^c, A)$	c_1	c_2	c_3	c_4
h_1	([0.3,0.2,0.1] [0.9,0.6,0.3] [0.7,0.5,0.3])	([0.7,0.6,0.5] [0.4,0.3,0.2] [0.8,0.5,0.2])	([0.6,0.4,0.2] [0.8,0.5,0.2] [0.7,0.5,0.3]	([0.8,0.5,0.2] [0.7,0.5,0.3] [0.6,0.4,0.2])
h_2	([0.6,0.4,0.2] [0.7,0.5,0.3] [0.8,0.7,0.6])	([0.6,0.5,0.4] [0.5,0.3,0.1] [0.7,0.6,0.5])	([0.7,0.6,0.5] [0.5,0.3,0.1] [0.4,0.3,0.2])	([0.7,0.6,0.5] [0.3,0.2,0.1] [0.6,0.5,0.4])
h_3	([0.8,0.6,0.4] [0.7,0.5,0.3] [0.4,0.3,0.2])	([0.8,0.5,0.2] [0.6,0.5,0.4] [0.3,0.2,0.1])	([0.7,0.6,0.5] [0.8,0.6,0.4] [0.4,0.3,0.2]	([0.7,0.6,0.5] [0.6,0.4,0.2] [0.7,0.5,0.3])

Table 4. **Tabular representation of the complement of triangular type-2 fuzzy soft set** $(\mathscr{Q}, B^c)$

Using the above mentioned definitions, we find the similarity measure, inclusion measure and entropy measure between triangular type-2 fuzzy soft sets. The calculated measure values are tabulated as below.

Parameters	ϵ_1	ϵ_2	ϵ_3	ϵ_4	Max.Value	Sel.parameter
$M_1((\mathscr{P}, A), (\mathscr{Q}, B))$	2.3105	1.8658	2.1300	2.2235	**2.3105**	c_1
$M_2((\mathscr{P}, A), (\mathscr{Q}, B))$	2.6749	2.5242	2.6516	2.6951	**2.6951**	c_4
$M_2((\mathscr{Q}, B), (\mathscr{P}, A))$	2.5364	2.1305	3.3831	2.4692	**2.5364**	c_1
$M_1((\mathscr{P}, A^c), (\mathscr{Q}, B^c))$	2.3828	1.7404	2.0927	2.1389	**2.3828**	c_1
$M_2((\mathscr{P}, A^c), (\mathscr{Q}, B^c))$	2.5699	2.0289	2.3488	2.4431	**2.5699**	c_1
$M_2((\mathscr{Q}, B^c), (\mathscr{P}, A^c))$	2.6942	2.4118	2.6446	2.6308	**2.6942**	c_1
$M_3((\mathscr{P}, A), (\mathscr{P}^c, A^c))$	1.5595	1.4205	1.5357	1.6200	**1.6200**	c_4
$M_3((\mathscr{Q}, B), (\mathscr{Q}, B^c))$	1.5658	1.6466	1.5886	1.6545	**1.6545**	c_4
$M_2(\mathscr{P} \cup \mathscr{P}^c, \mathscr{P} \cap \mathscr{P}^c)$	1.5595	1.4205	1.5357	1.6200	**1.6200**	c_4
$M_2(\mathscr{Q} \cup \mathscr{Q}^c, \mathscr{Q} \cap \mathscr{Q}^c)$	1.5658	1.6466	1.5886	1.6545	**1.6545**	c_4

Table 5. **Calculated values of similarity measure, inclusion measure and entropy measure**

- The similarity measure values of triangular type-2 fuzzy soft set and its complement are the same. symbolically, $M_1((\mathscr{P}, A), (\mathscr{Q}, B)) = M_1((\mathscr{P}, A^c), (\mathscr{Q}, B^c))$
- The inclusion measure values of triangular type-2 fuzzy soft set and its complement as follows, $M_2((\mathscr{P}, A), (\mathscr{Q}, B)) \neq M_2((\mathscr{P}, A^c), (\mathscr{Q}, B^c))$ $M_2((\mathscr{Q}, B), (\mathscr{P}, A)) = M_2((\mathscr{Q}, B^c), (\mathscr{P}, A^c))$
- The entropy measure values of triangular type-2 fuzzy soft set and its complement are the same. symbolically, $M_{3_j}((\mathscr{P}, A), (\mathscr{P}, A^c)) = M_{3_j}((\mathscr{Q}, B), (\mathscr{Q}, B^c))$
- The inter relationships between similarity measure, inclusion measure and entropy measure are verified, which are

given in sec. 3.4.

$$min\{M_2((\mathscr{P},A),(\mathscr{Q},B)),M_2((\mathscr{Q},A),(\mathscr{P},B))\} = M_1((\mathscr{P},A),(\mathscr{Q},B))$$

$$min\{M_2((\mathscr{P},A^c),(\mathscr{Q},B^c)),M_2((\mathscr{Q},B^c),(\mathscr{P},A^c))\} = M_1((\mathscr{P},A^c),(\mathscr{Q},B^c))$$

$$M_3((\mathscr{P},A),(\mathscr{P},A^c)) = M_2((\mathscr{P},A\cup A^c),(\mathscr{P},A\cap A^c))$$

$$M_3((\mathscr{Q},B),(\mathscr{Q},B^c)) = M_2((\mathscr{Q},B\cup B^c),(\mathscr{Q},B\cap B^c)$$

$$M_3((\mathscr{P},A),(\mathscr{P},A^c)) = M_1((\mathscr{P},A),(\mathscr{P},A^c))$$

$$M_3((\mathscr{Q},B),(\mathscr{Q},B^c)) = M_1((\mathscr{Q},B),(\mathscr{Q},B^c))$$

The parameter c_1 is having maximum measure value among four parameters. So that, the car (c_1) is selected from the sets.

5. Conclusion

In this paper, we have proved that the inter relationship between similarity measure, inclusion measure and entropy measure on triangular type-2 fuzzy soft sets. A decision making problem is solved to illustrate this relationship.In future work, other types of measures can also be used to find the solution of any decision making problem.

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