Int. J. Math. And Appl., 6(1–D)(2018), 603–609
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Critical Path with Fuzzy Measure

V. Anusuya[1,*] and P. Balasowandari[1]

1 PG and Research Department of Mathematics, Seethalakshmi Ramaswami College (Autonomous), Trichy, Tamil Nadu, India.

Abstract: This paper presents the comparison on fuzzy critical path problem with trapezoidal type-2 fuzzy numbers. New ranking method and distance based similarity measure are introduced to identify the fuzzy critical path in a acyclic project network. An illustrative example is also included to demonstrate our proposed approach.

Keywords: Fuzzy critical path, distance based similarity measure, Trapezoidal type-2 fuzzy number, acyclic project network.

1. Introduction

Network optimization is a very popular field among the well studied areas of operations research. The main aim of government agencies and Industrial organizations is to plan their project in order to maximize resource utility and minimize over all cost. This type of management problem can be very well tackled using the network techniques called critical path method. Since the activities in the network can be carried out is parallel, the minimum time to complete the project is the length of the longest path from the start of project to its finish. The longest path is the critical path of the network. The CPM is to identify critical activities on the Critical path. This paper analyze the critical path in a general project network with fuzzy activity time. We propose a ranking method for fuzzy numbers to a critical path method for fuzzy project network, where the duration time of each activity in a fuzzy project network is represented by a trapezoidal type-2 fuzzy number. We compare the possibility of meeting a fuzzy project in a specified time for different activities using proposed method, and fuzzy critical method based on ranking of fuzzy numbers.The fuzzy measures were introduced by sugeno [10]. As an important tool for determining the similarity between two objects, Zadeh [12] initiated fuzzy similarity measure, and later on, various similarity measure for fuzzy set have been sequentially proposed. The organization of the paper is as follows: In section 2, we have some basic concepts, section 3, gives some properties of total slack fuzzy time, Section 4, gives the network terminology. Section 5, gives an algorithm to find the critical path combined with trapezoidal type-2 fuzzy number using ranking method and distance based similarity measure. To illustrate the proposed algorithm the numerical example is solved in section 6.

2. Basic Concepts

Definition 2.1 (Type-2 Fuzzy number). *Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R, if the following conditions are satisfied, then $\tilde{A}$ is called a type-2 fuzzy number.*

* *E-mail: anusrctry@gmail.com*

(1). $\tilde{A}$ is normal.

(2). $\tilde{A}$ is a convex set.

(3). The support of $\tilde{A}$ is closed and bounded.

Definition 2.2 (Normal type-2 fuzzy number)**.** *A type-2 fuzzy number(T2fs), $\tilde{A}$ is said to be normal if its Foot of Uncertainty (FOU) is normal interval type-2 fuzzy numbers(IT2FS) and it has a primary membership function.*

Definition 2.3 (Addition on type-2 fuzzy numbers)**.** *Let*

$$\begin{aligned}\tilde{A} &= \bigcup_{forall\tilde{\alpha}} \tilde{\alpha}FOU(\tilde{A}_{\tilde{\alpha}}) = (A^L, A^M, A^N, A^U)\\ &= ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U) \ \ and\\ \tilde{B} &= \bigcup_{forall\tilde{\alpha}} \tilde{\alpha}FOU(\tilde{B}_{\tilde{\alpha}}) = (B^L, B^M, B^N, B^U)\\ &= ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^M, b_2^M, b_3^M, b_4^M)(b_1^N, b_2^N, b_3^N, b_4^N)(b_1^U, b_2^U, b_3^U, b_4^U)\end{aligned}$$

be two normal type-2 fuzzy numbers. By using extension principle, we have

$$\begin{aligned}\tilde{A}+\tilde{B} &= \left[\bigcup_{forall\tilde{\alpha}} \tilde{\alpha}FOU(\tilde{A}_{\tilde{\alpha}})\right] + \left[\bigcup_{forall\tilde{\alpha}} \tilde{\alpha}FOU(\tilde{B}_{\tilde{\alpha}})\right] = (A^L+B^L, A^M+B^M, A^N+B^N, A^U+B^U)\\ &= \begin{matrix}((a_1^L+b_1^L, a_2^L+b_2^L, a_3^L+b_3^L, a_4^L+b_4^L), (a_1^M+b_1^M, a_2^M+b_2^M, a_3^M+b_3^M, a_4^M+b_4^M)\\ (a_1^N+b_1^N, a_2^N+b_2^N, a_3^N+b_3^N, a_4^N+b_4^N)(a_1^U+b_1^U, a_2^U+b_2^U, a_3^U+b_3^U, a_4^U+b_4^U))\end{matrix}\end{aligned}$$

Definition 2.4. *Let $\tilde{A} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U)$ be a type-2 normal trapezoidal fuzzy number, then the ranking function is defined as,*

$$R(\tilde{A}) = \frac{(a_1^L + 2a_2^L + a_3^L + a_4^L + 2a_1^M + 4a_2^M + 4a_3^M + 2a_4^M + 2a_1^N + 4a_2^N + 4a_3^N + 2a_4^N + a_1^U + 2a_2^U + a_3^U + a_4^U)}{36}$$

Definition 2.5 (Fuzzy critical path)**.** *In a project network a path p_c such that $FCPM(p_c) = \min\{FCPM(p_k)/p_k \in P, k = 1\ tom\}$ is defined as a fuzzy critical path.*

Definition 2.6 (Fuzzy critical path length)**.** *The sum of the Fuzzy activity time of the corresponding path P_c is said to be the fuzzy critical path length.*

Definition 2.7. *Let $\tilde{A}$ and $\tilde{B}$ be two trapezoidal type-2 fuzzy numbers then the distance based similarity measure is defined as follows $S(\tilde{L}_j, \tilde{L}_{\min}) = \frac{1}{1+d(\tilde{L}_j, \tilde{L}_{\min})}$, for $j = 1, 2, \ldots, m$, where $d(\tilde{L}_j, \tilde{L}_{\min}) = \sum_{i=1}^{m} \left|\tilde{L}_j(x_i) - \tilde{L}_{\min}(x_i)\right|$ for $j = 1, 2, \ldots, m$.*

2.1. Notations

A_{ij}	The activity between node i and j.
$F\tilde{E}S_j$	The earliest starting fuzzy time of node j.
$F\tilde{L}F_i$	The latest finishing fuzzy time of node i.
$F\tilde{T}S_{ij}$	The total slack fuzzy time of A_{ij}.
p_k	The k^{th} fuzzy path.
P	The set of all fuzzy paths in a project network.
$FCPM(p_k)$	The total slack fuzzy time of path p_k in a project network.

3. Properties of Total Slack Fuzzy Time

Property 3.1 (Forward pass calculation)**.** *To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie)* $F\tilde{E}S_1 = (0.0, 0.0, 0.0, 0.0)$. $F\tilde{E}S_j = \max_i\{F\tilde{E}S_i + F\tilde{E}T_{ij}\}$, $j \neq i$, $j \in N$, $i =$ *number of preceding nodes.* ($F\tilde{E}S_j =$ *The earliest starting fuzzy time of node j). Ranking value is utilized to identify the maximum value. Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.*

Property 3.2 (Backward pass calculation)**.** *To calculate the latest finishing time in the project network set* $F\tilde{L}F_n = F\tilde{E}Sn.F\tilde{L}F_i = \min_j\{F\tilde{L}F_{ij}(-)F\tilde{E}T_{ij}\}$, $i \neq n$, $i \in N$, $j =$ *number of succeeding nodes. Ranking value is utilized to identify the minimum value. Latest starting fuzzy time= Latest finishing fuzzy time(-) Fuzzy activity time.*

Property 3.3. *For the activity* A_{ij}, $i < j$. *Total fuzzy slack:*

$$F\tilde{T}S_{ij} = F\tilde{L}F_j(-)(F\tilde{E}S_i(+)F\tilde{E}T_{ij})\ (or)\ (FLF_j(-)F\tilde{E}T_{ij})(-)F\tilde{E}S_i,\ 1 \leq i \leq j \leq n;\ i,j \in N.$$

Property 3.4. $FCPM(p_k) = \sum_{\substack{1 \leq i \leq j \leq n \\ i,j \in p_k}} F\tilde{T}S_{ij}$, $p_k \in P$, p_k *is the possible paths in a network from source node to the destination node,* $k = 1$ *to m.*

4. Network Terminology

Consider a directed acyclic project network $G(V, E)$ consisting of a finite set of nodes V=$\{1, 2, \ldots, n\}$ and a set of m directed edges $E \subseteq V \times V$. Each edge is denoted by an ordered pair (i, j) where $i, j \in V$ and $i \neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path p_{ij} as a sequence $p_{ij} = [i = i_1, (i_1, i_2), i_2, \ldots, i_{i-1}, i_l = j]$ of alternating nodes and edges. The existence of at least one path p_{si} in $G(V, E)$ is assumed for every node $i \in V - \{S\}.\tilde{d}_{ij}$ denotes a trapezoidal type-2 fuzzy number associated with the edge (i, j), corresponding to the length necessary to transverse (i, j) from i to j. The fuzzy length along the path P is denoted by $\tilde{d}(P)$ and is defined as $\tilde{d}(P) = \sum_{(i,j \in p)} \tilde{d}_{ij}$.

4.1. Algorithm: 1 (for finding critical path)

Step 1: Estimate the fuzzy activity time with respect to each activity.

Step 2: Let $F\tilde{E}S_1 = (0.0, 0.0, 0.0, 0.0)$ and calculate $F\tilde{E}S_j$, $j = 2, 3, \ldots, n$ by using Property 3.1.

Step 3: Let $F\tilde{L}F_n = F\tilde{E}Sn$ and calculate $F\tilde{L}F_i$, $i = n-1, n-2, \ldots, 2, 1$. By using Property 3.2.

Step 4: Calculate $F\tilde{T}S_{ij}$ with respect to each activity in a project network by using Property 3.3.

Step 5: Find all the possible paths and calculate $FCPM(p_k)$ by using Property 3.4.

Step 6: Identify the critical path by using Definition 2.5.

4.2. Algorithm : 2 (for finding critical path using similarity measure)

Step 1: Form the possible paths Lengths, $\tilde{L}_i$; $i = 1, 2, \ldots, n$ where

$$\tilde{L}_i = ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^M, b_2^M, b_3^M, b_4^M)(b_1^N, b_2^N, b_3^N, b_4^N)(b_1^U, b_2^U, b_3^U, b_4^U).$$

Step 2: Initialize

$$\tilde{L}_{\min} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U).$$
$$= \tilde{L}_1$$
$$\tilde{L}_2 = ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^M, b_2^M, b_3^M, b_4^M)(b_1^N, b_2^N, b_3^N, b_4^N)(b_1^U, b_2^U, b_3^U, b_4^U).$$

Step 3: $i = 2$.

Step 4: Compute

$$(a_1^L, a_2^L, a_3^L, a_4^L) = \min\{((a_1^L, a_2^L, a_3^L, a_4^L), (b_1^L, b_2^L, b_3^L, b_4^L)\}$$

$$(a_1^M, a_2^M, a_3^M, a_4^M) = \begin{cases} (a_1^M, a_2^M, a_3^M, a_4^M) if (a_1^M, a_2^M, a_3^M, a_4^M) \leq (b_1^L, b_2^L, b_3^L, b_4^L) \\ \frac{((a_1^M, a_2^M, a_3^M, a_4^M)\times(b_1^M, b_2^M, b_3^M, b_4^M)) - ((a_1^L, a_2^L, a_3^L, a_4^L)\times(b_1^L, b_2^L, b_3^L, b_4^L))}{((a_1^M, a_2^M, a_3^M, a_4^M)+(b_1^M, b_2^M, b_3^M, b_4^M)) - ((a_1^L, a_2^L, a_3^L, a_4^L)+(b_1^L, b_2^L, b_3^L, b_4^L))} \end{cases}$$

if .$(a_1^M, a_2^M, a_3^M, a_4^M) > (b_1^L, b_2^L, b_3^L, b_4^L)$

$$(a_1^N, a_2^N, a_3^N, a_4^N) = \begin{cases} (a_1^N, a_2^N, a_3^N, a_4^N) if (a_1^N, a_2^N, a_3^N, a_4^N) \leq (b_1^M, b_2^M, b_3^M, b_4^M) \\ \frac{((a_1^N, a_2^N, a_3^N, a_4^N)\times(b_1^N, b_2^N, b_3^N, b_4^N)) - ((a_1^M, a_2^M, a_3^M, a_4^M)\times(b_1^M, b_2^M, b_3^M, b_4^M))}{((a_1^N, a_2^N, a_3^N, a_4^N)+(b_1^N, b_2^N, b_3^N, b_4^N)) - ((a_1^M, a_2^M, a_3^M, a_4^M)+(b_1^M, b_2^M, b_3^M, b_4^M))} \end{cases}$$

if $(a_1^N, a_2^N, a_3^N, a_4^N) > (b_1^M, b_2^M, b_3^M, b_4^M)$.

$$(a_1^U, a_2^U, a_3^U, a_4^U) = \min\{(a_1^U, a_2^U, a_3^U, a_4^U), (b_1^N, b_2^N, b_3^N, b_4^N)\}$$

Step 5: $\tilde{L}_{\min} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U))$ as calculated in Step 4

Step 6: $i = i + 1$.

Step 7: If $i < n = 1$, go to Step 4.

Step 8: Find out all possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$; $i = 1, 2, \ldots, n$.

Step 9: Compute $\tilde{L}_{\min}$ by using type-2 fuzzy critical path length procedure.

Step 10: Find $d(\tilde{L}_j, \tilde{L}_{\min}) = \sum_{i=1}^{m} \left|\tilde{L}_j(x_i) - \tilde{L}_{\min}(x_i)\right|$. For $j = 1, 2, \ldots, m$ using Definition 2.7.

Step 11: Compute similarity measure for $j = 1, 2, \ldots, m$.

Step 12: The path which has the minimum similarity degree is the fuzzy critical path.

5. Numerical Example

The problem is to find the critical path between source node to destination node in the acyclic fuzzy project network having 6 vertices and 7 edges with trapezoidal type-2 fuzzy number.

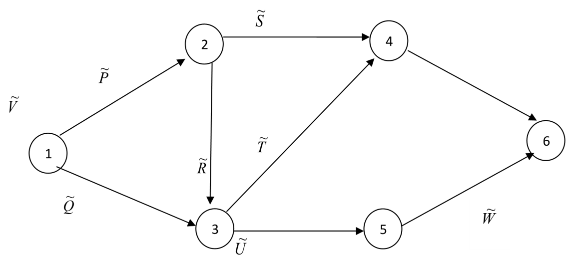

Figure 1.

Solution: The edge lengths are

$$\tilde{P} = ((1.9, 2.1, 2.3, 2.5; 0.3), (1.7, 1.9, 2.1, 2.3), (1.5, 1.8, 2.1, 2.6), (1.1, 1.3, 1.6, 2.0))$$

$$\tilde{Q} = ((1.8, 2.0, 2.2, 2.6; 0.4), (1.6, 1.8, 2.2, 2.4), (1.6, 1.9, 2.3, 2.7), (1.3, 1.5, 1.7, 2.1))$$

$$\tilde{R} = ((1.6, 1.8, 2.0, 2.2; 0.5), (1.4, 1.6, 2.2, 2.4), (1.3, 1.5, 1.7, 2.1), (1.1, 1.3, 1.5, 1.9))$$

$$\tilde{S} = ((1.7, 1.9, 2.1, 2.3; 0.6), (1.5, 1.7, 1.9, 2.1,), (1.4, 1.6, 1.8, 2.0), (1.2, 1.4, 1.6, 1.9))$$

$$\tilde{T} = ((1.5, 1.7, 1.9, 2.3; 0.7), (1.4, 1.6, 1.9, 2.1), (1.2, 1.4, 1.6, 1.8), (1.0, 1.3, 1.5, 1.7))$$

$$\tilde{U} = ((1.4, 1.7, 1.9, 2.1; 0.8), (1.3, 1.5, 1.7, 1.9), (1.1, 1.3, 1.5, 1.7), (1.0, 1.2, 1.5, 1.7))$$

$$\tilde{V} = ((1.7, 1.8, , 2.0, 2.2; 0.9), (1.5, 1.8, 2.1, 2.3), (1.3, 1.5, 1.7, 1.9), (1.1, 1.4, 1.5, 1.9))$$

$$\tilde{W} = ((1.9, 2.1, 2.3, 2.5; 0.6), (1.8, 2.2, 2.4, 2.6), (1.6, 1.9, 2.1, 2.4), (1.5, 1.8, 2.2, 2.5))$$

From the edge length, using properties we calculate the fuzzy durations and total slack fuzzy time for each activity as shown in Table 1.

Activity $(i-j)i<j$	*Duration* $F\tilde{E}T_{ij}$	$F\tilde{E}S_i$	$F\tilde{L}F_j$	$F\tilde{T}S_{ij}$
$1-2$	$((1.9, 2.1, 2.3, 2.5; 0.3),$ $(1.7, 1.9, 2.1, 2.3),$ $(1.5, 1.8, 2.1, 2.6),$ $(1.1, 1.3, 1.6, 2.0))$	$(0.0, 0.0, 0.0, 0.0)$	$((-0.5, 1.5, 2.9, 4.4),$ $(-0.7, 0.9, 3.1, 4.7),$ $(-0.7, 1.2, 2.7, 4.8),$ $(-1.4, 0.4, 2.5, 4.5))$	$((-3.0, -0.8, 0.8, 2.5),$ $(-3.0, -1.2, 1.2, 3.0),$ $(-3.3, -0.9, 0.9, 3.3),$ $(-0.6, -1.2, 1.3, 3.4))$
$1-3$	$((1.8, 2.0, 2.2, 2.4; 0.4),$ $(1.6, 1.8, 2.2, 2.4),$ $(1.6, 1.9, 2.3, 2.7),$ $(1.3, 1.5, 1.7, 2.1))$	$(0.0, 0.0, 0.0, 0.0)$	$((1.7, 3.5, 4.7, 6.0),$ $(1.7, 3.1, 4.7, 6.1),$ $(1.4, 2.9, 4.2, 6.1),$ $(0.5, 1.9, 3.8, 5.6))$	$((-0.7, 1.3, 2.7, 4.2),$ $(-0.7, 0.9, 2.9, 4.2),$ $(-1.3, 0.6, 2.3, 4.5),$ $(-1.6, 0.2, 2.3, 4.3))$
$2-3$	$((1.6, 1.8, 2.0, 2.2; 0.5),$ $(1.4, 1.6, 2.2, 2.4),$ $(1.3, 1.5, 1.7, 2.1),$ $(1.1, 1.3, 1.5, 1.9))$	$((1.9, 2.1, 2.3, 2.5; 0.0),$ $(1.7, 1.9, 2.1, 2.3),$ $(1.5, 1.8, 2.1, 2.6),$ $(1.1, 1.3, 1.6, 2.0))$	$((1.7, 3.5, 4.7, 6.0),$ $(1.7, 3.1, 4.7, 6.1),$ $(1.4, 2.9, 4.2, 6.1),$ $(0.5, 1.9, 3.8, 5.6))$	$((-3.0, -0.8, 0.8, 2.5),$ $(-3.0, -1.2, 1.2, 3.0),$ $(-3.3, -0.9, 0.9, 3.3),$ $(-3.4, -1.2, 1.2, 3.4))$
$2-4$	$((1.7, 1.9, 2.1, 2.3; 0.6),$ $(1.5, 1.7, 1.9, 2.1),$ $(1.4, 1.6, 1.8, 2.0),$ $(1.2, 1.4, 1.6, 1.9))$	$((1.9, 2.1, 2.3, 2.5; 0.0),$ $(1.7, 1.9, 2.1, 2.3),$ $(1.5, 1.8, 2.1, 2.6),$ $(1.1, 1.3, 1.6, 2.0))$	$((4.6, 5.7, 6.7, 7.6),$ $(3.9, 5.1, 6.6, 7.7),$ $(3.6, 4.8, 5.9, 7.5),$ $(2.8, 4.1, 5.4, 7.0))$	$((-0.2, 1.3, 2.7, 4.0),$ $(-0.5, 1.1, 3.0, 4.5),$ $(-1.0, 0.9, 2.5, 4.6),$ $(-1.1, 0.9, 2.7, 4.7))$
$3-4$	$((1.5, 1.7, 1.9, 2.3; 0.7),$ $(1.4, 1.6, 1.9, 2.1),$ $(1.2, 1.4, 1.6, 1.8),$ $(1.0, 1.3, 1.5, 1.7))$	$((3.5, 3.9, 4.3, 4.7; 0.0),$ $(3.1, 3.5, 4.3, 4.7),$ $(2.8, 3.3, 3.8, 4.7),$ $(2.2, 2.6, 3.1, 3.9))$	$((4.6, 5.7, 6.7, 7.6),$ $(3.9, 5.1, 6.6, 7.7),$ $(3.6, 4.8, 5.9, 7.5),$ $(2.8, 4.1, 5.4, 7.0))$	$((-2.4, -0.5, 1.1, 2.6),$ $(-2.9, -1.1, 1.5, 3.2),$ $(-2.9, -0.6, 1.2, 3.5),$ $(-2.8, -0.5, 1.5, 3.8))$
$3-5$	$((1.4, 1.7, 1.9, 2.1; 0.8),$ $(1.3, 1.5, 1.7, 1.9),$ $(1.1, 1.3, 1.5, 1.7),$ $(1.0, 1.2, 1.5, 1.7))$	$((3.5, 3.9, 4.3, 4.7; 0.0),$ $(3.1, 3.5, 4.3, 4.7),$ $(2.8, 3.3, 3.8, 4.7),$ $(2.2, 2.6, 3.1, 3.9))$	$((3.8, 5.4, 6.4, 7.4),$ $(3.6, 4.8, 6.2, 7.4),$ $(3.1, 4.4, 5.5, 7.2),$ $(2.2, 3.4, 5.0, 6.6))$	$((-3.0, -0.4, 0.8, 2.5),$ $(-3.0, -1.2, 1.2, 3.0),$ $(-3.3, -0.9, 0.9, 3.3),$ $(-3.4, -1.2, 1.2, 3.4))$

Activity $(i-j) i<j$	*Duration* $F\tilde{E}T_{ij}$	$F\tilde{E}S_i$	$F\tilde{L}F_j$	$F\tilde{T}S_{ij}$
$4-6$	$((1.7, 1.8, 2.0, 2.2; 0.9),$ $(1.5, 1.8, 2.1, 2.3),$ $(1.3, 1.5, 1.7, 1.9),$ $(1.1, 1.4, 1.5, 1.9))$	$((5.0, 5.6, 6.2, 7.0; 0.0),$ $(4.5, 5.1, 6.2, 6.8),$ $(4.0, 4.7, 5.4, 6.5),$ $(3.2, 3.9, 4.6, 5.6))$	$((6.8, 7.7, 8.5, 9.3),$ $(6.2, 7.2, 8.4, 9.2),$ $(5.5, 6.5, 7.4, 8.8),$ $(4.7, 5.6, 6.8, 8.1))$	$((-2.4, -0.5, 1.1, 2.6),$ $(-2.9, -1.1, 1.5, 3.2),$ $(-2.9, -0.6, 1.2, 3.5),$ $(-2.8, -0.5, 1.5, 3.8))$
$5-6$	$((1.9, 2.1, 2.3, 2.5; 0.6),$ $(1.8, 2.2, 2.4, 2.6),$ $(1.6, 1.9, 2.1, 2.4),$ $(1.5, 1.8, 2.2, 2.5))$	$((4.9, 5.6, 6.2, 6.8; 0.0),$ $(4.4, 5.0, 6.0, 6.6),$ $(3.9, 4.6, 5.3, 6.4),$ $(3.2, 3.8, 4.6, 5.6))$	$((6.8, 7.7, 8.5, 9.3),$ $(6.2, 7.2, 8.4, 9.2),$ $(5.5, 6.5, 7.4, 8.8),$ $(4.7, 5.6, 6.8, 8.1))$	$((-2.5, -0.8, 0.8, 2.5),$ $(-3.0, -1.2, 1.2, 3.0),$ $(-3.3, -0.9, 0.-, 3.3),$ $(-3.4, -1.2, 1.2, 3.4))$

Table 1. **Activities, fuzzy durations and total slack fuzzy time for each activity**

Find all the possible paths and calculate FCPM(p_k) by using Property 3.4. The possible paths are $(1-2-4-6)$, $(1-2-3-5-6)$, $(1-3-4-6)$, $(1-3-5-6)$, $(1-2-3-4-6)$. Which are denoted by p_1, p_2, p_3, p_4, p_5. Using Algorithm 1 and 2 we have the following table.

Path	Ranking value (using Algorithm 1)	Similarity measure (using Algorithm 2)
$(1-2-4-6)$	2.308	1
$(1-2-3-5-6)$	0.06388	0.0127
$(1-3-4-6)$	2.2167	0.0323
$(1-3-5-6)$	1.6583	0.0294
$(1-2-3-4-6)$	0.6222	0.0132

Table 2.

- Table 2 gives the ranking value and distance based similarity measure for the possible paths in a given fuzzy acyclic project network.
- In both the cases, the path $(1-2-3-5-6)$ has a minimum degree.
- Hence the path $(1-2-3-5-6)$ is identified as a fuzzy critical path in a given fuzzy acyclic project network.

6. Conclusion

In this work, ranking method and distance based similarity measure are used to find the fuzzy critical path in a acyclic fuzzy project network. We found that the critical path which is obtained by using ranking method and distance based similarity measure are the same.

References

[1] V.Anusuya and R.Sathya, *Fuzzy shortest Path Problem by inclusion Measure On type-2 fuzzy Number*, Bulletin of Mathematics And Statistics Research, 2(4)(2014).

[2] V.Anusuya and R.Sathya, *Type-2 fuzzy shortest path*, International Journal of Fuzzy Mathematical Archive, 2(2013), 36-42.

[3] V.Anusuya and P.Balasowandari, *Fuzzy critical path with type-2 trapezoidal fuzzy numbers*, National Conference on Graph Theory, Fuzzy Graph Theory, and their Applications, Jamal Academic Research Journal: An interdisciplinary, Special Issue (2016), 171-176.

[4] G.Bortolan and R.Degani, *A review of some methods for ranking fuzzy subsets*, Fuzzy Sets and Systems, 15(1985), 1-19.

[5] S.Chanas and P.Zielinski, *Critical path analysis in the network with fuzzy activity times*, Fuzzy Sets and Systems, 122(2001), 195-204.

[6] S.Elizabeth and L.Sujatha, *Fuzzy critical path problem for project network*, International Journal of Pure and Applied Mathematics, 85(2013), 223-240.

[7] G.S.Liang and T.-Chenhan, *Fuzzy critical path project network*, Information and Management Sciences, 15(2004), 29-40.

[8] S.H.Nasution, *Fuzzy critical path method*, IEEE Trans. System Man Cybernetics, 24(1994), 48-57.

[9] J.J.O'Brien, *CPM in Construction Management*, New York, Mc-Graw-Hill, (1993).

[10] M.Sugeno, *Fuzzy measures and fuzzy integrals-a survey*, In Gupta, saridis and Gaines, (1977), 89-102.

[11] S.Yao and FT.Lin, *Fuzzy critical path method based on signed distance ranking of fuzzy members*, IEEE Transactions on Systems, Man and Cybernetics-Part A: Systems and Humans, 30(1)(2000), 76-82.

[12] L.A.Zadeah, *Fuzzy sets*, Information and Control, 8(1965), 138-353.

[13] L.A.Zadeah, *The concepts of a linguistic variable and its application to approximate reasoning part-I*, Information Sciences, 8(3)(1975), 199-249.

[14] L.A.Zadeah, *The concepts of a linguistic variable and its application to approximate reasoning part-II*, Information Sciences, 8 (4)(1975), 301-357.

[15] L.A.Zadeah, *The concepts of a linguistic variable and its application to approximate reasoning part-III*, Information Sciences, 9 (1)(1975), 43-80.