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Double Crown Related E Cordial Graphs

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Abstract: WE obtain different families of graphs by attaching two pendent edges at each vertex of G. We call these graphs as double crown graphs and denote them by G^{++}. We show that C_n^{++} and S_4^{++} and W_{n+1}^{++} are E-cordial families.

MSC: 05C78.

Keywords: Double crown, E-cordial, double, pendent edge, wheel, shel labeling.

1. Introduction

In 1997 Yilmaz and Cahit [4] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let G be a graph with vertex set V and edge set E. Let f be a function that maps E into $\{0, 1\}$. Define f on V by $f(v) = \sum\{f(uv)/(uv) \in E\}(mod\ 2)$. The function f is called as E cordial labeling if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. Where $e_f(i)$ is the number of edges labeled with $i = 0, 1$ and $v_f(i)$ is the number of vertices labeled with $i = 0, 1$. We also use $v_f(0, 1) = (a, b)$ to denote the number of vertices labeled with 0 are a and that with 1 are b. Similarly $e_f(0, 1) = (x, y)$ to denote number of edges labeled with 0 are x and that labeled with 1 are y respectively. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of cordial labelings. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit has shown that Trees T_n with n vertices and Complete graphs K_n on n vertices are E-cordial if and only if n is not congruent to 2 (modulo 4). Friendship graph $C_3^{(n)}$ for all n and fans Fn for n not congruent to 1 (mod 4). One may refer A Dynamic survey of graph labeling for more details on completed work. In this paper we show that C_n^{++} and S_4^{++} and W_{n+1}^{++} are E-cordial families. For definitions and terminology we refer [2, 3, 5].

2. Definitions

Definition 2.1 (Crown Graph)**.** *Initially this was defined for Cycle graph and denoted by C_n^+. It was obtained by attaching an pendent edge each to every vertex of C_n. We develop the concept for any graph G and denote it by G^+. It is obtained from a graph G by attaching a pendent edge at each vertex of G. Note that $|V(G^+)| = 2.|V(G)|$ and for edges $|E(G^+)| = |E(G)| + |V(G)|$.*

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Definition 2.2 (Double crown Of $G = (p, q)$ graph)**.** *We obtain it by attaching two pendent edges at each vertex of G. We denote it by* G^{++}*. Note that* $|E(G^{++})| = 2p + q$*,* $|V(G^{++})| = 3p$*.*

Definition 2.3 (Shel graph S_n)**.** *It is obtained by taking chords from a fixed point of* C_n *to every other vertex of* C_n *other than neighbouring two vertices. It has* $2n - 3$ *edges and n vertices.*

3. Main Results

Theorem 3.1. *$G = C_n^{++}$ is E-cordial.*

Proof. Define C_n^{++} in terms of vertex set and Edge set as follows: $V(G) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{v_{i,j}, i = 1, 2, \ldots, n \text{ and } j = 1, 2\}$; $E(G) = \{e_i = (v_i v_{i+1})/i = 1, 2, \ldots, n; i + 1 \text{ taken modulo } n\} \cup \{e_{i,j} = (v_i v_{i,j})/i = 1, 2, \ldots, n \text{ and } j = 1, 2\}$. Note that $|V(G)| = 3n$ and $|E(G)| = 3n$. Define a function $f : E(G) \to \{0, 1\}$ given by

Case 1: n is divisible by 4. Take $t = \frac{n}{4}$; $n = 2x$.

$$f(e_i) = 0 \text{ for } i \equiv 2, 3(mod\ 3) \text{ and } i \le 3t;$$
$$f(e_i) = 1 \text{ for all other i}$$
$$f(e_{i,j}) = 0 \text{ for all } i = 1, \ldots, x$$
$$f(e_{i,j}) = 1 \text{ for all } x + 1 \le i \le n$$

Note that $v_f(0, 1) = (\frac{3n}{2}, \frac{3n}{2}), ef(0, 1) = (\frac{3n}{2}, \frac{3n}{2})$.

Case 2: $n \equiv 1(mod\ 4)$

$$f(e_i) = 0 \text{ for } i \equiv 2, 3(mod\ 3) \text{ and } i \le 3t \text{ where } t = [\frac{n}{4}]$$
$$f(e_i) = 1 \text{ for all other i}$$
$$f(e_{11}) = 0,$$
$$f(e_{12}) = 1,$$
$$f(e_{i,j}) = 0 \text{ for all } i = 2, \ldots, q + 1 \text{ (where } q = \frac{n-1}{2})$$
$$f(e_{i,j}) = 1 \text{ for all other i}$$

Note that $v_f(0, 1) = (q + 1, q), e_f(0, 1) = (\frac{3n-1}{2}, \frac{3n-1}{2} + 1)$.

Case 3: $n \equiv 3(mod\ 4)$.

$$f(e_i) = 0 \text{ for } i \equiv 2, 3(mod\ 4) \text{ and } 1 \le i \le 3t \text{ where } t = [\frac{n}{4}] + 1$$
$$f(e_i) = 1 \text{ for rest of i}$$
$$f(e_{i,j}) = 0 \text{ for } 1 \le i \le \frac{n+3}{2} \text{ for } j = 1, 2 \text{ and } i \ne 2$$
$$f(e_{i,j}) = 1 \text{ for rest of } i, j = 1, 2,\ i \ne 2 \text{ for } i = 2,$$
$$f(e_{i,1}) = 0 \text{ and}$$
$$f(e_{i,2}) = 1$$

Note that number distribution is $e_f(0, 1) = (\frac{3n+1}{2}, \frac{3n-1}{2}) = v_f(0, 1)$.

Case 4: $n \equiv 2(mod\ 4)$ the desired labeling does not exist.

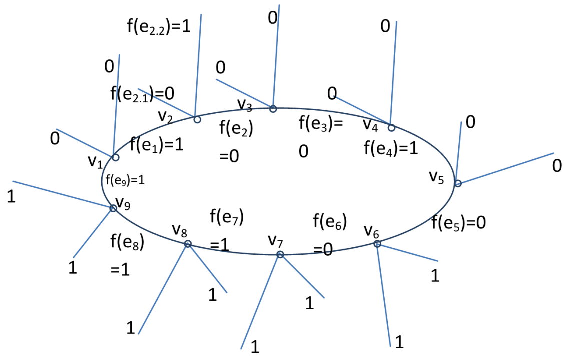

Figure 1. C_9^{++} **is E-cordial actual edge labels are shown**

□

Theorem 3.2. *S_n^{++} which is double crown of shel S_n is E-cordial graph.*

Proof. We define $G = S_4^{++}$ in terms of vertex set and Edge set as follows: $V(G) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{v_{i,j}, i = 1, 2, \ldots, n \ \ and \ \ j = 1, 2\}$; $E(G) = \{e_i = (v_i v_{i+1})/i = 1, 2, \ldots, n, i+1 \ \ taken \ modulo \ n\} \cup \{e_{i,j} = (v_i v_{i,j})/i = 1, 2, \ldots, n \ and \ j = 1, 2\} \cup \{c_i = (v_1 v_i)/i = 3, 4, \ldots, n-1\}$. Define a function $f : E(G) \to \{0, 1\}$ given by

Case 1: $n \equiv 3(mod\ 4)$. Let $t = \frac{n-1}{2}$,

$$f(e_i) = 0 \ \text{ for } i = 1, 2, \ldots, t$$

$$f(e_i) = 1 \ \text{ otherwise}$$

$$f(c_i) = 0 \ \text{ for } i = 1, 2, \ldots, t', \text{ where } t' = \frac{n-3}{2}$$

$$f(c_i) = 1 \ \text{ otherwise}$$

$$f(e_{i,j}) = 0 \ \text{ for } i = 1 \text{ to } \frac{n+1}{2}$$

$$f(e_{i,j}) = 1 \ \text{ for } i = \frac{n+3}{2}, \ldots, n$$

Note that $v_f(0,1) = (\frac{3n+1}{2}, \frac{3n-1}{2}), e_f(0,1) = (2n-1, 2n-2)$.

Case 2: $n \equiv 1(mod\ 4)$. Let $t = \frac{n-1}{2}$.

$$f(e_i) = 0 \ \text{ for } i = 1, 2, \ldots, t$$

$$f(e_i) = 1 \ \text{ otherwise}$$

$$f(c_i) = 0 \ \text{ for } i = 1, 2, \ldots, t'$$

$$f(c_i) = 1 \ \text{ otherwise } t' = \frac{n-3}{2}$$

$$f(e_{i,j}) = 0 \ \text{ for } i = 2 \text{ to } \frac{n+1}{2}, \ j = 1, 2$$

$$f(e_{1,1}) = 0,$$

$$f(e_{1,2}) = 1$$

$$f(e_{i,j}) = 1 \ \text{ for } i = \frac{n+3}{2}, \ldots, n, \ j = 1, 2.$$

Note that the label numbers are $v_f(0,1) = (\frac{3n+1}{2}, \frac{3n-1}{2}), e_f(0,1) = (2n-2, 2n-1)$ for $n \equiv 2(mod\ 4)$ the desired labeling dose't exists.

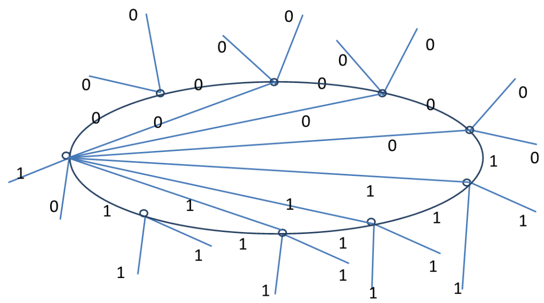

Figure 2. **E-cordial labeling copy of S_9^{++}**

Case 3: $n \equiv 0(mod\ 4)$. On n we take three subcases as $n \equiv 4(mod\ 12)$, $n \equiv 8(mod\ 4)$ and $n \equiv 12(mod\ 4)$. We define $G = S_4^{++}$ in terms of vertex set and Edge set as follows: $V(G) = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{v_{i,j}, i = 1, 2, \ldots, n\ and\ j = 1, 2\}$; $E(G) = \{e_i = (v_i v_{i+1})/i = 1, 2, \ldots, n,$ where $i + 1$ taken modulo n$\} \cup \{e_{i,j} = (v_i v_{i,j})/i = 1, 2, \ldots, n\ and\ j = 1, 2\} \cup \{c_i = (v_1 v_i)/i = 3, 4, \ldots, n-1\}$. Define a function $f : E(G) \to \{0, 1\}$ given by Case 1: $n \equiv 0(mod\ 4)$.

Subcase 1: $n \equiv 4(mod\ 12)$. This gives $n = 12x + 4$ for suitable $x = 0, 1, 2, \ldots$. In this case we have $|V| = 3n = 36x + 12$. For E-cordial labeling we must have $v_f(0, 1) = (18x + 6, 18x + 6)$ and on edges we have $|E| = 4n - 3 = 48x + 13$ and $e_f(0, 1) = (2n - 1, 2n - 2) = (24x + 7, 24x + 6)$ or $e_f(0, 1) = (2n - 2, 2n - 1) = (24x + 6, 24x + 7)$. We choose $e_f(0, 1) = (2n - 1, 2n - 2) = (24x + 7, 24x + 6)$. Define a function $f : E(G) \to \{0, 1\}$ as follows: $f(e_{i,j}) = 1$ for $j = 1, 2$ and $i = 1, 2, 3, \ldots, 9x + 3$. $f(e_{i,j}) = 0$ for $j = 1, 2$ and $i = 9x + 4, 9x + 5, \ldots, n$. Choose $2x$ triangles on S_n not having common edge but having apex vertex of S_n as a common vertex, Take label on all these triangle edges as 1. Label all rest of edges on S_n^{++} as 0. The label distribution is $v_f(0, 1) = (\frac{3n}{2}, \frac{3n}{2}), e_f(0, 1) = (2n - 1, 2n - 2)$.

Subcase 2: $n \equiv 8(mod\ 12)$. $n = 12x + 8$ for suitable $x = 0, 1, 2, \ldots$. In this case we have $|V| = 3n = 36x + 24$. For E-cordial labeling we must have $v_f(0, 1) = (18x + 12, 18x + 12)$ and on edges we have $|E| = 4n - 3 = 48x + 29$ and $e_f(0, 1) = (2n - 1, 2n - 2) = (24x + 15, 24x + 14)$ or $e_f(0, 1) = (2n - 2, 2n - 1) = (24x + 14, 24x + 15)$. We choose $e_f(0, 1) = (2n - 1, 2n - 2) = (24x + 14, 24x + 15)$. Let $t = 3x$. Define a function $f : E(G) \to \{0, 1\}$ as follows: $f(e_{i,j}) = 1$ for $j = 1, 2$ and $i = 1, 2, 3, \ldots, 9x + 6$. $f(e_{i,j}) = 0$ for $j = 1, 2$ and $i = 9x + 7, 9x + 8, \ldots, n$. Choose $2x + 1$ triangles on S_n not having common edge but having apex vertex of S_n as a common vertex. Take label on all these triangle edges as 1. Label all rest of edges on S_n^{++} as 0. The label distribution is $v_f(0, 1) = (\frac{3n}{2}, \frac{3n}{2}), e_f(0, 1) = (2n - 2, 2n - 1)$.

Subcase 3: $n \equiv 12(mod\ 12)$; $n = 12x + 12$; for suitable $x = 0, 1, 2, 3, \ldots$. In this case we have $|V| = 3n = 36x + 36$. For E-cordial labeling we must have $v_f(0, 1) = (18x + 18, 18x + 18)$ and on edges we have $|E| = 4n - 3 = 48x + 48$ and $e_f(0, 1) = (2n - 1, 2n - 2) = (24x + 23, 24x + 22)$ or $e_f(0, 1) = (2n - 2, 2n - 1) = (24x + 22, 24x + 23)$. We choose $e_f(0, 1) = (2n - 1, 2n - 2) = (24x + 7, 24x + 6)$. Define a function $f : E(G) \to \{0, 1\}$ as follows: $f(e_{i,j}) = 1$ for $j = 1, 2$ and $i = 1, 2, 3, \ldots, 9x + 9$. $f(e_{i,j}) = 0$ for $j = 1, 2$ and $i = 9x + 10, 9x + 11, \ldots, n$. Choose one square and $2x$ triangles on S_n not having common edge but having apex vertex of S_n as a common vertex, take label on all these triangle edges as 1. Label all rest of edges on S_n^{++} as 0. The label distribution is $v_f(0, 1) = (\frac{3n}{2}, \frac{3n}{2}), e_f(0, 1) = (2n - 1, 2n - 2)$. □

Theorem 3.3. *$G = W_{n+1}^{++}$ is E-cordial.*

Proof. It can be defined as: take a cycle on length n (C_n). Take a new vertex w and join it to each vertex of C_n by an edge each. To each vertex v_i of cycle two pendent edges are attached ($i = 1, 2, \ldots, n$). We define G in the following way in terms of $V(G)$ and $E(G)$. $V(G) = \{v_1, v_2, \ldots, V_n\} \cup \{u_{i,j}/\ \ for\ \ i = 1, 2, \ldots, n\ \ and\ \ j = 1, 2\} \cup \{w\} U \{w', w''\}$. $E(G) = \{ei = (v_i v_{i+1})/i = 1, 2, \ldots, n$ where $i + 1$ is taken modulo n $\} \cup \{ei, j = (viuj)/i = 1, 2, \ldots, n\ and\ j = 1, 2.\} \cup \{wi = (wv_i)/i = 1, 2, \ldots, n\} \cup \{(ww'), (ww'')\}$. Define a function $f : E(G) \to \{1, 0\}$ as follows.

Case 1: $(n+1) \equiv 0(mod\ 4)$.

$$f(v_i) = 0 \text{ for } i = 1, 2, \ldots, 2x-1 \text{ and}$$
$$f(v_i) = 1 \text{ otherwise}$$
$$f(w_i) = 0 \text{ for } i = 2, 3, \ldots, 2x+2$$
$$f(w_i) = 1 \text{ otherwise}$$
$$f(e_{i,j}) = 0 \text{ for all } j = 1, 2 \text{ and } i = 1, 2, \ldots, 2x$$
$$f(e_{ij}) = 1 \text{ for all } i = 2x+1, \ldots, n \text{ and } j = 1, 2$$
$$f(ww') = 1$$
$$f(ww'') = 1$$

We observe that label numbers are $v_f(0,1) = (6x, 6x)$ and $e_f(0,1) = (2n+1, 2n+1)$.

Case 2: $n+1 \equiv 1(mod\ 4)$. Define a function $f : E(G) \to \{1, 0\}$ as follows.

$$f(e_i) = 0 \text{ for } i = 1, 2, \ldots, \frac{n}{2}$$
$$f(e_i) = 1 \text{ for } \frac{n}{2}+1, \ldots, n$$
$$f(w_i) = 0 \text{ for } i = 1, 2, \ldots, \frac{n}{2}$$
$$f(w_i) = 1 \text{ for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$$
$$f(e_{ij}) = 1 \text{ for all } j = 1, 2 \text{ and } i = 1, 2, \ldots, \frac{n}{2}$$
$$f(e_{ij}) = 0 \text{ for all } j = 1, 2 \text{ and } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$$
$$f(ww') = 0,$$
$$f(ww'') = 1$$

We observe that label numbers are $v_f(0,1) = (\frac{3n-1}{2}, \frac{3n+1}{2})$ and $e_f(0,1) = (2n+1, 2n+1)$.

Case 3: $n+1 \equiv 3(mod\ 4)$

$$f(e_i) = 0 \text{ for } i = 1, 2, \ldots, \frac{n}{2}$$
$$f(e_i) = 1 \text{ for } \frac{n}{2}+1, \ldots, n$$
$$f(w_i) = 0 \text{ for } i = 1, 2, \ldots, \frac{n}{2}$$
$$f(w_i) = 1 \text{ for } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$$
$$f(e_{ij}) = 1 \text{ for all } j = 1, 2 \text{ and } i = 1, 2, \ldots, \frac{n}{2}$$
$$f(e_{ij}) = 0 \text{ for al } j = 1, 2 \text{ and } i = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$$
$$f(ww') = 0,$$
$$f(ww'') = 1$$

We observe that label numbers are $v_f(0,1) = (\frac{3n+1}{2}, \frac{3n-1}{2})$ and $e_f(0,1) = (2n+1, 2n+1)$. When $n+1 \equiv 2(mod\ 4)$ there is no E-cordial labeling.

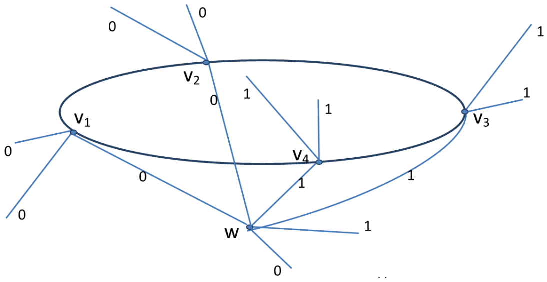

Figure 3. **Labeled copy of W_5^{++}**

□

4. Conclusion

We have shown three types of cycle related graphs to be E-cordial. They are namely C_n^{++}, S_n^{++}, W_{n+1}^{++}. This work makes us to construct $G^{++\cdots+}$ (for t times) and we think that these new graphs for $G = C_n$, S_n, W_{n+1} are E-cordial under certain constraints as above. It is necessary to investigate E-cordiality for more graphs.

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