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Soft $b^{\#}$-open Sets and Soft $b^{\#}$-continuous Functions in Soft Topological Spaces

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Abstract: Soft set theory was firstly introduced by Molodtsov in 1999 as a general Mathematical tool for dealing with uncertainty. He has shown several applications of this theory in solving many practical problems in Economics, Engineering,Social science, Medical science, etc. The purpose of this paper is to introduce a new type of soft open set is called Soft $b^{\#}$-open sets in Soft Topological spaces. Also, we discussed soft $b^{\#}$-continuous in Soft Topological spaces.

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1. Introduction and Preliminaries

It is known that Topology is an important area of Mathematics with many applications in the domains of Computer science and Physical sciences. Soft topology is a relatively new and promising domain which can lead to the development of new Mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in natural sciences. In 1999, D. Molodtsov [13] introduced the notion of soft set. He applied the soft theory in several fields such as smoothness of functions, Game theory, Probability, Perron integration, Riemann integration, theory of measurement. The concept of soft set is used to solve complicated problems in other sciences such as, Engineering, Economics etc.. Maji [12] described an application of soft set theory to a decision-making problem. As a generalization of closed sets, the notion of Soft b-closed sets were introduced and studied by Metin Akdag , Alkan Ozkan [1]. In the present paper, we introduce some new type of soft open set is called soft $b^{\#}$-open sets in soft topological spaces and also we study about soft $b^{\#}$-interior, soft $b^{\#}$-closure, soft $b^{\#}$-continuous functions in soft topological spaces.

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let $A\widetilde{\subseteq}E$.

Definition 1.1 ([12, 13])**.** *A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \Rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular $e \in (F, A)$. F(e) may be considered the set of e-approximate elements of the soft set (F, A).*

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Definition 1.2 ([12, 13]). *For two soft sets (F, A) and $(G, B))$ over a common universe U, we say that (F, A) is a soft subset of $(G, B))$ if*

(1). $A \widetilde{\subseteq} B$, and

(2). $\forall\ e \widetilde{\in} A$, $F(e) \widetilde{\subseteq} G(e)$.

We write $(F, A) \widetilde{\subseteq} (G, B))$. (F, A) is said to be a soft super set of $(G, B))$, if $(G, B))$ is a soft subset of (F, A). We denote it by $(F, A) \widetilde{\supseteq} (G, B))$.

Definition 1.3 ([12, 13]). *A soft set (F, A) over U is said to be*

(1). null soft set denoted by $\widetilde{\emptyset}$ if $\forall\ e \widetilde{\in} (F, A)$, $F(e) = \widetilde{\phi}$.

(2). absolute soft set denoted by A, if $\forall\ e \widetilde{\in} (F, A)$, $F(e) = U$.

Definition 1.4 ([12, 13]). *The union of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \widetilde{\cup} B$ and $\forall\ e \widetilde{\in} C$, $H(e) = F(e)$ if $e \widetilde{\in} A - B$, $G(e)$ if $e \widetilde{\in} B - A$, $F(e) \widetilde{\cup} G(e)$ if $e \widetilde{\in} A \widetilde{\cap} B$. We write $(F, A) \widetilde{\cup} (G, B) = (H, C)$. The intersection (H, C) of (F, A) and (G, B) over X, denoted $(F, A) \widetilde{\cap} (G, B)$ is defined as $C = A \widetilde{\cap} B$, and $H(e) = F(e) \widetilde{\cap} G(e)$ for all $e \widetilde{\in} C$.*

Definition 1.5 ([12, 13]). *Let Y be a non-empty subset of X, then denotes the soft set (Y, E) over X for which $Y(e) = Y$, $\forall\ e \widetilde{\in} E$. In particular, (X, E), will be denoted by $\widetilde{X}$.*

Definition 1.6 ([12, 13]). *For a soft set (F, A) over the universe U, the relative complement of (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$, where $F' : A \to P(U)$ is a mapping defined by $F'(e) = U - F(e)$ for all $e \widetilde{\in} A$.*

Definition 1.7 ([12, 13]). *Let $\widetilde{\tau}$ be the collection of soft sets over X, then τ is called a soft topology on X if $\widetilde{\tau}$ satisfies the following axioms:*

(1). $\widetilde{\phi}$, $\widetilde{X}$ belongs to $\widetilde{\tau}$.

(2). The union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

(3). The intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet $(X, \widetilde{\tau}, E)$ is called a soft topological space over X.

Definition 1.8. *Let U be an initial universe and E be a set of parameters. Let $F_A \widetilde{\in} S(U)$. A soft topology on F_A, denoted by $\widetilde{\tau}$, is a collection of soft subsets of F_A having the following properties:*

(1). F_ϕ, F_A belong to $\widetilde{\tau}$.

(2). $\{F_{A_i} \widetilde{\subseteq} F_A : i \widetilde{\in} I\} \widetilde{\subseteq} \widetilde{\tau} \Rightarrow \widetilde{\cup}_i \widetilde{\in} I F_{A_i} \widetilde{\in} \widetilde{\tau}$.

(3). $\{F_{A_i} \widetilde{\subseteq} F_A : 1 \leq i \leq n, \widetilde{\in} N\} \widetilde{\in} \widetilde{\tau} \Rightarrow \widetilde{\cap}_{i=0}^{n} \{F_{A_i} \widetilde{\in} \widetilde{\tau}\}$. The pair $(F_A, \widetilde{\tau})$ is called a soft topological spaces. The members of $\widetilde{\tau}$ are called Soft Open sets in X and complements of them are called Soft Closed sets in X.

Example 1.9. *Suppose that there are Three type of higher secondary students like to join in professional studies $U = \{P_1, P_2, P_3\}$, under consideration and that $E = \{x_1, x_2, x_3\}$ is a set of decision parameters. The x_i $(i = 1, 2, 3)$ stand for the parameters of like 'Medical', 'Engineering', 'Agri, colleges respectively. Consider the mapping f_A given by 'persons (.)', where (.) is to be filled in by one of the parameters $x_i \widetilde{\in} E$. For instance, $f_A(x_1)$ means person 1 (like to join "Medical"), and its* *functional value is the set* $\{p_1\widetilde{\in}U$: *person 1 like to join "Medical" course*$\}$. *Let* $U = \{P_1, P_2, P_3\}$, $E = \{x_1, x_2, x_3\}$, *and* $A = \{x_1, x_2\}$ *Then, we can view the soft set* F_A *as consisting of the following collection of approximations Let* $U = \{p_1, p_2, p_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}\widetilde{\subseteq}E$, *and* $F_A = \{(x_1, \{p_1, p_2\}), (x_2, \{p_2, p_3\})\}$. *Then,* $F_{A_1} = \{(x_1, \{p_1\})\}$, $F_{A_2} = \{(x_1, \{p_2\})\}$, $F_{A_3} = \{(x_1, \{p_1, p_2\})\}$, $F_{A_4} = \{(x_2, \{p_2\})\}$, $F_{A_5} = \{(x_2, \{p_3\})\}$, $F_{A_6} = \{(x_2, \{p_2, p_3\})\}$, $F_{A_7} = \{(x_1, \{p_1\}), (x_2, \{p_2\})\}$, $F_{A_8} = \{(x_1, \{p_1\}), (x_2, \{p_3\})\}$, $F_{A_9} = \{(x_1, \{p_1\}), (x_2, \{p_2, p_3\})\}$, $F_{A_{10}} = \{(x_1, \{p_2\}), (x_2, \{p_2\})\}$, $F_{A_{11}} = \{(x_1, \{p_2\}), (x_2, \{p_3\})\}$, $F_{A_{12}} = \{(x_1, \{p_2\}), (x_2, \{p_2, p_3\})\}$, $F_{A_{13}} = \{(x_1, \{p_1, p_2\}), (x_2, \{p_2\})\}$, $F_{A_{14}} = \{(x_1, \{p_1, p_2\}), (x_2, \{p_3\})\}$, $F_{A_{15}} = F_A, F_{A_{16}} = F_\phi$. *Then the soft topology* $\widetilde{\tau} = \{F_A, F_\phi, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$.

Definition 1.10 ([12, 13]). *Let* $(X, \widetilde{\tau}, E)$ *be soft space over X. A soft set* (F, E) *over X is said to be soft closed in X, if its relative complement* $(F, E)'$ *belongs to* τ. *The relative complement is a mapping* $F' : E \to P(X)$ *defined by* $F'(e) = X - F(e)$ *for all* $e\widetilde{\in}A$.

Definition 1.11 ([12, 13]). *Let X be an initial universe set , E be the set of parameters and* $\widetilde{\tau} = \{\widetilde{\phi}, \widetilde{X}\}$. *Then* $\widetilde{\tau}$ *is called the soft indiscrete topology on X and* $(X, \widetilde{\tau}, E)$ *is said to be a soft indiscrete space over X. If* $\widetilde{\tau}$ *is the collection of all soft sets which can be defined over X, then* $\widetilde{\tau}$ *is called the soft discrete topology on X and* $(X, \widetilde{\tau}, E)$ *is said to be a soft discrete space over X.*

Definition 1.12 ([12, 13]). *Let* $(X, \widetilde{\tau}, E)$ *be a soft topological space over X and the soft interior of* (F, E) *denoted by soft* $int(F, E)$ *is the union of all soft open subsets of* (F, E). *Clearly,* (F, E) *is the largest soft open set over X which is contained in* (F, E). *The soft closure of* (F, E) *denoted by soft* $Cl(F, E)$ *is the intersection of all soft closed sets containing* (F, E). *Clearly ,* (F, E) *is smallest soft closed set containing* (F, E).
soft Int(F,E) $= \widetilde{\cup}$ {*(O,E):(O,E) is soft open and (O,E)* $\widetilde{\subseteq}$ *(F,E)*}.
soft Cl(F,E)$=\widetilde{\cap}$ {*(O,E):(O,E) is soft closed and (F,E)* $\widetilde{\subseteq}$*(O,E)*}.

Definition 1.13. *A Soft subset* (F, A) *of a space* $(X, \widetilde{\tau}, E)$ *is called:*

(1). Soft α*-open [2] if* (F, A) $\widetilde{\subseteq}$*S int(S cl(S int((F, A)))) and Soft* α*-closed if Scl(Sint(Scl((F, A))))* $\widetilde{\subseteq}$ (F, A)

(2). Soft semi-open [4] if (F, A) $\widetilde{\subseteq}$ *Scl(S int(F, A)) and Soft semi-closed if Sint(Scl(F, A))* $\widetilde{\subseteq}$*(F, A),*

(3). Soft pre- open [10] if (F, A) $\widetilde{\subseteq}$*Sint(Scl((F, A))) and Soft pre-closed if Scl(Sint(((F, A)))* $\widetilde{\subseteq}$*(F, A)*

(4). Soft regular open [19] if (F, A) *=Sint(Scl((F, A))) and Soft regular-closed if Scl(Sint(((F, A)))=(F, A)*

(5). Soft b-open [1] if $(F, A)$$\widetilde{\subseteq}$*Scl(Sint(F, A))* $\widetilde{\cup}$*Sint(Scl(F, A)),and Soft b-closed if Scl(Sint(F, A))* $\widetilde{\cap}$*Sint(Scl(F, A))*$\widetilde{\subseteq}$ *(F, A)*

Proposition 1.14. *Let (F, A) be a soft subset of a space (X,*$\widetilde{\tau}$*, E) . Then*

(1). Soft α*-cl((F, A)) = (F, A)*$\widetilde{\cup}$ *Scl(Sint(Scl((F, A)))),*

(2). Soft α*-int((F, A)) = (F, A)*$\widetilde{\cap}$ *Sint(Scl(Sint ((F, A)))),*

(3). Soft semi cl ((F, A)) = (F, A)$\widetilde{\cup}$ *Sint(Scl((F, A))),*

(4). Soft semi int ((F, A)) = (F, A)$\widetilde{\cap}$ *Scl(Sint((F, A))),*

(5). Soft precl ((F, A)) = (F, A) $\widetilde{\cup}$*Scl(Sint((F, A))),*

(6). Soft pre int ((F, A)) = (F, A)$\widetilde{\cap}$ *Sint(Scl((F, A))),*

2. Soft b$^{\#}$-open Sets

We introduce the following definition.

Definition 2.1. *We introduce a new definition as follows: Soft* $b^{\#}$*-open if* (F, A)=*Scl(S int*$((F, A))$*)* $\tilde{\cup}$*Sint(Scl*$((F, A))$*).*

Definition 2.2 ([2]). *A Soft subset (F, A) of a space* $(X,\tilde{\tau}, E)$ *is called:*

The union of all Soft $b^{\#}$*-open sets, each contained in a set (F, A) in a space X is called the Soft* $b^{\#}$*-interior of (F, A) and is denoted by Soft* $b^{\#}$*-Int((F, A)).*

Remark 2.3. *Every Soft* $b^{\#}$*-open set is Soft b-open set but not conversely.*

Example 2.4. *Let* $(X,\tilde{\tau}, E)$ *be a soft topological space and soft topology* $\tilde{\tau}=\{F_A, F_\phi, F_{A_1}, F_{A_3}, F_{A_7}, F_{A_9}, F_{A_{13}}\}$*.* F_{A_5} *is Soft b-open set but not Soft* $b^{\#}$*-open set.*

Theorem 2.5. *For a subset (F, A) of a space* $(X,\tilde{\tau}, E)$ *, the following are equivalent:*

(1). (F, A) is Soft $b^{\#}$*-open.*

(2). (F, A) is Soft b-open.

(3). (F, A) is Soft pre-closed, Soft semi-closed and (F, A)=Soft preint((F, A)) $\tilde{\cup}$ *Soft semi int((F, A)).*

(4). (F, A) is Soft semi-closed and (F, A)= Soft precl(Soft preint((F, A))).

Proof. (F, A) is Soft b$^{\#}$-open $\Leftrightarrow$ Scl(Sint ((F, A))) $\tilde{\cup}$ Sint(Scl((F, A)))=(F, A) $\Leftrightarrow$ Scl(Sint ((F, A))) $\tilde{\cup}$Sint(Scl((F, A)))$\tilde{\subseteq}$(F, A) and (F, A)$\tilde{\subseteq}$ Scl(Sint((F, A))) $\tilde{\cup}$Sint(Scl((F, A))) $\Leftrightarrow$ (F, A) is Soft b-open. This proves (i) $\Leftrightarrow$ (ii). To prove (ii)$\Rightarrow$ (iii), let (F, A) be Soft b-open. Since (F, A) is Scl(Sint ((F, A))) $\tilde{\cup}$ Sint(Scl((F, A))) $\tilde{\subseteq}$ (F, A). It follows that Scl(Sint((F, A))) $\tilde{\subseteq}$ (F, A) and Sint(Scl((F, A))) $\tilde{\subseteq}$ (F, A). Thus (F, A) is Soft pre-closed and Soft semi-closed. Now since (F, A) is Soft b-open,(F, A) $\tilde{\subseteq}$ Scl(Sint((F, A))) $\tilde{\cup}$Sint(Scl((F, A))). Then by using Proposition 1.14,Soft pre int((F, A)) $\tilde{\cup}$ soft semi int((F, A))=((F, A)$\tilde{\cap}$ Sint(Scl(F, A)) $\tilde{\cup}$ ((F, A)$\tilde{\cap}$ Scl(Sint((F, A)))=(F, A)$\tilde{\cap}$ (Sint(Scl((F, A))) $\tilde{\cup}$ (Scl(Sint((F, A))))=(F, A). This proves (ii) $\Rightarrow$ (iii). Now to prove (iii) $\Rightarrow$ (iv). Suppose (iii) holds. (F, A)= soft preint((F, A)) $\tilde{\cup}$ Soft sem int((F, A))=Soft preint((F, A)) $\tilde{\cup}$ ((F, A)$\tilde{\cap}$ Scl(Sint((F, A)))=(soft preint((F, A))$\tilde{\cup}$ (F, A)) $\tilde{\cap}$ (soft preint((F, A))$\tilde{\cup}$ Scl(Sint(F, A)) $=(F, A)$ $\tilde{\cap}$ (soft preint$(F, A)$$\tilde{\cup}$ Scl(Sint((F, A)))= Soft pre int((F, A))$\tilde{\cup}$ ((F, A)$\tilde{\cap}$Scl(Sint(F, A)). Since (F, A) is soft pre-closed it follows that (F, A)=Soft pre int((F, A))$\tilde{\cup}$ Scl(Sint((F, A)))= Soft precl(Soft pre int((F, A))). This proves (iii) $\Rightarrow$ (iv). To prove (iv) $\Rightarrow$ (i). Suppose (iii) holds. Since (F, A) is soft semi-closed that is Sint(Scl((F, A))) $\tilde{\subseteq}$ (F, A). Using Proposition 1.14, (F, A)=soft precl(soft preint((F, A))) =Soft preint((F, A)) $\tilde{\cup}$Scl(Sint((F, A)))=((F, A)$\tilde{\cap}$Sint(Scl((F, A)))) $\tilde{\cup}$ Scl(Sint((F, A))) = Sint(Scl((F, A))) $\tilde{\cup}$Scl(Sint((F, A))). This completes the proof. □

Lemma 2.6. *(i). If (F, A) is Soft* $b^{\#}$*-open and co-dense then (F, A)is soft regular open.*

(ii). If (F, A) is Soft $b^{\#}$*-open and nowhere dense then(F, A) is soft regular closed*

Proof. Suppose (F, A) is Soft b$^{\#}$-open and co-dense. Then(F, A)=Scl(Sint(F, A)) $\tilde{\cup}$Sint(Scl((F, A)))=$\tilde{\phi}$ $\tilde{\cup}$ Sint(Scl((F, A)))=S int(Scl((F, A))) that implies (F, A) is regular open. This proves (i). Now suppose (F, A) is Soft b$^{\#}$-open and nowhere dense. Then (F, A)=Scl(Sint ((F, A))) $\tilde{\cup}$ Sint(Scl((F, A)))=Scl(Sint ((F, A))) $\tilde{\cup}$ $\tilde{\phi}$=Scl(Sint ((F, A))) that implies (F, A) is soft regular closed. This proves (ii). □

Theorem 2.7. *Let* (F, A) *and (F,B) be soft subsets of* $(X,\tilde{\tau}, E)$ *Then* *(i). Soft $b^{\#}$-int(X)=X and Soft $b^{\#}$-int($\widetilde{\phi}$)=$\widetilde{\phi}$*

(ii). Soft $b^{\#}$-int(F, A) $\widetilde{\subset}$ (F, A).

(iii). If (F,B) is any Soft $b^{\#}$-open set contained in (F, A), then (F,B) $\widetilde{\subset}$ Soft $b^{\#}$-int(F, A).

(iv). If (F, A) $\widetilde{\subset}$ (F,B), then Soft $b^{\#}$-int(F, A) $\widetilde{\subset}$ Soft $b^{\#}$-int((F,B)).

(v). Soft $b^{\#}$-int(Soft $b^{\#}$-int(F, A))=Soft $b^{\#}$-int(F, A).

Proof. (i). Since X and $\widetilde{\phi}$ are Soft $b^{\#}$open sets, Soft $b^{\#}$-int(X)=$\widetilde{\cup}$ {((F, H)):(F, H)) is a Soft $b^{\#}$-open,((F, H))$\widetilde{\subset}$X} =X$\widetilde{\cup}$ {(F, A) is a $b^{\#}$open sets }=X. (ie) Soft $b^{\#}$ int(X)=X.Since $\widetilde{\phi}$ is the only Soft $b^{\#}$ open set contained in $\widetilde{\phi}$, Soft $b^{\#}$-int$\widetilde{\in}\phi$)=$\widetilde{\phi}$

(ii). Let x is a Soft $b^{\#}$-int(F, A) soft interior point of (F, A). Let x$\widetilde{\in}$Soft $b^{\#}$-int(F, A) $\Rightarrow$ x is a soft interior point of (F, A). $\Rightarrow$$(F, A)$ is a soft nbhd of x. $\Rightarrow$x $\widetilde{\in}$ (F, A). Thus, x$\widetilde{\in}$ Soft $b^{\#}$-int(F, A) $\Rightarrow$x$\widetilde{\in}$$(F, A)$. Hence Soft $b^{\#}$-int$(F, A)$$\widetilde{\in}$$(F, A)$.

(iii). Let (F,B) be any Soft $b^{\#}$-open sets such that (F,B) $\widetilde{\subset}$ (F, A).Let Let x$\widetilde{\in}$(F,B).Since (F,B) is a Soft $b^{\#}$ -open set contained in (F, A). x is a Soft $b^{\#}$-interior point of (F, A). (ie) (F,B) is a Soft $b^{\#}$-int(F, A).Hence (F,B)$\widetilde{\subset}$ Soft $b^{\#}$-int(F, A).

(iv). Let (F, A) and (F,B) be subsets of (X, $\widetilde{\tau}$) such that $(F, A)$$\widetilde{\subset}$(F,B). Let x$\widetilde{\in}$ Soft $b^{\#}$-int(F, A).Then x is a Soft $b^{\#}$-interior point of (F, A) and so (F, A) is a Soft $b^{\#}$-nbhd of x.Since (F,B)$\widetilde{\supset}$ (F, A),(F,B) is also Soft $b^{\#}$-nbhd of x. $\Rightarrow$x$\widetilde{\in}$ Soft $b^{\#}$-int((F,B)). Thus we have shown that x$\widetilde{\in}$ Soft $b^{\#}$-int(F, A) $\Rightarrow$x$\widetilde{\in}$ Soft $b^{\#}$-int((F,B)).

(v). Let (F, A) be any subset of X. By definition of Soft $b^{\#}$-interior, Soft $b^{\#}$-int(F, A) is Soft $b^{\#}$ open and hence Soft $b^{\#}$-int(Soft $b^{\#}$-int(F, A))=Soft $b^{\#}$-int(F, A). □

Theorem 2.8. *If (F, A) is a soft subset of (X,$\widetilde{\tau}$, E), then Sint(F, A)$\widetilde{\subset}$ Soft $b^{\#}$-int (F, A).*

Proof. Let (F, A) be a subset of X. Let x$\widetilde{\in}$ Sint (F, A) $\Rightarrow$x$\widetilde{\cup}${ (F, H) : (F, H) is soft open, (F, H)$\widetilde{\subset}$ (F, A) }.
$\Rightarrow$there exists an soft open set (F, H) such that x$\widetilde{\in}$ (F, H) $\widetilde{\subset}$(F, A) .
$\Rightarrow$there exist a Soft $b^{\#}$-open set (F, H) such that x∈ (F, H) $\widetilde{\subset}$(F, A) , as every soft open set is a Soft $b^{\#}$-open set in X .
$\Rightarrow$x $\widetilde{\in}\widetilde{\cup}${ (F, H) : (F, H) is Soft $b^{\#}$-open, (F, H)$\widetilde{\subset}$ (F, A) }.
$\Rightarrow$ x$\widetilde{\in}$ Soft $b^{\#}$-int (F, A).
Thus x$\widetilde{\in}$ Sint (F, A) $\Rightarrow$x$\widetilde{\in}$ Soft $b^{\#}$- int (F, A). Hence Sint (F, A)$\widetilde{\subset}$ Soft $b^{\#}$- int (F, A). □

3. Soft $b^{\#}$-closed Sets

Definition 3.1. *A soft sub set (F, A) of a space (X,$\widetilde{\tau}$, E) is called Soft $b^{\#}$-closed if $X\backslash(F, A)$ is Soft $b^{\#}$-open. That is (F, A) is Soft $b^{\#}$-closed if and only if Soft $b^{\#}$-closed if Scl(S int((F, A))) $\widetilde{\cap}$ Sint(Scl((F, A)))=(F, A)*

Definition 3.2 ([2])**.** *A Soft subset (F, A) of a space (X,$\widetilde{\tau}$, E) is called: The intersection of all Soft $b^{\#}$-closed sets containing a set (F, A) in a space X is Called the Soft $b^{\#}$-closure of (F, A) and is denoted by Soft $b^{\#}$-Cl((F, A)).*

Remark 3.3. *Every Soft $b^{\#}$-closed set is Soft b-closed set but not conversely.*

Example 3.4. *Let (X,$\widetilde{\tau}$, E) be a soft topological space and soft topology $\widetilde{\tau}^c$={F_A, F_ϕ,F_{A_2}, F_{A_3}, F_{A_6},$F_{A_{11}}$,$F_{A_{12}}$}. F_{A_3} is Soft b-closed set but not Soft $b^{\#}$- closed set*

Theorem 3.5. *For a subset (F, A) of a space $(X,\widetilde{\tau}, E)$ the following are equivalent.*

(i). (F, A) is Soft $b^{\#}$-closed.

(ii). (F, A) is Soft b-closed.

(iii). (F, A) is Soft pre-open, Soft semi-open and (F, A)=soft precl(F, A) $\widetilde{\cap}$ Soft semicl((F, A)).

(iv). (F, A) is Soft semi-open and (F, A)=Soft preint(Soft precl(((F, A)))).

Proof. Follows from Theorem 2.5. and Definition 3.1. □

Theorem 3.6. *If (F, A) ,(F, B)are subsets $(X,\widetilde{\tau}, E)$. Then*

(i). Soft $b^{\#}$-cl(X)=X and Soft $b^{\#}$-cl($\widetilde{\phi}$)=$\widetilde{\phi}$

(ii). $(F, A)\widetilde{\subseteq}$ Soft $b^{\#}$ -cl(F, A).

(iii). If (F, B) is any Soft $b^{\#}$-closed set containing A, then Soft $b^{\#}$ -cl(F, A) $\widetilde{\subseteq}$ (F, B).

(iv). If $(F, A)\widetilde{\subseteq}$ (F, B)then Soft $b^{\#}$-cl(F, A) $\widetilde{\subseteq}$ Soft $b^{\#}$-cl(F, B).

(v). Soft $b^{\#}$-cl(Soft $b^{\#}$-cl(F, A)) = Soft $b^{\#}$-cl(F, A).

Proof. (i). By the definition of Soft $b^{\#}$-closure, x is the only Soft $b^{\#}$-closed set containing X. Therefore Soft $b^{\#}$cl(X)=Intersection of all the Soft $b^{\#}$-closed sets containing X=$\widetilde{\cap}${X} = X.That is Soft $b^{\#}$-cl(X)=X.

By the definition of Soft $b^{\#}$-closure, Soft $b^{\#}$-cl($\widetilde{\phi}$)=Intersection of all the Soft $b^{\#}$-closed sets containing $\widetilde{\phi}$ =$\widetilde{\phi}$ $\widetilde{\cap}$ Soft $b^{\#}$-closed sets containing $\widetilde{\phi}$ = $\widetilde{\phi}$.That is Soft $b^{\#}$-cl($\widetilde{\phi}$)=$\widetilde{\phi}$.

(ii). By the definition of Soft $b^{\#}$-closure of (F, A), it is obvious that (F, A) $\widetilde{\subseteq}$ Soft $b^{\#}$-cl(F, A).

(iii). Let (F, B) be any Soft $b^{\#}$-closed set containing (F, A). Since Soft $b^{\#}$-cl(F, A) is the intersection of all Soft $b^{\#}$-closed sets containing (F, A), Soft $b^{\#}$-cl(F, A) is contained in every Soft $b^{\#}$-closed set containing (F, A).Hence in particular Soft $b^{\#}$-cl(F, A)$\widetilde{\subseteq}$(F, B)

(iv). Let (F, A) and (F, B) be Soft subsets of X such that $(F, A)\subset$ (F, B). By the definition Soft $b^{\#}$-cl((F, B))=$\widetilde{\cap}${ G: (F, B)$\widetilde{\subseteq}$F$\in$ Soft $b^{\#}$C(X)}. If (F, B)$\widetilde{\subseteq}$ G$\in$ Soft $b^{\#}$ C(X),then Soft $b^{\#}$-cl((F, B))$\widetilde{\subseteq}$ G. Since (F, A) $\widetilde{\subseteq}$(F, B), $(F, A)\widetilde{\subseteq}$ ((F, B)$\widetilde{\subseteq}$F$\in$ Soft $b^{\#}$C(X), we have Soft $b^{\#}$-cl(F, A) $\widetilde{\subseteq}$G. Therefore Soft $b^{\#}$-cl(F, A)$\widetilde{\subseteq}\widetilde{\cap}${G:(F, B)$\widetilde{\subseteq}$ G$\in$ Soft $b^{\#}$ C(X)}=Soft $b^{\#}$-cl(B). (i.e) Soft $b^{\#}$-cl(F, A) $\widetilde{\subseteq}$ Soft $b^{\#}$-cl(F, A).

(v). Let (F, B) be a Soft $b^{\#}$-closed set containing (F, A). Then by definition Soft $b^{\#}$-cl(F, A)$\subset$(F, B).Since(F, B) is Soft $b^{\#}$-closed set and contains Soft $b^{\#}$cl(F, A) which contained in every Soft $b^{\#}$-closed set containing (F, A). It follows that Soft $b^{\#}$Cl(Soft $b^{\#}$cl(F, A))$\widetilde{\subseteq}$ Soft $b^{\#}$cl(F, A). Therefore Soft $b^{\#}$-Cl(Soft $b^{\#}$-cl(F, A)) = Soft $b^{\#}$-cl(F, A). □

4. Soft $b^{\#}$-continuous and Soft $b^{\#}$-irresolute

Definition 4.1. *A function $f : X \Rightarrow Y$ is said to be*

(1). soft $b^{\#}$-continuous if for every soft closed set in Y, its inverse image is soft $b^{\#}$- closed in X.

(2). soft $b^{\#}$-irresolute if for every soft $b^{\#}$-closed set in Y, its inverse image is soft $b^{\#}$- closed in X.

Example 4.2. *$U = \{P_1, P_2, P_3\}$, $E = \{x_1, x_2, x_3\}$, $andA = \{x_1, x_2\}$ Then, we can view the soft set F_A as consisting of the following collection of approximations Let U=$\{p_1, p_2, p_3\}$, E=$\{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \tilde{\subseteq} E$, and $F_A = \{(x_1, \{p_1, p_2\}), (x_2, \{p_2, p_3\})\}$. Then, $F_{A_1} = \{(x_1, \{p_1\})\}$, F_{A_3}=$\{(x_1, \{p_1, p_2\})\}$,$F_{A_7} = \{(x_1,\{p_1\}), (x_2, \{p_2\})\}$, $F_{A_9} = \{(x_1, \{p_1\}), (x_2, \{p_2, p_3\})\}$, $F_{A_{13}} = \{(x_1, \{p_1, p_2\}), (x_2, p_2\})\}$,$F_{A_{14}}$=$\{(x_1,\{p_1, p_2\}), (x_2, \{p_3\})\}$, $F_{A_{15}} = F_A$, $F_{A_{16}} = F_\phi$ Then the soft topology $\tilde{\tau}$=$\{F_A$, F_ϕ,F_{A_1}, F_{A_3}, F_{A_7},F_{A_9},$F_{A_{13}}\}$*

$V = \{a, b, c\}\}$, $E = \{e_1, e_2, e_3\}$, $andA = \{e_1, e_2\}$ Then, we can view the soft set F_A as consisting of the following collection of approximations Let U=$\{a, b, c\}$, $E = \{e_1, e_2, e_3\}$, A=$\{e_1, e_2\} \tilde{\subseteq} E$, and $F_A = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$. Then, F_{A_5}=$\{(e_2, \{c\})\}$, $F_{A_7} = \{(e_1,\{a\}), (e_2, \{b\})$, F_{A_9}=$\{(e_1, \{a\}), (e_2, \{b, c\})\}$,$F_{A_{16}} = F_\phi\}$ Then the soft topology $\tilde{\sigma}$=$\{F_A$, F_ϕ,F_{A_5}, F_{A_7}, $F_{A_9}\}$ Let (X, $\tilde{\tau}$) and (Y, $\tilde{\sigma}$) be two Soft topological spaces and f be a Soft map from X to Y. Then f is a Soft b-continuous map but not Soft $b^{\#}$-continuous map

Theorem 4.3. *Let (X, $\tilde{\tau}$) and (Y, $\tilde{\sigma}$) be two Soft topological spaces and f be a map from X to Y. Then the following are equivalent.*

(i). f is a Soft $b^{\#}$-continuous map.

(ii). The inverse image of a soft closed set in Y is a Soft $b^{\#}$-closed set in X

(iii). Soft $b^{\#}$-cl(f^{-1}((F, B)))$\tilde{\subseteq}$ f^{-1}(cl((F, B))) for every subset (F, B) of Y.

(iv). f(Soft $b^{\#}$-cl((F, A))) $\tilde{\subseteq}$ cl(f((F, A)) for every sub set (F, A) of X.

(v). f^{-1}(int((F, B))) $\tilde{\subseteq}$Soft b $^{\#}$-int(f^{-1}((F, B))) for every sub set (F, B) of Y.

Proof. (i) ⇒ (ii) Let (F, A) be a Soft closed set in Y, then Y\(F, A) is Soft open Y. Then f^{-1}(Y\(F, A))is Soft $b^{\#}$-open in X. It follows that f^{-1}((F, A)) is Soft $b^{\#}$-closed in X. To prove (ii) ⇒ (iii). Let (F, B)be any sub set of Y. Since Scl((F, B)) is Soft closed in Y, then f^{-1}(cl((F, B))) is Soft $b^{\#}$-closed in X. Therefore Soft $b^{\#}$-cl(f^{-1}((F, B))) $\tilde{\subseteq}$ Soft $b^{\#}$-cl(f^{-1}(cl((F, B)))= f^{-1}(cl((F, B))). To prove (iii) ⇒ (iv). Let (F, A) be any sub set of X. By (iii), we have f^{-1}(cl(f((F, A)))) $\tilde{\supseteq}$ Soft $b^{\#}$-cl(f^{-1}(f((F, A)))) $\tilde{\supseteq}$ Soft $b^{\#}$-cl((F, A)). Therefore f(Soft $b^{\#}$-cl((F, A))) $\tilde{\subseteq}$Scl(f((F, A)). To prove (iv) ⇒ (v). Let (F, B) be any sub set of Y. By (iv), f(Soft $b^{\#}$-cl(X\f^{-1}((F, B)))) $\tilde{\subseteq}$ cl(f(X\ f^{-1}((F, B))) that implies f(X\ Soft $b^{\#}$-int(f^{-1}((F, B)))) $\tilde{\subseteq}$ cl(Y\(F, B))=Y\int((F, B)). Therefore we have X\Soft $b^{\#}$-int(f^{-1}((F, B))) $\tilde{\subseteq}$ f^{-1}(Y\S int((F, B))) that implies f^{-1}(Sint((F, B))) $\tilde{\subseteq}$ Soft $b^{\#}$-int(f-1((F, B))). To prove (v) ⇒ (i). Let (F, A) be any Soft open sub set of Y. Then f^{-1}(Sint((F, B))) $\tilde{\subseteq}$ Soft $b^{\#}$-int(f^{-1}((F, B))). Then f^{-1}((F, B)) $\tilde{\subseteq}$ Soft $b^{\#}$-int(f^{-1}((F, B))). But Soft $b^{\#}$-int(f^{-1}((F, B))) $\tilde{\subseteq}$ f^{-1}((F, B)) that implies f^{-1}((F, B)) = Soft $b^{\#}$-int(f^{-1}((F, B))). This completes the proof. □

Theorem 4.4. *Let (X, $\tilde{\tau}$) and (Y, $\tilde{\sigma}$) be two Soft topological spaces and f be a map from X to Y. Then f is soft $b^{\#}$-irresolute if and only if the inverse image of a soft $b^{\#}$-closed set in Y is soft $b^{\#}$-closed in X.*

Theorem 4.5. *Every Soft $b^{\#}$-irresolute mapping is Soft $b^{\#}$-continuous mapping.*

Proof. Let f : X →Y is Soft $b^{\#}$-irresolute mapping. Let (F, K) be a soft closed set in Y, then (F, K) is Soft $b^{\#}$-closed set in Y. Since f is Soft $b^{\#}$-irresolute mapping, f^{-1}(F, K)) is a Soft $b^{\#}$-closed set in X. Hence, f is Soft $b^{\#}$-continuous mapping. □

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